

487 We want to find the smallest number in $0, \dots, n$ with property p . Linear search solves the problem. But evaluating p is expensive; let us say it takes time 1, and all else is free. The fastest solution is to evaluate p on all n numbers concurrently, and then find the smallest number that has the property. Write a program without concurrency for which the sequential to concurrent transformation gives the desired computation.

After trying the question, scroll down to the solution.

§ We introduce array $A: [n*bin]$. We define the desired result R , invariant Ii , and helper specification P as follows.

$$R = \neg(\exists j: 0..h'. p.j) \wedge (p.h' \vee h'=n)$$

$$Ii = \forall j: 0..i. A.j = p.j$$

$$P = In \wedge \neg(\exists j: 0..h. p.j) \Rightarrow R$$

Now the program is

$$R \Leftarrow I0 \Rightarrow I'n. h:=0. P$$

$$I0 \Rightarrow I'n \Leftarrow \mathbf{for} \ i:=0;..n \ \mathbf{do} \ Ii \Rightarrow I'(i+1) \ \mathbf{od}$$

$$Ii \Rightarrow I'(i+1) \Leftarrow A.i:=p.i$$

$$P \Leftarrow \mathbf{if} \ h=n \ \mathbf{then} \ \mathbf{ok} \ \mathbf{else} \ \mathbf{if} \ A.h \ \mathbf{then} \ \mathbf{ok} \ \mathbf{else} \ h:=h+1. \ \mathbf{P} \ \mathbf{fi} \ \mathbf{fi}$$

The n iterations of the **for**-loop can be executed concurrently.

We can express the result of the sequential to concurrent transformation at source as follows.

$$R \Leftarrow I0 \Rightarrow I'n. h:=0. P$$

$$I0 \Rightarrow I'n \Leftarrow i:=0. Ii \Rightarrow I'n$$

$$Ii \Rightarrow I'n \Leftarrow \mathbf{if} \ i=n \ \mathbf{then} \ \mathbf{ok} \ \mathbf{else} \ A.i:=p.i \parallel (i:=i+1. Ii \Rightarrow I'n) \ \mathbf{fi}$$

$$P \Leftarrow \mathbf{if} \ h=n \ \mathbf{then} \ \mathbf{ok} \ \mathbf{else} \ \mathbf{if} \ A.h \ \mathbf{then} \ \mathbf{ok} \ \mathbf{else} \ h:=h+1. \ \mathbf{P} \ \mathbf{fi} \ \mathbf{fi}$$

To understand the execution, it might help to unroll the recursion a little: in the refinement of $Ii \Rightarrow I'n$, replace the recursive call $Ii \Rightarrow I'n$ by what it calls:

$$\mathbf{if} \ i=n \ \mathbf{then} \ \mathbf{ok} \ \mathbf{else} \ A.i:=p.i \parallel (i:=i+1. Ii \Rightarrow I'n) \ \mathbf{fi}$$

And maybe do the same once more.