64 Simplify (no proof)

(a) 0→1 | 1→2 | 2→3 | 3→4 | 4→5 | [0;..5]

(b) [0][0][0][0]

(c) ((3;2)→[10;..15] | 3→[5;..10] | [0;..5]) 3

(d) ([0;..5] [3; 4]) 1

(e) (2;2)→“j” | [[“abc”]; [“de”]; [“fghi”]]

(f) #[*nat*]

(g) #[*3]

(h) [3; 4]: [3*4**int*]

(i) [3; 4]: [3; *int*]

(j) [3, 4; 5]: [2**int*]

(k) [(3, 4); 5]: [2**int*]

(l) [3; (4, 5); 6; (7, 8, 9)] ‘ [3; 4; (5, 6); (7, 8)]

After trying the question, scroll down to the solution.

(a) $0\to1 \mid 1\to2 \mid 2\to3 \mid 3\to4 \mid 4\to5 \mid [0;..5]$

§ $0\to1 \mid 1\to2 \mid 2\to3 \mid 3\to4 \mid 4\to5 \mid [0; 1; 2; 3; 4]$

$= 0\to1 \mid 1\to2 \mid 2\to3 \mid 3\to4 \mid [0; 1; 2; 3; 5]$

$= 0\to1 \mid 1\to2 \mid 2\to3 \mid [0; 1; 2; 4; 5]$

$= 0\to1 \mid 1\to2 \mid [0; 1; 3; 4; 5]$

$= 0\to1 \mid [0; 2; 3; 4; 5]$

$= [1; 2; 3; 4; 5]$

$= [1;..6]$

(b) $[0][0][0][0]$

§ First I prove $[0][0] = [0]$.

$[0][0]$ Indexing with a list of indexes gives a list of items.

$= [[0]0]$ $[0]0 = 0$

$= [0]$

Now the main proof.

$[0][0][0][0]$ Adjacency associates from left to right.

$= ([0][0])[0][0]$ $[0][0] = [0]$

$= [0][0][0]$ Adjacency associates from left to right.

$= ([0][0])[0]$ $[0][0] = [0]$

$= [0][0]$ $[0][0] = [0]$

$= [0]$

(c) $((3;2)\to[10;..15] \mid 3\to[5;..10] \mid [0;..5])\ 3$

§ $((3;2)\to[10;..15] \mid 3\to[5;..10] \mid [0;..5])\ 3$

$= ((3;2)\to[10;..15] \mid 3\to[5;..10] \mid [0; 1; 2; 3; 4])\ 3$

$= ((3;2)\to[10;..15] \mid [0; 1; 2; [5;..10]; 4])\ 3$

$= ((3;2)\to[10;..15] \mid [0; 1; 2; [5; 6; 7; 8; 9]; 4])\ 3$

$= [0; 1; 2; [5; 6; [10;..15]; 8; 9]; 4]\ 3$

$= [5; 6; [10;..15]; 8; 9]$

(d) $([0;..5]\ [3; 4])\ 1$

§ One way:

$([0;..5]\ [3; 4])\ 1$

$= [[0;..5]\ 3;\ [0;..5]\ 4]\ 1$

$= [0;..5]\ 4$

$= 4$

Another way:

$([0;..5]\ [3; 4])\ 1$

$= [0;..5]\ ([3; 4]\ 1)$

$= [0;..5]\ 4$

$= 4$

(e) $(2;2)\to$"j" $\mid$ [["abc"]; ["de"]; ["fghi"]]

§ Item 2 of [["abc"]; ["de"]; ["fghi"]] is ["fghi"] and its item 2 is "h" so replacing item 2;2 or [["abc"]; ["de"]; ["fghi"]] with "j" gives

[["abc"]; ["de"]; ["fgji"]]

(f) #[*nat*]

§ 1 because "A nonempty bunch of items is also an item." page 17

or, informally

#[*nat*]

= #[0, 1, 2, 3, ...]
= #([0], [1], [2], [3], ...)
= #[0], #[1], #[2], #[3], ...
= 1, 1, 1, 1, ...
= 1

This is the sort of "proof" that mathematicians accept, but it's not a formal proof because the three dots mean "guess what goes here". Anyway, the question did not ask for proof.

(g) #[*3]
§ #[*3]
= #[*nil*, 3, 3;3, 3;3;3, ...]
= #([*nil*], [3], [3;3], [3;3;3], ...)
= #[*nil*], #[3], #[3;3], #[3;3;3], ...
= 0, 1, 2, 3, ...
= *nat*

Again, an informal "proof", but the question did not ask for proof.

(h) [3; 4]: [3*4**int*]
§ 4**int* = *int*; *int*; *int*; *int*
3*4**int* = *int*; *int*; *int*; *int*; *int*; *int*; *int*; *int*; *int*; *int*; *int*; *int*
[3*4**int*] = [*int*; *int*; *int*; *int*; *int*; *int*; *int*; *int*; *int*; *int*; *int*; *int*]

which is all lists of 12 integers, and [3; 4] is not a list of 12 integers, so the answer is
⊥

(i) [3; 4]: [3; *int*]
§ [3; *int*] includes all lists of length 2 whose item 0 is 3 and whose item 1 is in *int* . The list [3; 4] is one of them, so the answer is
⊤

(j) [3, 4; 5]: [2**int*]
§ [2**int*] includes all lists of length 2 both of whose items are in *int* .
[3, 4; 5]
= [3, (4; 5)]
= [3], [4; 5]

and [3] is not a list of length 2 , so the answer is
⊥

(k) [(3, 4); 5]: [2**int*]
§ [(3, 4); 5] = [3;5, 4;5] = [3; 5], [4; 5]

and both these lists are of length 2 and both items of each are in *int* so the answer is
⊤

(l) [3; (4, 5); 6; (7, 8, 9)] ‘ [3; 4; (5, 6); (7, 8)]
§ [3; (4, 5); 6; (7, 8, 9)] ‘ [3; 4; (5, 6); (7, 8)] distribution
= ([3; 4; 6; 7], [3; 5; 6; 7], [3; 4; 6; 8], [3; 5; 6; 8], [3; 4; 6; 9], [3; 5; 6; 9])
‘ ([3; 4; 5; 7], [3; 4; 6; 7], [3; 4; 5; 8], [3; 4; 6; 8]) intersection
= [3; 4; 6; 7], [3; 4; 6; 8] factor (distribution law)
= [3; 4; 6; (7, 8)]