

58 (prefix order) Give axioms to define the prefix partial order on strings. String  $S$  comes before string  $T$  in this order if and only if  $S$  is an initial segment of  $T$ .

After trying the question, scroll down to the solution.

§ Use  $\preceq$  for the prefix partial order. It can be defined as

$$nil \preceq S$$

$$S \preceq nil = S = nil$$

$$i;S \preceq j;T = i=j \wedge S \preceq T$$

where  $S$  and  $T$  are strings, and  $i$  and  $j$  are items. Or it can be defined as

$$S \preceq T = \Leftrightarrow S \leq \Leftrightarrow T \wedge S = T_{0;\dots \Leftrightarrow S}$$

After we meet quantification it can be defined as

$$S \preceq T = \exists U: *X. S;U = T$$

where  $X$  is the alphabet of symbols.