482 (disjoint composition) Concurrent composition $P||Q$ requires that P and Q have no variables in common, although each can make use of the initial values of the other's variables by making a private copy. An alternative, let's say disjoint composition, is to allow both P and Q to use all the variables with no restrictions, and then to choose disjoint sets of variables v and w and define

$$P\ |v|w|\ Q \;=\; (P.\ v'=v) \land (Q.\ w'=w)$$

(a) Describe how $P\ |v|w|\ Q$ can be executed.

(b) Prove that if P and Q are implementable specifications, then $P\ |v|w|\ Q$ is implementable.

After trying the question, scroll down to the solution.

(a) Describe how $P\ |v|w|\ Q$ can be executed.

§ Make a copy of all variables. Execute P using the original set of variables and concurrently execute Q using the copies. Then copy back from the copy w to the original w. Then throw away the copies. There may be variables x other than v and w; if so, their final values are arbitrary, and this implementation makes them be what P says they should be. Formally, using application $\langle v\cdot\ P\rangle\ x$ as the formal notation for (substitute x for v in P),

$$\textbf{var}\ cv{:=}\ v\cdot\ \textbf{var}\ cw{:=}\ w\cdot\ \textbf{var}\ cx{:=}\ x\cdot$$
$$(P \parallel \langle v, w, x, v', w', x'\cdot\ Q\rangle\ cv\ cw\ cx\ cv'\ cw'\ cx').\ \ w{:=}\ cw$$

(b) Prove that if P and Q are implementable specifications, then $P\ |v|w|\ Q$ is implementable.

§ First, a lemma.

	$P.\ \ v'{=}v$	expand sequential composition
$=$	$\exists v'', w'', x''\cdot\ \langle v', w', x'\cdot\ P\rangle\ v''\ w''\ x''\ \ \wedge\ \ v'{=}v''$	one-point v''
$=$	$\exists w'', x''\cdot\ \langle v', w', x'\cdot\ P\rangle\ v'\ w''\ x''$	rename w'', x'' to w', x'
$=$	$\exists w', x'\cdot\ \langle v', w', x'\cdot\ P\rangle\ v'\ w'\ x'$	simplify
$=$	$\exists w', x'\cdot\ P$	

So $\ P\ |v|w|\ Q\ \ =\ \ (P.\ \ v'{=}v) \wedge (Q.\ \ w'{=}w)\ \ =\ \ (\exists w', x'\cdot\ P) \wedge (\exists v', x'\cdot\ Q)$

Now the main proof.

	($P\ |v|w|\ Q$ is implementable)	definition of implementable
$=$	$\forall v, w, x\cdot\ \exists v', w', x'\cdot\ P\ |v|w|\ Q$	use previous result
$=$	$\forall v, w, x\cdot\ \exists v', w', x'\cdot\ (\exists w', x'\cdot\ P) \wedge (\exists v', x'\cdot\ Q)$	identity for x'
$=$	$\forall v, w, x\cdot\ \exists v', w'\cdot\ (\exists w', x'\cdot\ P) \wedge (\exists v', x'\cdot\ Q)$	
$=$	$\forall v, w, x\cdot\ \exists v'\cdot\ \exists w'\cdot\ (\exists w', x'\cdot\ P) \wedge (\exists v', x'\cdot\ Q)$	distribution (factoring)
$=$	$\forall v, w, x\cdot\ \exists v'\cdot\ (\exists w', x'\cdot\ P) \wedge (\exists w'\cdot\ \exists v', x'\cdot\ Q)$	distribution (factoring)
$=$	$\forall v, w, x\cdot\ (\exists v'\cdot\ \exists w', x'\cdot\ P) \wedge (\exists w'\cdot\ \exists v', x'\cdot\ Q)$	
$=$	$\forall v, w, x\cdot\ (\exists v', w', x'\cdot\ P) \wedge (\exists v', w', x'\cdot\ Q)$	splitting law
$=$	$(\forall v, w, x\cdot\ \exists v', w', x'\cdot\ P) \wedge (\forall v, w, x\cdot\ \exists v', w', x'\cdot\ Q)$	definition of implementable
$=$	(P is implementable) $\wedge$ (Q is implementable)	