

489 Exercise 161 asks for a program to compute cumulative sums (running total). Write a program that can be transformed from sequential to concurrent execution with $\log n$ time where n is the length of the list.

After trying the question, scroll down to the solution.

§ Let L be a list variable. The result we want is $R 0 (\#L)$ where
$$R i k = \#L' = \#L \wedge \forall j: i, ..k \cdot L' j = \Sigma L [i; ..j+1]$$

We refine it as follows.

$$R i k \Leftarrow \mathbf{if} \ k-i \leq 1 \ \mathbf{then} \ ok$$
$$\qquad \mathbf{else} \ \mathbf{new} \ m := \mathit{div} (k+i) \ 2.$$
$$\qquad \qquad (R i m \parallel R m k).$$
$$\qquad \qquad \mathbf{for} \ j := m; ..k \ \mathbf{do} \ L j := L j + L (m-1) \ \mathbf{od} \ \mathbf{fi}$$

The final **for**-loop can be concurrent, so that it takes no time. The computation then takes $\log (\#L)$ time.