X5.1 A **value** expression has the form P **value** e and the axiom

$P.\ (P$ **value** $e)=e$

except that $(P$ **value** $e)$ is not subject to double-priming in sequential composition, nor to substitution when using the Substitution Law. An alternative syntax is

value x: $T\cdot P$

where x is a new variable, T is its type, and P is a specification. The value of this expression is the final value of local variable x.

(a) What is its axiom?

(b) Apply your axiom to

$x:= x + ($**value** y: $nat\cdot\ x:= x+1.\ \ y:= x+2)$

After trying the question, scroll down to the solution.

(a)§ The axiom is

new x: T· P. (**value** x: T· P)=x

or

∃x, x': T· P. (**value** x: T· P)=x

except that (**value** x: T· P) is not subject to double-priming in sequential composition, nor to substitution when using the Substitution Law.

(b) Apply your axiom to

x:= x + (**value** y: *nat*· x:= x+1. y:= x+2)

§ First, just the value expression.

∃y, y': *nat*· x:= x+1. y:= x+2. (**value** y: *nat*· x:= x+1. y:= x+2)=y

Substitution Law twice, but don't change (**value** y: *nat*· x:= x+1. y:= x+2)

= ∃y, y': *nat*· (**value** y: *nat*· x:= x+1. y:= x+2)=x+1+2

Now y and y' no longer appear outside (**value** y: *nat*· x:= x+1. y:= x+2)

= (**value** y: *nat*· x:= x+1. y:= x+2)=x+3

So

x:= x + (**value** y: *nat*· x:= x+1. y:= x+2)

= x:= x+x+3