

65 Simplify, assuming  $i, L$ :  $\square L$

(a)  $i \rightarrow L i \mid L$

(b)  $(L i \rightarrow i \mid L) i$

(c)  $L [0;..i] ;; [x] ;; L [i+1;..#L]$

After trying the question, scroll down to the solution.

$$\begin{array}{l} \text{(a)} \quad i \rightarrow L i \mid L \\ \S \quad L \end{array}$$

$$\begin{array}{l} \text{(b)} \quad (L i \rightarrow i \mid L) i \\ \S \quad L i \end{array}$$

Proof:

$$\begin{array}{l} (L i \rightarrow i \mid L) i \\ = \mathbf{if} \ L i = i \ \mathbf{then} \ i \ \mathbf{else} \ L i \ \mathbf{fi} \\ = \mathbf{if} \ L i = i \ \mathbf{then} \ L i \ \mathbf{else} \ L i \ \mathbf{fi} \\ = L i \end{array}$$

use law  $(n \rightarrow i \mid L) m = \mathbf{if} \ n = m \ \mathbf{then} \ i \ \mathbf{else} \ L m \ \mathbf{fi}$   
context  
case idempotent

$$\begin{array}{l} \text{(c)} \quad L [0;..i] ;; [x] ;; L [i+1;..#L] \\ \S \quad i \rightarrow x \mid L \end{array}$$