

70 Some mathematicians like to use a notation like  $\exists! x: D \cdot P x$  to mean “there is a unique  $x$  in  $D$  such that  $P x$  holds”. Define  $\exists!$  formally.

After trying the question, scroll down to the solution.

§ The word “unique” means “there is exactly one value of  $x$  such that  $P x$  holds.

$(\exists x: D \cdot P x) \wedge \neg(\exists x, y: D \cdot x \neq y \wedge P x \wedge P y)$   
or  $\exists x: D \cdot P x \wedge \neg \exists y: D \cdot x \neq y \wedge P y$   
or  $(\exists x: D \cdot P x) \wedge (\forall x, y: D \cdot P x \wedge P y \Rightarrow x=y)$   
or  $\exists x: D \cdot P x \wedge \forall y: D \cdot P y \Rightarrow x=y$   
or  $\exists x: D \cdot \forall y: D \cdot P y = (x=y)$   
or  $\wp(\{x: D \cdot P x\}) = 1$