

# Image Features

# Image Features

- Image features are useful descriptions of local or global image properties designed to accomplish a certain task
- You may want to choose different features for different tasks
- Depending on the problem we need to typically answer three questions:
  - **Where** to extract image features?
  - **What** to extract (what's the content of the feature)?
  - **How to match** them?

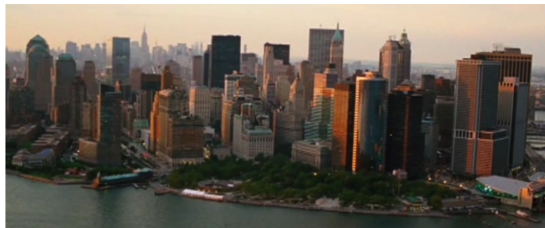
# Image Features

- Let's watch a video clip



# Image Features

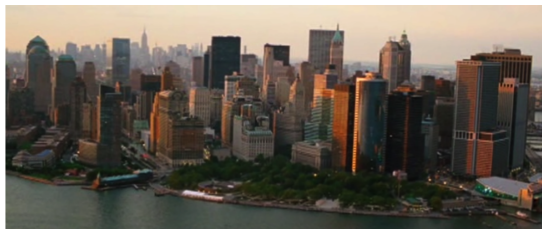
- Where is the movie taking place?





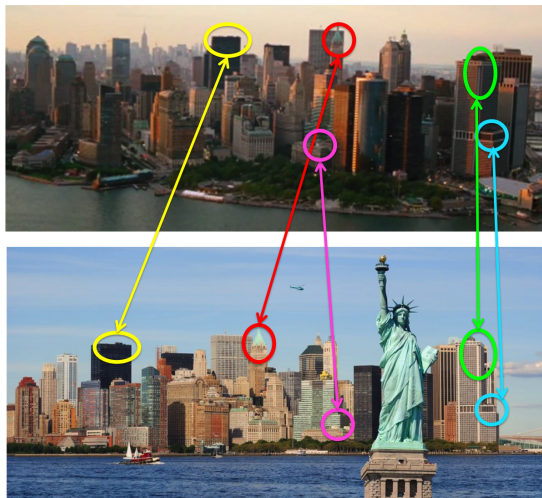
# Image Features

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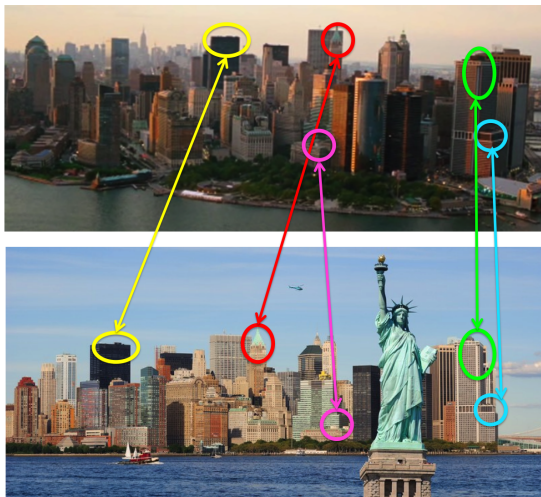


# Image Features

- Where is the movie taking place?

We matched in:

- Distinctive locations:  
**keypoints**
- Distinctive features:  
**descriptors**



# Image Features

- **Tracking:** Where to did the scene/actors move?



Where did it each point originate from the previous frame?

# Image Features

- **Tracking:** Where to did the scene/actors move?

We matched:

- Quite distinctive locations
- Quite distinctive features



Where did it each point originate from the previous frame?

# Image Features

- A shot in a movie is a clip with a coherent camera (no sudden viewpoint changes)

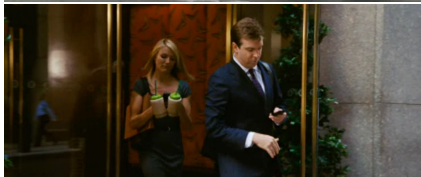


# Image Features

- A shot in a movie is a clip with a coherent camera (no sudden viewpoint changes)

We matched:

- **Globally** – one descriptor for full image
- Descriptor can be simple, e.g. **color**

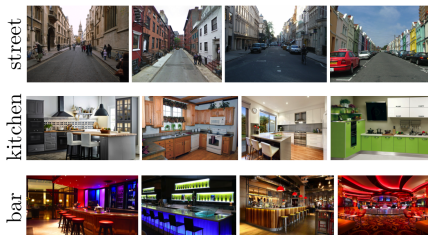


# Image Features

- How could we tell which type of scene it is?



What kind of scene is behind the actors?  
Kitchen? Bedroom? Street? Bar?





# Image Features

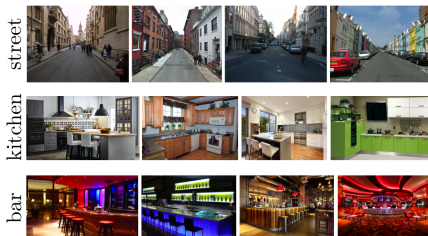
- How could we tell which type of scene it is?

We matched:

- **Globally** – one descriptor for full image (?)
- More complex descriptor: color, gradients...



What kind of scene is behind the actors?  
Kitchen? Bedroom? Street? Bar?



# Image Features

- How would we solve this?



Are these two cups of the same type?

# Image Features

- How would we solve this?

We matched:

- One descriptor for full **patch**
- Descriptor can be simple, e.g. **color**



Are these two cups of the same type?

# Image Features

- How would we solve this?



Where can I find this pattern? .....→ LAKE BELL

# Image Features

- How would we solve this?

We matched:

- At each location
- Compared pixel values



Where can I find this pattern? .....→ LAKE BELL

# Image Features

- How would we solve this?



Where can I find this pattern? .....



# Image Features

- How would we solve this?

We matched:

- Distinctive locations
- Distinctive features
- Affine invariant



Where can I find this pattern? .....



# Image Features

- How would we solve this?





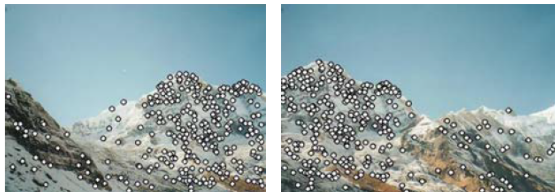
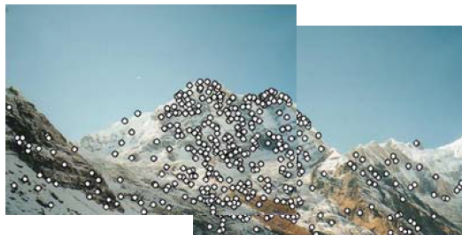
- **Detection: Where** to extract image features?
  - “Interesting” locations (keypoints)
  - In each location (densely)
- **Description: What** to extract?
  - What’s the spatial scope of the feature?
  - What’s the content of the feature?
- **Matching: How to match** them?

- **Detection: Where** to extract image features?
  - “Interesting” locations (keypoints) **TODAY**
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Image Features:

# Interest Point (Keypoint) Detection

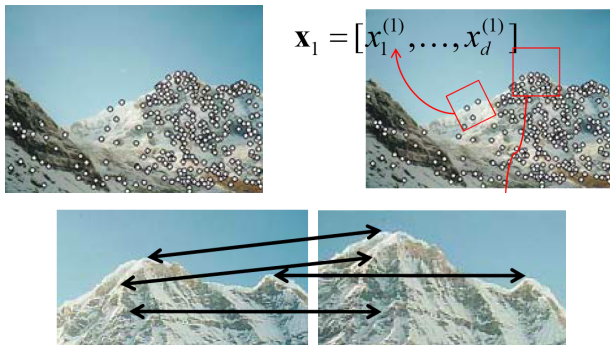
# Application Example: Image Stitching



[Source: K. Grauman]

# Local Features

- **Detection:** Identify the interest points.
- **Description:** Extract **feature vector** descriptor around each interest point.
- **Matching:** Determine correspondence between descriptors in two views.



[Source: K. Grauman]

# Goal: Repeatability of the Interest Point Operator

- Our goal is to detect (at least some of) the same points in both images
- We have to be able to run the detection procedure independently per image
- We need to generate enough points to increase our chances of detecting matching points
- We shouldn't generate too many or our matching algorithm will be too slow

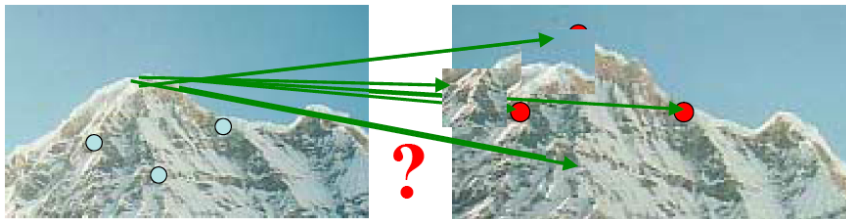


Figure: Too few keypoints  $\rightarrow$  little chance to find the true matches

[Source: K. Grauman, slide credit: R. Urtasun]

# Goal: Distinctiveness of the Keypoints

- We want to be able to **reliably** determine which point goes with which.



[Source: K. Grauman, slide credit: R. Urtasun]

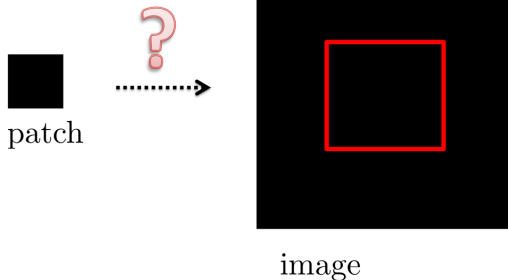
# What Points to Choose?



[Source: K. Grauman]



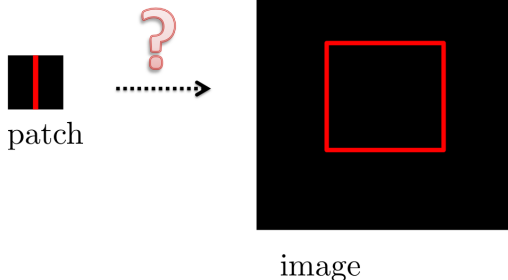
# What Points to Choose?



- Textureless patches are nearly impossible to localize.

[Adopted from: R. Urtasun]

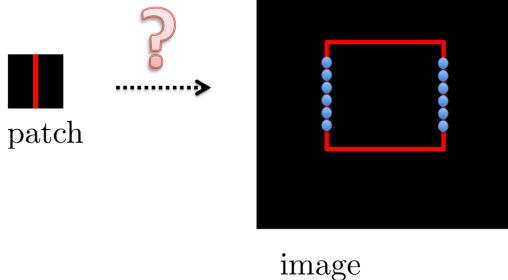
# What Points to Choose?



- Textureless patches are nearly impossible to localize.
- Patches with large contrast changes (gradients) are easier to localize.

[Adopted from: R. Urtasun]

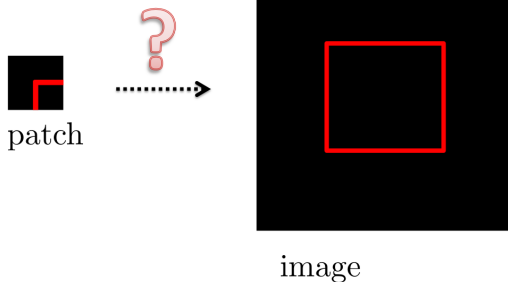
# What Points to Choose?



- Textureless patches are nearly impossible to localize.
- Patches with large contrast changes (gradients) are easier to localize.
- But straight line segments cannot be localized on lines segments with the same orientation (aperture problem)

[Adopted from: R. Urtasun]

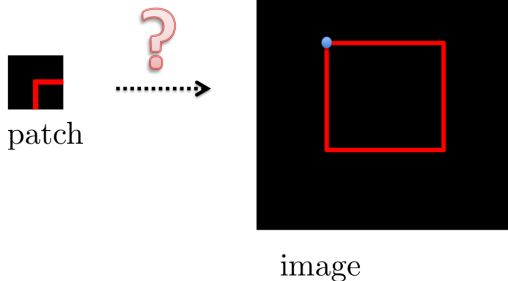
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- Gradients in at least two different orientations are easiest, e.g., **corners!**

[Adopted from: R. Urtasun]

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[Adopted from: R. Urtasun]

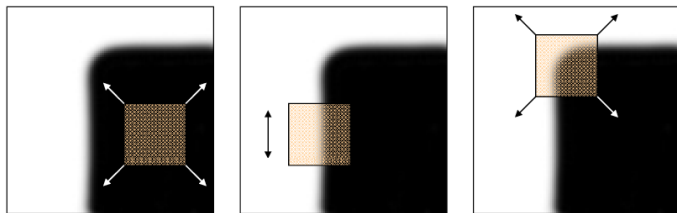
# Interest Points: Corners

- How can we find corners in an image?



# Interest Points: Corners

- We should easily recognize the point by looking through a small window.
- Shifting a window in any direction should give a large change in intensity.



**Figure:** (left) flat region: no change in all directions, (center) edge: no change along the edge direction, (right) corner: significant change in all directions

[Source: Alyosha Efros, Darya Frolova, Denis Simakov]

# Interest Points: Corners

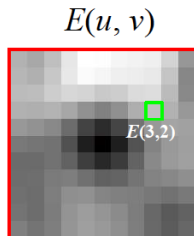
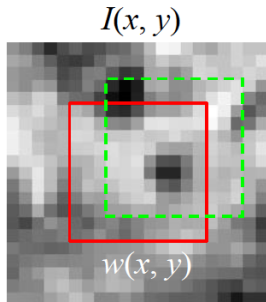
- Compare two image patches using (weighted) summed square difference
- Measures change in appearance of window  $w(x, y)$  for the shift

$$E_{\text{WSSD}}(u, v) = \sum_x \sum_y w(x, y) [I(x + u, y + v) - I(x, y)]^2$$

↑  
window function

↑  
shifted intensity

↑  
intensity



[Source: J. Hays]



# Interest Points: Corners

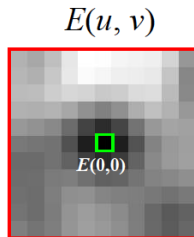
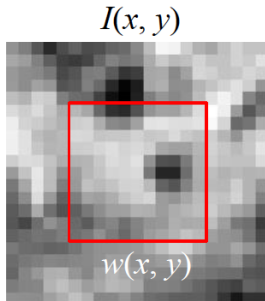
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window function

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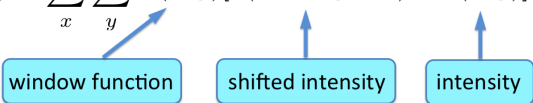
intensity



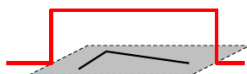
[Source: J. Hays]

# Interest Points: Corners

- Compare two image patches using (weighted) summed square difference
- Measures change in appearance of window  $w(x, y)$  for the shift

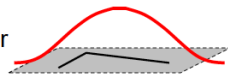
$$E_{\text{WSSD}}(u, v) = \sum_x \sum_y w(x, y) [I(x + u, y + v) - I(x, y)]^2$$


Window function  $w(x, y) =$



1 in window, 0 outside

or



Gaussian

[Source: J. Hays]

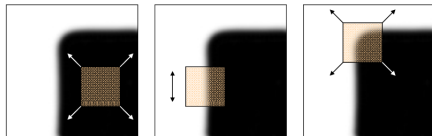
# Interest Points: Corners

- Let's look at  $E_{\text{WSSD}}$
- We want to find out how this function behaves for small shifts

$$E(u, v)$$



- Remember our goal to detect corners:



# Interest Points: Corners

- Using a simple first-order Taylor Series expansion:

$$I(x + u, y + v) \approx I(x, y) + u \cdot \frac{\partial I}{\partial x}(x, y) + v \cdot \frac{\partial I}{\partial y}(x, y)$$

- And plugging it in our expression for  $E_{\text{WSSD}}$ :

$$\begin{aligned} E_{\text{WSSD}}(u, v) &= \sum_x \sum_y w(x, y) \left( I(x + u, y + v) - I(x, y) \right)^2 \\ &\approx \sum_x \sum_y w(x, y) \left( I(x, y) + u \cdot I_x + v \cdot I_y - I(x, y) \right)^2 \\ &= \sum_x \sum_y w(x, y) \left( u^2 I_x^2 + 2u \cdot v \cdot I_x \cdot I_y + v^2 I_y^2 \right) \\ &= \sum_x \sum_y w(x, y) \cdot \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} I_x^2 & I_x \cdot I_y \\ I_x \cdot I_y & I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \end{aligned}$$

# Interest Points: Corners

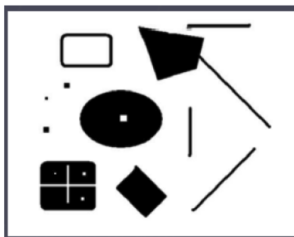
- Since  $(u, v)$  doesn't depend on  $(x, y)$  we can rewriting it slightly:

$$\begin{aligned} E_{\text{WSSD}}(u, v) &= \sum_x \sum_y w(x, y) [u \quad v] \begin{bmatrix} I_x^2 & I_x \cdot I_y \\ I_x \cdot I_y & I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \\ &= [u \quad v] \underbrace{\left( \sum_x \sum_y w(x, y) \begin{bmatrix} I_x^2 & I_x \cdot I_y \\ I_x \cdot I_y & I_y^2 \end{bmatrix} \right)}_{\text{Let's denote this with } M} \begin{bmatrix} u \\ v \end{bmatrix} \\ &= [u \quad v] M \begin{bmatrix} u \\ v \end{bmatrix} \end{aligned}$$

- $M$  is a  $2 \times 2$  *second moment matrix* computed from image gradients:

$$M = \sum_x \sum_y w(x, y) \begin{bmatrix} I_x^2 & I_x \cdot I_y \\ I_x \cdot I_y & I_y^2 \end{bmatrix}$$

# How Do I Compute $M$ ?

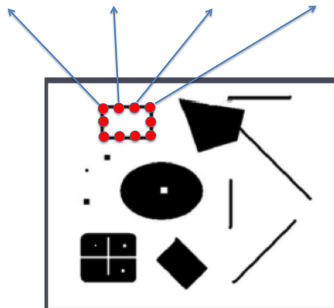


image

- Let's say I have this image

# How Do I Compute $M$ ?

$M = ?$   $M = ?$   $M = ?$   $M = ?$

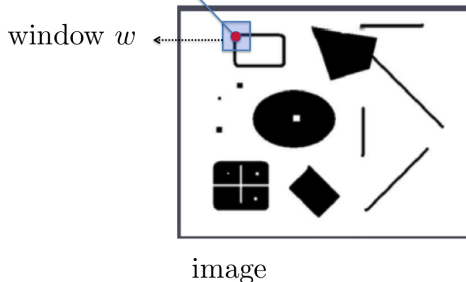


image

- Let's say I have this image
- I need to compute a  $2 \times 2$  second moment matrix in each image location

# How Do I Compute $M$ ?

$$M = \sum_x \sum_y w(x, y) \begin{bmatrix} I_x^2 & I_x \cdot I_y \\ I_x \cdot I_y & I_y^2 \end{bmatrix}$$

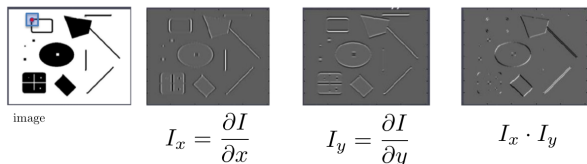


- Let's say I have this image
- I need to compute a  $2 \times 2$  second moment matrix in each image location
- In a particular location I need to compute  $M$  as a weighted average of gradients in a window



# How Do I Compute $M$ ?

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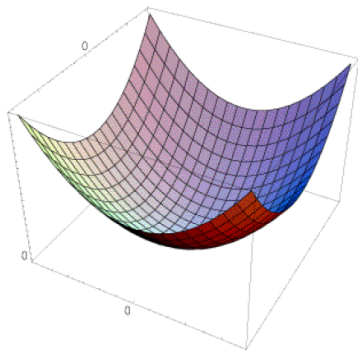
- Let's say I have this image
- I need to compute a  $2 \times 2$  second moment matrix in each image location
- In a particular location I need to compute  $M$  as a weighted average of gradients in a window
- I can do this efficiently by computing three matrices,  $I_x^2$ ,  $I_y^2$  and  $I_x \cdot I_y$ , and convolving each one with a filter, e.g. a box or Gaussian filter

# Interest Points: Corners

- We now have  $M$  computed in each image location
- Our  $E_{\text{WSSD}}$  is a **quadratic function** where  $M$  implies its shape

$$E_{\text{WSSD}}(u, v) = [u \quad v] M \begin{bmatrix} u \\ v \end{bmatrix}$$

$$M = \sum_x \sum_y w(x, y) \begin{bmatrix} I_x^2 & I_x \cdot I_y \\ I_x \cdot I_y & I_y^2 \end{bmatrix}$$



[Source: J. Hays]

# Interest Points: Corners

- Let's take a horizontal "slice" of  $E_{\text{WSSD}}(u, v)$ :

$$\begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$$

- This is the equation of an ellipse

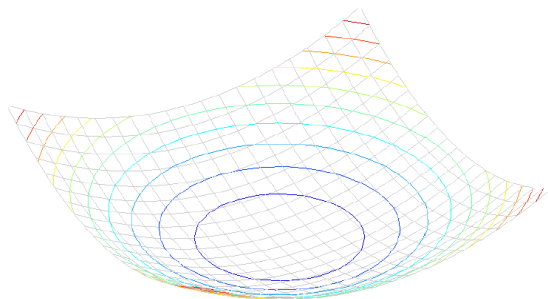


Figure: Different ellipses obtain by different horizontal "slices"

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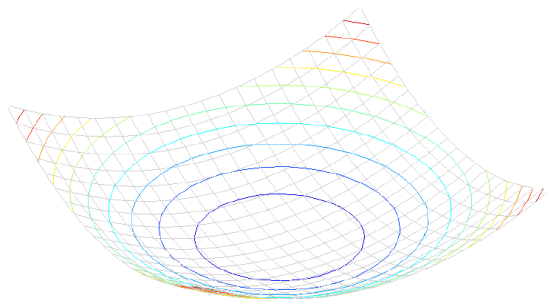


Figure: Different ellipses obtain by different horizontal "slices"

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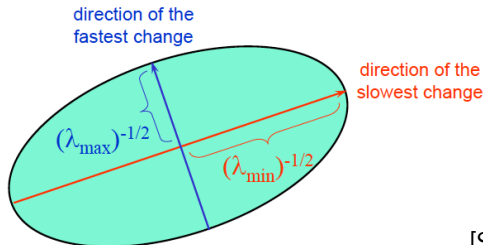
- Our matrix  $M$  is symmetric:

$$M = \sum_x \sum_y w(x,y) \begin{bmatrix} I_x^2 & I_x \cdot I_y \\ I_x \cdot I_y & I_y^2 \end{bmatrix}$$

- And thus we can diagonalize it (in Matlab:  $[V,D] = \text{EIG}(M)$ ):

$$M = V \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} V^{-1}$$

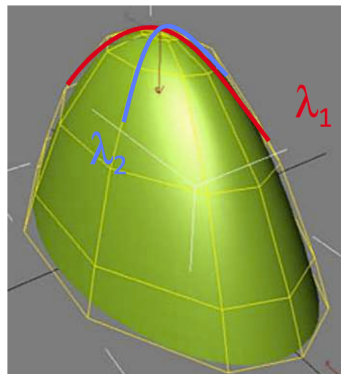
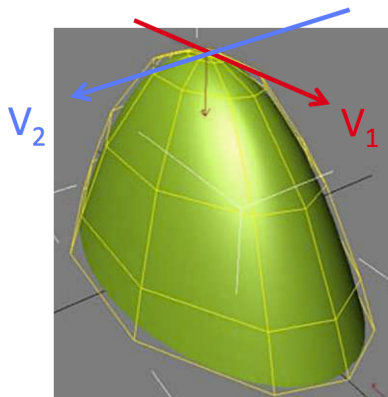
- Columns of  $V$  are major and minor axes of ellipse,  $1/\lambda^{-1}$  are radius



[Source: J. Hays]

# Interest Points: Corners

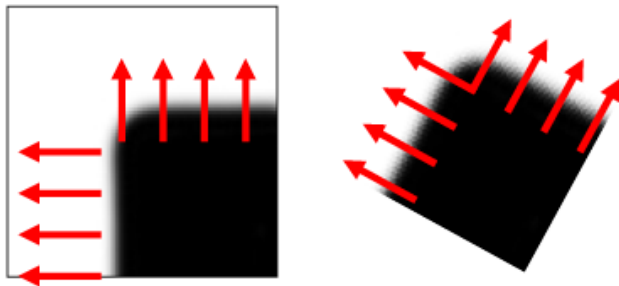
- Columns of  $V$  are **principal directions**
- $\lambda_1, \lambda_2$  are **principal curvatures**



[Source: F. Flores-Mangas]

## Interest Points: Corners

- The eigenvalues of  $M$  ( $\lambda_1, \lambda_2$ ) reveal the amount of intensity change in the two principal orthogonal gradient directions in the window



[Source: R. Szeliski, slide credit: R. Urtasun]

# Interest Points: Corners

- How do the ellipses look like for this image?

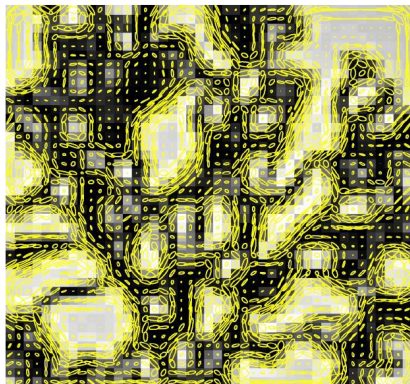


[Source: J. Hays]



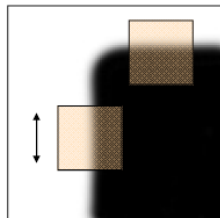
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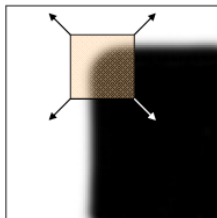
# Interest Points: Corners



“edge”:

$$\lambda_1 \gg \lambda_2$$

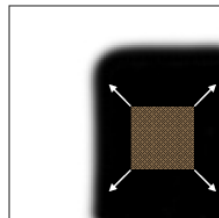
$$\lambda_2 \gg \lambda_1$$



“corner”:

$\lambda_1$  and  $\lambda_2$  are large,

$$\lambda_1 \sim \lambda_2;$$



“flat” region

$\lambda_1$  and  $\lambda_2$  are small;

[Source: K. Grauman, slide credit: R. Urtasun]

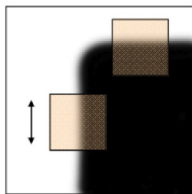
# Interest Points: Corners

## Criteria to find corners:

- Harris and Stephens, 88 is rotationally invariant and downweights edge-like features where  $\lambda_1 \gg \lambda_0$

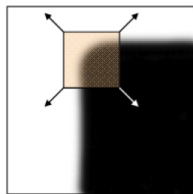
$$R = \det(M) - \alpha \cdot \text{trace}(M)^2 = \lambda_0 \lambda_1 - \alpha (\lambda_0 + \lambda_1)^2$$

- $\alpha$  a constant (0.04 to 0.06)



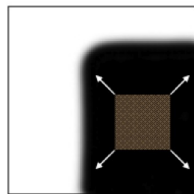
“edge”:

$$R < 0$$



“corner”:

$$R > 0$$



“flat” region

$$|R| \text{ small}$$

- The corresponding detector is called **Harris corner detector**

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- Shi and Tomasi, 94 proposed the smallest eigenvalue of  $\mathbf{A}$ , i.e.,  $\lambda_0^{-1/2}$ .
- Triggs, 04 suggested

$$\lambda_0 - \alpha \lambda_1$$

also reduces the response at 1D edges, where aliasing errors sometimes inflate the smaller eigenvalue

- Brown et al, 05 use the harmonic mean

$$\frac{\det(\mathbf{A})}{\text{trace}(\mathbf{A})} = \frac{\lambda_0 \lambda_1}{\lambda_0 + \lambda_1}$$

[Source R. Urtasun]

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- Shi and Tomasi, 94 proposed the smallest eigenvalue of  $\mathbf{A}$ , i.e.,  $\lambda_0^{-1/2}$ .
- Triggs, 04 suggested

$$\lambda_0 - \alpha \lambda_1$$

also reduces the response at 1D edges, where aliasing errors sometimes inflate the smaller eigenvalue

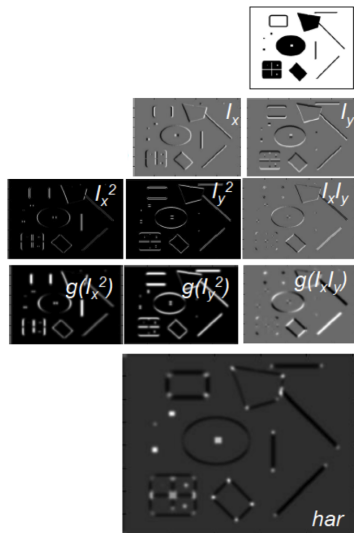
- Brown et al, 05 use the harmonic mean

$$\frac{\det(\mathbf{A})}{\text{trace}(\mathbf{A})} = \frac{\lambda_0 \lambda_1}{\lambda_0 + \lambda_1}$$

[Source R. Urtasun]

# Harris Corner detector

- 1 Compute gradients  $I_x$  and  $I_y$
- 2 Compute  $I_x^2$ ,  $I_y^2$ ,  $I_x \cdot I_y$
- 3 Average (Gaussian)  $\rightarrow$  gives  $M$
- 4 Compute  $R = \det(M) - \alpha \text{trace}(M)^2$  for each image window (*cornerness* score)
- 5 Find points with large  $R$  ( $R >$  threshold).
- 6 Take only points of local maxima, i.e., perform non-maximum suppression

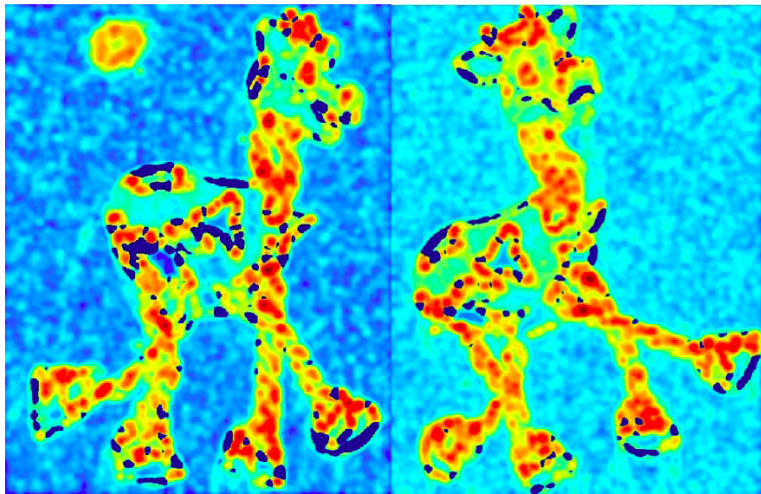


# Example



[Source: K. Grauman]

# 1) Compute Cornerness



[Source: K. Grauman]

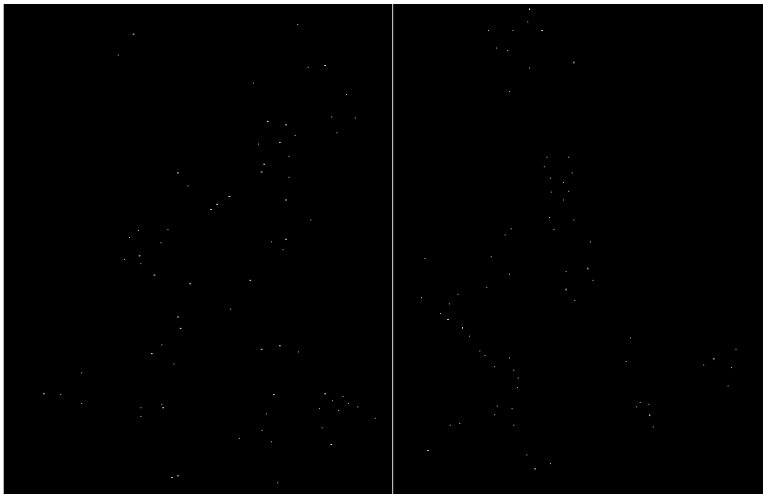


## 2) Find High Response



[Source: K. Grauman]

### 3) Non-maxima Suppression



[Source: K. Grauman]

# Results



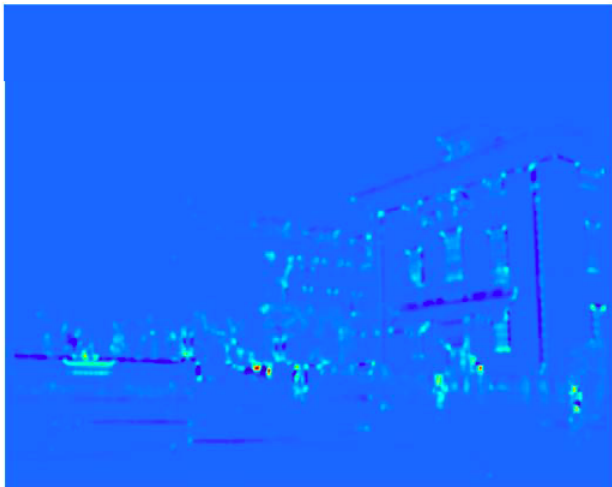
[Source: K. Grauman]

# Another Example



[Source: K. Grauman]

# Cornerness



[Source: K. Grauman]

# Interest Points



[Source: K. Grauman]

# Interest Points – Ideal Properties?

- We want corner locations to be **invariant** to photometric transformations and **covariant** to geometric transformations

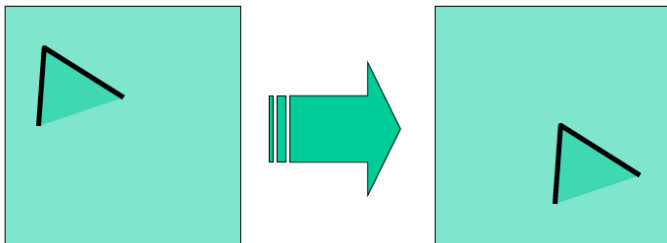
**Invariance** : Image is transformed and corner locations do not change

**Covariance** : If we have two transformed versions of the same image, features should be detected in corresponding locations



# Properties of Harris Corner Detector

- Shift invariant?



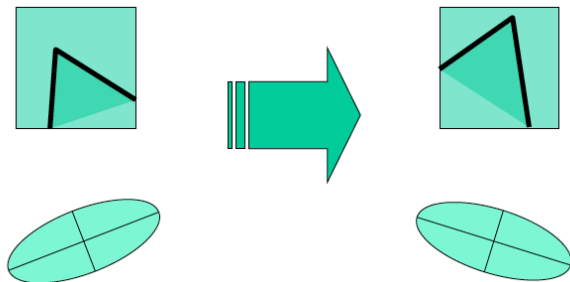
- Derivatives and window function **are shift-invariant**

[Source: J. Hays]



# Properties of Harris Corner Detector

- Rotation invariant?

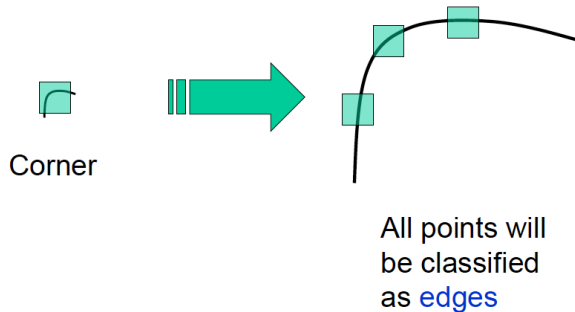


- Second moment ellipse rotates but its shape (i.e. eigenvalues) remains the same → **are rotation-invariant**

[Source: J. Hays]

# Properties of Harris Corner Detector

- Scale invariant?



- Corner location is **not scale invariant!**

[Source: J. Hays]

## Next Time

- Can we also define keypoints that are shift, rotation and scale invariant?
- What should be our **description** around keypoint?