52 The union axiom says

$$x: A, B \;=\; x: A \;\lor\; x: B$$

There are 16 two-operand binary operators that could sit where $\lor$ sits in this axiom if we just replace bunch union (,) by a corresponding bunch operator. Which of the 16 two-operand binary operators correspond to useful bunch operators?

After trying the question, scroll down to the solution.

§ What is "useful"? It's not a well-defined question. I suppose any non-degenerate operator is useful (which means it uses both its operands; on the value table below, if the comment to the right mentions both A and B then the operator is not degenerate). One could argue that the degenerate operators are useful for throwing away information, or that they aren't useful because there is a perfectly good zero-operand or one-operand operator that could be used in their place.

Let $\backslash A$ be the complement of bunch A (those elements that are not in A , $\backslash$ has precedence 2), defined formally by

$$x: \backslash A = \neg\, x: A$$

	⊤⊤	⊤⊥	⊥⊤	⊥⊥	
	⊤	⊤	⊤	⊤	$\backslash null$ (universal bunch)
$\vee$	⊤	⊤	⊤	⊥	A , B
$\Leftarrow$	⊤	⊤	⊥	⊤	$A , \backslash B$
	⊤	⊤	⊥	⊥	A
$\Rightarrow$	⊤	⊥	⊤	⊤	$\backslash A , B$
	⊤	⊥	⊤	⊥	B
$=$	⊤	⊥	⊥	⊤	$A‘B$, $\backslash A‘\backslash B$
$\wedge$	⊤	⊥	⊥	⊥	$A‘B$
	⊥	⊤	⊤	⊤	$\backslash A , \backslash B$
$\neq$	⊥	⊤	⊤	⊥	$A‘\backslash B$, $\backslash A‘B$
	⊥	⊤	⊥	⊤	$\backslash B$
	⊥	⊤	⊥	⊥	$A‘\backslash B$
	⊥	⊥	⊤	⊤	$\backslash A$
	⊥	⊥	⊤	⊥	$\backslash A‘B$
	⊥	⊥	⊥	⊤	$\backslash A‘\backslash B$
	⊥	⊥	⊥	⊥	$null$