Image Features:

Local Descriptors
Local Features

- **Detection**: Identify the interest points.
- **Description**: Extract a feature descriptor around each interest point.
- **Matching**: Determine correspondence between descriptors in two views.

\[ x_1 = [x_1^{(1)}, \ldots, x_d^{(1)}] \]

[Source: K. Grauman]
The Ideal Feature Descriptor

- **Repeatable**: Invariant to rotation, scale, photometric variations
- **Distinctive**: We will need to match it to lots of images/objects!
- **Compact**: Should capture rich information yet not be too high-dimensional (otherwise matching will be slow)
- **Efficient**: We would like to compute it (close-to) real-time
Invariances

[Source: T. Tuytelaars]
Invariances

[Source: T. Tuytelaars]
What If We Just Took Pixels?

- The simplest way is to write down the list of intensities to form a feature vector, and normalize them (i.e., mean 0, variance 1).
- Why normalization?
- But this is very sensitive to even small shifts, rotations and any affine transformation.
Tones Of Better Options

- SIFT
- PCA-SIFT
- GLOH
- HOG
- SURF
- DAISY
- LBP
- Shape Contexts
- Color Histograms
Tones Of Better Options

- SIFT **TODAY**
- PCA-SIFT
- GLOH
- HOG
- SURF
- DAISY
- LBP
- Shape Contexts
- Color Histograms
SIFT Descriptor [Lowe 2004]

- SIFT stands for Scale Invariant Feature Transform
- Invented by David Lowe, who also did DoG scale invariant interest points
- Actually in the same paper, which you should read:

David G. Lowe

*Distinctive image features from scale-invariant keypoints*

International Journal of Computer Vision, 2004


(a) image gradients

(b) keypoint descriptor
SIFT Descriptor

Our scale invariant interest point detector gives scale $\rho$ for each keypoint

[Adopted from: F. Flores-Mangas]
For each keypoint, we take the Gaussian-blurred image at corresponding scale $\rho$

\[
\begin{align*}
I_s &= I \ast G_{\sigma_6} \\
I_2 &= I \ast G_{\sigma_{6}}^{\text{first octave}} \\
I_1 &= I \ast G_{\sigma_6} \\
I_0 &= I \ast G_{\sigma_6} \\
D(x,y,\sigma)&=D(x,y,\sigma_6)
\end{align*}
\]
Compute the gradient magnitude and orientation in neighborhood of each keypoint.

\[ I \ast G_\rho \]  

Gaussian smoothed image at scale of keypoint

compute magnitude and orientation of gradients in neighborhood

16 \times 16 pixel patch

[Adopted from: F. Flores-Mangas]
3. Compute the gradient magnitude and orientation in neighborhood of each keypoint

\[
|\nabla I(x, y)| = \sqrt{\left( \frac{\partial (I(x, y) \ast G_\rho)}{\partial x} \right)^2 + \left( \frac{\partial (I(x, y) \ast G_\rho)}{\partial y} \right)^2}
\]

magnitude of gradient:

\[
\theta(x, y) = \arctan \left( \frac{\partial I \ast G_\rho}{\partial y} / \frac{\partial I \ast G_\rho}{\partial x} \right)
\]

gradient orientation:

(in case you forgot ;))
SIFT Descriptor

Compute dominant orientation of each keypoint. How?

\[ I \ast G_\rho \]

Gaussian smoothed image at scale of keypoint

compute magnitude and orientation of gradients in neighborhood

16 × 16 pixel patch

[Adopted from: F. Flores-Mangas]
SIFT Descriptor: Computing Dominant Orientation

- Compute a histogram of gradient orientations, each bin covers $10^\circ$.

$16 \times 16$ compute histograms of orientations by orientation increments of $10^\circ$.

[Adopted from: F. Flores-Mangas]
SIFT Descriptor: Computing Dominant Orientation

- Compute a histogram of gradient orientations, each bin covers 10°
- Orientations closer to the keypoint center should contribute more

$G_{1.5\rho}$

weight influence of orientation based on distance from center

$|\nabla I(x, y)| \cdot G_{1.5\rho}(d)$

[Adopted from: F. Flores-Mangas]
SIFT Descriptor: Computing Dominant Orientation

- Compute a histogram of gradient orientations, each bin covers $10^\circ$
- Orientations closer to the keypoint center should contribute more
- Orientation giving the peak in the histogram is the keypoint's orientation

[Adopted from: F. Flores-Mangas]
Compute dominant orientation

compute magnitude and orientation of gradients in neighborhood

16 × 16 pixel patch

[Adopted from: F. Flores-Mangas]
Compute a 128 dimensional descriptor: $4 \times 4$ grid, each cell is a histogram of 8 orientation bins relative to dominant orientation.

$$P_i = (x_i, y_i, \rho_i, \vartheta_i) \quad \text{and} \quad f_i \ldots \quad \text{128 dim vector}$$

[Adopted from: F. Flores-Mangas]
SIFT Descriptor: Computing the Feature Vector

- Compute the orientations relative to the dominant orientation

16 × 16 patch centered in \((x_i, y_i)\)

[Adopted from: F. Flores-Mangas]
Compute the orientations relative to the dominant orientation

16 × 16 patch centered in \((x_i, y_i)\)

[Adopted from: F. Flores-Mangas]
SIFT Descriptor: Computing the Feature Vector

- Compute the orientations **relative** to the **dominant orientation**
- Form a $4 \times 4$ grid. For each grid cell compute a histogram of orientations for 8 orientation bins spaced apart by $45^\circ$

16 $\times$ 16 patch centered in $(x_i, y_i)$

SIFT descriptor

compute histogram of orientations this time 8 bins spaced by $45^\circ$

[Adopted from: F. Flores-Mangas]
SIFT Descriptor: Computing the Feature Vector

- Compute the orientations relative to the dominant orientation
- Form a $4 \times 4$ grid. For each grid cell compute a histogram of orientations for 8 orientation bins spaced apart by 45°

$16 \times 16$ patch centered in $(x_i, y_i)$

again weigh contributions this time: $|\nabla I(x, y)| \cdot G_{0.5}$

[Adopted from: F. Flores-Mangas]
SIFT Descriptor: Computing the Feature Vector

- Compute the orientations relative to the dominant orientation
- Form a $4 \times 4$ grid. For each grid cell compute a histogram of orientations for 8 orientation bins spaced apart by $45^\circ$
- Form the 128 dimensional feature vector

$\mathbf{f}_i = \begin{bmatrix} \vdots \end{bmatrix}$

[Adopted from: F. Flores-Mangas]
The resulting 128 non-negative values form a raw version of the SIFT descriptor vector.

To reduce the effects of contrast or gain (additive variations are already removed by the gradient), the 128-D vector is normalized to unit length: $f_i = f_i / ||f_i||$
The resulting 128 non-negative values form a raw version of the SIFT descriptor vector.

To reduce the effects of contrast or gain (additive variations are already removed by the gradient), the 128-D vector is normalized to unit length: $f_i = f_i / \| f_i \|$

To further make the descriptor robust to other photometric variations, values are clipped to 0.2 and the resulting vector is once again renormalized to unit length.
The resulting 128 non-negative values form a **raw version** of the SIFT descriptor vector.

To reduce the **effects of contrast or gain** (additive variations are already removed by the gradient), the 128-D vector is normalized to unit length: $$f_i = f_i / \|f_i\|$$

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Great engineering effort!
The resulting 128 non-negative values form a **raw version** of the SIFT descriptor vector.

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Great engineering effort!

What is SIFT invariant to?
The resulting 128 non-negative values form a raw version of the SIFT descriptor vector.

To reduce the effects of contrast or gain (additive variations are already removed by the gradient), the 128-D vector is normalized to unit length: \( f_i = f_i / \| f_i \| \)

To further make the descriptor robust to other photometric variations, values are clipped to 0.2 and the resulting vector is once again renormalized to unit length.

Great engineering effort!

What is SIFT invariant to?
Properties of SIFT

Invariant to:
- Scale
- Rotation

Partially invariant to:
- Illumination changes (sometimes even day vs. night)
- Camera viewpoint (up to about 60 degrees of out-of-plane rotation)
- Occlusion, clutter (why?)

Also important:
- Fast and efficient – can run in real time
- Lots of code available
Examples

Figure: Matching in day / night under viewpoint change

[Source: S. Seitz]
Example

**Figure:** NASA Mars Rover images with SIFT feature matches

[Source: N. Snavely]
PCA-SIFT

- The dimensionality of SIFT is pretty high, i.e., 128D for each keypoint
- Reduce the dimensionality using linear dimensionality reduction
- In this case, principal component analysis (PCA)
- Use 10D or so descriptor

[Source: R. Urtasun]
Gradient location-orientation histogram (GLOH)

- Developed by Mikolajczyk and Schmid (2005): variant of SIFT that uses a log-polar binning structure instead of the four quadrants.
- The spatial bins are 11, and 15, with eight angular bins (except for the central region), for a total of 17 spatial bins and 16 orientation bins.
- The 272D histogram is then projected onto a 128D descriptor using PCA trained on a large database.

[Source: R. Szeliski]
Other Descriptors

- SURF
- DAISY
- LBP
- HOG
- Shape Contexts
- Color Histograms
Local Features

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[Source: K. Grauman]
Image Features:

Matching the Local Descriptors
Matching the Local Descriptors

Once we have extracted keypoints and their descriptors, we want to match the features between pairs of images.

- Ideally a match is a correspondence between a local part of the object on one image to the same local part of the object in another image.
- How should we compute a match?

Figure: Images from K. Grauman
Matching the Local Descriptors

Once we have extracted keypoints and their descriptors, we want to match the features between pairs of images.

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- How should we compute a match?

Figure: Images from K. Grauman
Matching the Local Descriptors

- Simple: **Compare them all**, compute Euclidean distance
Matching the Local Descriptors

- Simple: **Compare them all**, compute Euclidean distance

\[
\begin{align*}
\|f_1 - f'_1\| \\
\|f_1 - f'_2\| \\
\|f_1 - f'_3\| \\
&\vdots \\
\|f_1 - f'_{k'}\| 
\end{align*}
\]
Matching the Local Descriptors

- Find closest match (min distance). How do we know if match is reliable?

\[
\begin{align*}
    f_1 & = \cdots \\
    f_2 & = \cdots \\
    f_3 & = \cdots \\
    f_{k-1} & = \cdots \\
    f_k & = \cdots \\
    f'_1 & = \cdots \\
    f'_2 & = \cdots \\
    f'_3 & = \cdots \\
    f'_{k-1} & = \cdots \\
    f'_{k} & = \cdots \\
\end{align*}
\]

\[
\begin{align*}
    ||f_1 - f'_1|| & = \min (closest \ match) \\
    ||f_1 - f'_2|| & \\
    ||f_1 - f'_3|| & \\
    ||f_1 - f'_{k-1}|| & \\
    ||f_1 - f'_{k}|| & \\
\end{align*}
\]
Matching the Local Descriptors

- Find also the second closest match. Match reliable if first distance "much" smaller than second distance.
Matching the Local Descriptors

- Compute the ratio:

\[
\phi_i = \frac{||f_i - f'_i||}{||f_i - f'_{i}^{**}||}
\]

where \( f'_i \) is the closest and \( f'_{i}^{**} \) second closest match to \( f_i \).
Which Threshold to Use?

- Setting the threshold too high results in too many false positives, i.e., incorrect matches being returned.
- Setting the threshold too low results in too many false negatives, i.e., too many correct matches being missed.

Figure: Images from R. Szeliski
Which Threshold to Use?

- Threshold ratio of nearest to 2nd nearest descriptor
- Typically: $\phi_i < 0.8$

Figure: Images from D. Lowe

[Source: K. Grauman]
Applications of Local Invariant Features

- Wide baseline stereo
- Motion tracking
- Panorama stitching
- Mobile robot navigation
- 3D reconstruction
- Recognition
- Retrieval

[Source: K. Grauman]
Wide Baseline Stereo

[Source: T. Tuytelaars]
Recognizing the Same Object

Schmid and Mohr 1997

Sivic and Zisserman, 2003

Rothganger et al. 2003

Lowe 2002

[Source: K. Grauman]
Motion Tracking

Figure: Images from J. Pilet
Now What

- Now we know how to extract scale and rotation invariant features
- We even know how to match features across images
- Can we use this to find Waldo in an even more sneaky scenario?
Now What

- Now we know how to extract scale and rotation invariant features
- We even know how to match features across images
- Can we use this to find Waldo in an even more sneaky scenario?

Waldo on the road
Now What

- Now we know how to extract scale and rotation invariant features
- We even know how to match features across images
- Can we use this to find Waldo in an even more sneaky scenario?

He comes closer... We know how to solve this
Now What

- Now we know how to extract scale and rotation invariant features
- We even know how to match features across images
- Can we use this to find Waldo in an even more sneaky scenario?

Someone takes a (weird) picture of him!
More interesting: If we have DVD covers (e.g., from Amazon), can we match them to DVDs in real scenes?
Matching Planar Objects In New Viewpoints
What Kind of Transformation Happened To My DVD?

$T$
What Kind of Transformation Happened To My DVD?

- Rectangle goes to a parallelogram (almost but not really, but let’s believe that for now)
All 2D Linear Transformations

Linear transformations are combinations of

- Scale,
- Rotation
- Shear
- Mirror

\[
\begin{bmatrix}
x' \\
y'
\end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}
\]

[Source: N. Snavely]
Properties of linear transformations:

- Origin maps to origin
- Lines map to lines
All 2D Linear Transformations

Properties of linear transformations:

- Origin maps to origin
- Lines map to lines
- Parallel lines remain parallel
All 2D Linear Transformations

Properties of linear transformations:

- Origin maps to origin
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- Ratios are preserved
Properties of linear transformations:

- Origin maps to origin
- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition
All 2D Linear Transformations

Properties of linear transformations:

- Origin maps to origin
- Lines map to lines
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- Closed under composition

\[
\begin{bmatrix}
  x' \\
  y'
\end{bmatrix}
= \begin{bmatrix}
  a & b \\
  c & d
\end{bmatrix}
\begin{bmatrix}
  e & f \\
  g & h
\end{bmatrix}
\begin{bmatrix}
  i & j \\
  k & l
\end{bmatrix}
\begin{bmatrix}
  x \\
  y
\end{bmatrix}
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All 2D Linear Transformations

Properties of linear transformations:

- Origin maps to origin
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\[
\begin{bmatrix}
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y'
\end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} i & j \\ k & l \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}
\]

What about the translation?

[Source: N. Snavely]
All 2D Linear Transformations

Properties of linear transformations:

- Origin maps to origin
- Lines map to lines
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\[
\begin{bmatrix}
  x' \\
  y'
\end{bmatrix} = \begin{bmatrix}
  a & b \\
  c & d
\end{bmatrix} \begin{bmatrix}
  e & f \\
  g & h
\end{bmatrix} \begin{bmatrix}
  i & j \\
  k & l
\end{bmatrix} \begin{bmatrix}
  x \\
  y
\end{bmatrix}
\]

What about the translation?

[Source: N. Snavely]
Affine Transformations

Affine transformations are combinations of

- **Linear transformations**, and
- **Translations**

\[
\begin{bmatrix}
x' \\
y'
\end{bmatrix} = \begin{bmatrix}
a & b \\
c & d
\end{bmatrix} \begin{bmatrix}
x \\
y
\end{bmatrix} + \begin{bmatrix}
e \\
f
\end{bmatrix}
\]

same as:

\[
\begin{bmatrix}
x' \\
y'
\end{bmatrix} = \begin{bmatrix}
a & b & e \\
c & d & f
\end{bmatrix} \begin{bmatrix}
x \\
y \\
1
\end{bmatrix}
\]
Affine Transformations

Affine transformations are combinations of

- Linear transformations, and

- Translations

\[
\begin{bmatrix}
x' \\
y'
\end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix}
\]

same as:

\[
\begin{bmatrix}
x' \\
y'
\end{bmatrix} = \begin{bmatrix} a & b & e \\ c & d & f \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}
\]
Affine Transformations

Affine transformations are combinations of
- Linear transformations, and
- Translations

\[
\begin{bmatrix}
  x' \\
  y'
\end{bmatrix} = \begin{bmatrix} a & b & e \\ c & d & f \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}
\]

Properties of affine transformations:
- Origin does not necessarily map to origin
- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition
- Rectangles go to parallelograms

[Source: N. Snavely]
2D Image Transformations

These transformations are a nested set of groups

Closed under composition and inverse is a member
Affine transformation approximates viewpoint changes for roughly planar objects and roughly orthographic cameras (more about these later in class)

DVD went affine!
Computing the (Affine) Transformation

Given a set of matches between images I and J

- How can we compute the affine transformation $A$ from $I$ to $J$?
- Find transform $A$ that best agrees with the matches

[Source: N. Snavely]
Computing the (Affine) Transformation

Given a set of matches between images I and J

- How can we compute the affine transformation $A$ from I to J?
- Find transform $A$ that best agrees with the matches

[Source: N. Snavely]
Computing the Affine Transformation

Let \((x_i, y_i)\) be a point on the reference (model) image, and \((x'_i, y'_i)\) its match in the test image.

An affine transformation \(A\) maps \((x_i, y_i)\) to \((x'_i, y'_i)\):

\[
\begin{bmatrix}
  x'_i \\
  y'_i
\end{bmatrix} = \begin{bmatrix}
  a & b & e \\
  c & d & f
\end{bmatrix} \begin{bmatrix}
  x_i \\
  y_i \\
  1
\end{bmatrix}
\]

We can rewrite this into a simple linear system:

\[
\begin{bmatrix}
  x_i & y_i & 0 & 0 & 1 & 0 \\
  0 & 0 & x_i & y_i & 0 & 1
\end{bmatrix} \begin{bmatrix}
  a \\
  b \\
  c \\
  d \\
  e \\
  f
\end{bmatrix} = \begin{bmatrix}
  x'_i \\
  y'_i
\end{bmatrix}
\]
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  1
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\end{bmatrix} \begin{bmatrix}
  a \\
  b \\
  c \\
  d \\
  e \\
  f
\end{bmatrix} = \begin{bmatrix}
  x'_i \\
  y'_i
\end{bmatrix}
\]
Computing the Affine Transformation

- But we have many matches:

$$\begin{bmatrix}
\vdots \\
x_i & y_i & 0 & 0 & 1 & 0 \\
0 & 0 & x_i & y_i & 0 & 1 \\
\vdots 
\end{bmatrix}
\begin{bmatrix}
a \\
b \\
c \\
d \\
e \\
f 
\end{bmatrix}
= \begin{bmatrix}
\vdots \\
x'_i \\
y'_i \\
\vdots 
\end{bmatrix}$$

- For each match we have two more equations

- How many matches do we need to compute $A$?

- But the more, the better (more reliable)
Computing the Affine Transformation

- But we have many matches:

\[
\begin{bmatrix}
\vdots \\
x_i & y_i & 0 & 0 & 1 & 0 \\
0 & 0 & x_i & y_i & 0 & 1 \\
\vdots \\
\end{bmatrix}
\begin{bmatrix}
a \\
b \\
c \\
d \\
e \\
f \\
\end{bmatrix}
= \begin{bmatrix}
\vdots \\
x'_i \\
y'_i \\
\vdots \\
\end{bmatrix}
\]

- For each match we have two more equations

- How many matches do we need to compute \( A \)?
Computing the Affine Transformation

- But we have many matches:

\[
\begin{bmatrix}
\vdots \\
x_i & y_i & 0 & 0 & 1 & 0 \\
0 & 0 & x_i & y_i & 0 & 1 \\
\vdots \\
\end{bmatrix}
\begin{bmatrix}
a \\
b \\
c \\
d \\
e \\
f \\
\end{bmatrix}
= 
\begin{bmatrix}
\vdots \\
x_i' \\
y_i' \\
\vdots \\
\end{bmatrix}
\]

- For each match we have two more equations

- How many matches do we need to compute A?
  - 6 parameters → 3 matches
  - But the more, the better (more reliable)
  - How do we compute A?
Computing the Affine Transformation

- But we have many matches:

\[
\begin{bmatrix}
\vdots \\
x_i & y_i & 0 & 0 & 1 & 0 \\
0 & 0 & x_i & y_i & 0 & 1 \\
\vdots \\
\end{bmatrix}
\begin{bmatrix}
a \\
b \\
c \\
d \\
e \\
f \\
\end{bmatrix}
= \begin{bmatrix}
\vdots \\
x'_i \\
y'_i \\
\vdots \\
\end{bmatrix}
\]

- For each match we have two more equations
- How many matches do we need to compute A?
- 6 parameters → 3 matches
- But the more, the better (more reliable)
- How do we compute A?
Computing the Affine Transformation

If we have 3 matches, then computing $A$ is really easy:

$$a = P^{-1}P'$$

If we have more than 3, then we do least-squares estimation:

$$\min_{a,b,\cdots,f} \|Pa - P'\|_2^2$$

Which has a closed form solution:

$$a = (P^TP)^{-1}P^TP'$$
Image Alignment Algorithm: Affine Case

Given images $I$ and $J$

1. Compute image features for $I$ and $J$
2. Match features between $I$ and $J$
3. Compute affine transformation $A$ between $I$ and $J$ using least squares on the set of matches

Is there a problem with this?

[Source: N. Snavely]
Image Alignment Algorithm: Affine Case

Given images $I$ and $J$

1. Compute image features for $I$ and $J$
2. Match features between $I$ and $J$
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Is there a problem with this?

[Source: N. Snavely]
Robustness

[Source: N. Snavely]
Simple Case

- Let's consider a simpler example ... Fit a line to the points below!

Problem: Fit a line to these datapoints

Least squares fit

- How can we fix this?

[Source: N. Snavely]
Let's consider a simpler example ... Fit a line to the points below!

Problem: Fit a line to these datapoints
Least squares fit

How can we fix this?

[Source: N. Snavely]
Simple Idea: RANSAC

- Take the minimal number of points to compute what we want. In the line example, two points (in our affine example, three matches)

- By “take” we mean choose at random from all points
Simple Idea: RANSAC

- Take the minimal number of points to compute what we want. In the line example, two points (in our affine example, three matches)
- By “take” we mean choose at random from all points
- Fit a line to the selected pair of points
Simple Idea: RANSAC

- Take the minimal number of points to compute what we want. In the line example, two points (in our affine example, three matches).
- By “take” we mean choose at random from all points.
- Fit a line to the selected pair of points.
- Count the number of all points that “agree” with the line: We call the agreeing points inliers.
Simple Idea: RANSAC

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- By “take” we mean choose at random from all points
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- “Agree” = within a small distance of the line
Simple Idea: RANSAC

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- Fit a line to the selected pair of points
- Count the number of all points that “agree” with the line: We call the agreeing points **inliers**
- “Agree” = within a small distance of the line
- Repeat this many times, remember the number of inliers for each trial
- Among several trials, select the one with the largest number of inliers

This procedure is called **RA**ndom **SA**mple **C**onsensus
Simple Idea: RANSAC

- Take the minimal number of points to compute what we want. In the line example, two points (in our affine example, three matches)

- By “take” we mean choose at random from all points

- Fit a line to the selected pair of points

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- “Agree” = within a small distance of the line

- Repeat this many times, remember the number of inliers for each trial

- Among several trials, select the one with the largest number of inliers

This procedure is called Random Sample Consensus
RANSAC for Line Fitting Example

1. Randomly select minimal subset of points
2. Hypothesize a model

[Source: R. Raguram]
RANSAC for Line Fitting Example

1. Randomly select minimal subset of points
2. Hypothesize a model
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[Source: R. Raguram]
RANSAC for Line Fitting Example

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[Source: R. Raguram]
Translations

[Source: N. Snavely]
Select one match at random, count inliers

[Source: N. Snavely]
Select another match at random, count inliers

[Source: N. Snavely]
Output the translation with the highest number of inliers

[Source: N. Snavely]
RANSAC

- All the inliers will agree with each other on the translation vector; the (hopefully small) number of outliers will (hopefully) disagree with each other.

- RANSAC only has guarantees if there are < 50% outliers.
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RANSAC

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  - Often model noise as Gaussian with some standard deviation (e.g., 3 pixels)
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How many rounds do we need?

[Source: R. Urtasun]
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- Sufficient number of trials $S$ must be tried.
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- Applicable to many different problems
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[Source: N. Snavely, slide credit: R. Urtasun]
Ransac Verification

[Source: K. Grauman, slide credit: R. Urtasun]
To match image $I$ and $J$ under affine transformation:

- Compute scale and rotation invariant keypoints in both images
- Compute a (rotation invariant) feature vector in each keypoint (e.g., SIFT)
- Match all features in $I$ to all features in $J$
- For each feature in reference image $I$ find closest match in $J$
- If ratio between closest and second closest match is $< 0.8$, keep match
- Do RANSAC to compute affine transformation $A$:
  - Select 3 matches at random
  - Compute $A$
  - Compute the number of inliers
  - Repeat
  - Find $A$ that gave the most inliers