

512 (*T*-strings) Let us call a string S : *("a", "b", "c") a *T*-string if no two adjacent nonempty segments are identical:

$$\neg \exists i, j, k. 0 \leq i < j < k \leq \text{len}(S) \wedge S_{i..j} = S_{j..k}$$

Write a program to output all *T*-strings in alphabetical order. (The mathematician Axel Thue proved that there are infinitely many *T*-strings.)

After trying the question, scroll down to the solution.

§ Define $R =$ (print all T -strings in alphabetical order) .
 Define $Z =$ (print all T -strings from S on in alphabetical order) .
 Define $T =$ (S is a T -string) $= \neg \exists i, j, k. 0 \leq i < j < k \leq \#S \wedge S_{i..j} = S_{j..k}$.
 Define $U =$ (S has no adjacent nonempty identical segments of length $< l$)
 $= \neg \exists i, j, k. 0 \leq i < j < k \leq \#S \wedge j - i < l \wedge S_{i..j} = S_{j..k}$.
 $R \Leftarrow S := ""$. $T \Rightarrow Z$
 $T \Rightarrow Z \Leftarrow !S. S := S; "a"$. Z
 $Z \Leftarrow l := 1$. $U \Rightarrow Z$
 $U \Rightarrow Z \Leftarrow$

if $\#S \geq 2 \times l$
then **if** $S_{\leftrightarrow S - 2 \times l; \dots \leftrightarrow S - l} = S_{\leftrightarrow S - l; \dots \leftrightarrow S}$
then $S :=$ (the alphabetically next text that is not longer) . Z
else $l := l + 1$. $U \Rightarrow Z$ **fi**
else $T \Rightarrow Z$ **fi**

$S :=$ (the alphabetically next text that is not longer) \Leftarrow

if $S_{\leftrightarrow S - 1} = "a"$ **then** $S := S_{0; \dots \leftrightarrow S - 1}; "b"$
else if $S_{\leftrightarrow S - 1} = "b"$ **then** $S := S_{0; \dots \leftrightarrow S - 1}; "c"$

else $S := S_{0; \dots \leftrightarrow S - 1}$. $S :=$ (the alphabetically next text that is not longer) **fi fi**

The one insight is the fact that a non- T -string cannot be made into a T -string by extending it, hence the assignment $S :=$ (the alphabetically next text that is not longer) . We are assured that there is one by Thue.