Functional Programming

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- – assignment
- – sequential composition
- + functions

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+ functions

specification = function from input to output

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specification = function from input to output

program = implemented specification

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\+ functions

specification = function from input to output

program = implemented specification

application ✔

composition ✔

selective union ✔

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quantifiers ✗

Functional Programming

– assignment

– sequential composition

\+ functions

specification = function from input to output

program = implemented specification

application ✔

composition ✔

selective union ✔

quantifiers ✗

program + inputs = function + arguments

example specification $\langle L\colon [*rat]\cdot \Sigma L\rangle$

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$\Sigma L \;=$

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$$\Sigma L \;=\; \langle n\colon 0,..\#L{+}1\cdot \Sigma\, L\,[n;..\#L]\rangle\; 0$$

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example specification $\langle L\text{: [*}rat\text{]}\cdot\ \Sigma L\rangle$

$$\Sigma L \;=\; \langle n\text{: } 0,..\#L{+}1\cdot\ \Sigma\ L\ [n;..\#L]\rangle\ 0$$

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$$\Sigma\, L\, [n+1;..\#L] \;=\; \langle n: 0,..\#L+1\cdot \Sigma\, L\, [n;..\#L]\rangle\, (n+1)$$

time specification $\langle L: [*rat]\cdot \#L\rangle$

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time specification $\langle L: [*rat] \cdot \#L \rangle$

$$\#L \;=\; \langle n: 0,..\#L{+}1 \cdot \Sigma\, L\, [n;..\#L] \rangle\, 0$$

$$\uparrow$$

$$\langle n: 0,..\#L{+}1 \cdot \Sigma\, L\, [n;..\#L] \rangle$$

$$=\quad \langle n: 0,..\#L \;,\; \#L \cdot \Sigma\, L\, [n;..\#L] \rangle$$

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$$\Sigma\, L\, [n+1;..\#L] \;=\; \langle n: 0,..\#L+1\cdot \Sigma\, L\, [n;..\#L]\rangle\, (n+1)$$

time specification $\langle L{:}\ [*rat]\cdot \#L\rangle$

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$$\langle n{:}\ 0,..\#L{+}1\cdot \Sigma\, L\, [n;..\#L]\rangle$$

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time specification $\langle L\colon [*rat]\cdot \#L\rangle$

$$\#L \;=\; \langle n\colon 0,..\#L{+}1\cdot \#L{-}n\rangle\, 0$$

$$\downarrow$$

$$\langle n\colon 0,..\#L{+}1\cdot \Sigma\, L\, [n;..\#L]\rangle$$

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$$\Sigma\, L\, [n+1;..\#L] \quad = \quad \langle n\text{: } 0,..\#L+1\cdot \Sigma\, L\, [n;..\#L]\rangle\, (n+1)$$

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time

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$\langle n: 0,..\#L\cdot \#L-n\rangle$

$= \quad \langle n: 0,..\#L\cdot 1 + \#L-n-1\rangle$

$\langle n: \#L\cdot \Sigma\, L\, [n;..\#L]\rangle \;=\; \langle n: \#L\cdot 0\rangle$

$\Sigma\, L\, [n+1;..\#L] \;=\; \langle n: 0,..\#L+1\cdot \Sigma\, L\, [n;..\#L]\rangle\, (n+1)$

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$$\langle n: 0,..\#L\cdot \#L-n\rangle$$

$$=\quad \langle n: 0,..\#L\cdot 1+\#L-n-1\rangle$$

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$\langle n: \#L\cdot \#L–n\rangle \;=\; \langle n: \#L\cdot 0\rangle$

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↑

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$$\langle n: 0,..\#L\cdot \#L–n\rangle$$

$$= \quad \langle n: 0,..\#L\cdot 1 + \#L–n–1\rangle$$

$$\langle n: \#L\cdot \#L–n\rangle \;=\; \langle n: \#L\cdot 0\rangle$$

$$\#L–n–1 \;=\; \langle n: 0,..\#L+1\cdot \#L–n\rangle\,(n+1)$$

time specification $\langle L: [*rat]\cdot \#L\rangle$

$\#L \;=\; \langle n: 0,..\#L+1\cdot \#L-n\rangle\ 0$

$\langle n: 0,..\#L+1\cdot \#L-n\rangle$

$= \quad \langle n: 0,..\#L\ ,\ \#L \cdot \#L-n\rangle$

$= \quad \langle n: 0,..\#L\cdot \#L-n\rangle \mid \langle n: \#L\cdot \#L-n\rangle$

$\langle n: 0,..\#L\cdot \#L-n\rangle$

$= \quad \langle n: 0,..\#L\cdot\ 1 + \#L-n-1\rangle$

↑

$\langle n: \#L\cdot \#L-n\rangle \;=\; \langle n: \#L\cdot 0\rangle$

$\#L-n-1 \;=\; \langle n: 0,..\#L+1\cdot \#L-n\rangle\ (n+1)$

time

specification $\langle L\colon [*rat]\cdot \#L\rangle$

$$\#L \;=\; \langle n\colon 0,..\#L{+}1\cdot \#L{-}n\rangle\ 0$$

$$\langle n\colon 0,..\#L{+}1\cdot \#L{-}n\rangle$$

$$=\quad \langle n\colon 0,..\#L\ \ ,\ \#L\cdot \#L{-}n\rangle$$

$$=\quad \langle n\colon 0,..\#L\cdot \#L{-}n\rangle \mid \langle n\colon \#L\cdot \#L{-}n\rangle$$

$$\langle n\colon 0,..\#L\cdot \#L{-}n\rangle$$

$$=\quad \langle n\colon 0,..\#L\cdot\ 1 + \#L{-}n{-}1\rangle$$

$$\uparrow$$

$$\langle n\colon \#L\cdot \#L{-}n\rangle \;=\; \langle n\colon \#L\cdot 0\rangle$$

$$\downarrow$$

$$\#L{-}n{-}1 \;=\; \langle n\colon 0,..\#L{+}1\cdot \#L{-}n\rangle\ (n{+}1)$$

time specification $\langle L: [*rat]\cdot \#L\rangle$

$$\#L \;=\; \langle n: 0,..\#L+1\cdot \#L-n\rangle\, 0$$

$$\begin{aligned}
& \langle n: 0,..\#L+1\cdot \#L-n\rangle \\
= \quad & \langle n: 0,..\#L \;\;,\; \#L\cdot \#L-n\rangle \\
= \quad & \langle n: 0,..\#L\cdot \#L-n\rangle \mid \langle n: \#L\cdot \#L-n\rangle
\end{aligned}$$

$$\begin{aligned}
& \langle n: 0,..\#L\cdot \#L-n\rangle \\
= \quad & \langle n: 0,..\#L\cdot \#L-n\rangle
\end{aligned}$$

$\uparrow$

$$\langle n: \#L\cdot \#L-n\rangle \quad = \quad \langle n: \#L\cdot 0\rangle$$

$\downarrow$

$$\#L-n \quad = \quad 1 + \langle n: 0,..\#L+1\cdot \#L-n\rangle\,(n+1)$$

Function Refinement

Function Refinement

Specification S is unsatisfiable for domain element x : ¢ $S\,x < 1$

Function Refinement

Specification S is unsatisfiable for domain element x : $¢\, S\, x \ < \ 1$

Specification S is satisfiable for domain element x : $¢\, S\, x \ \geq \ 1$

Function Refinement

Specification S is unsatisfiable for domain element x : ¢ $S\,x < 1$

Specification S is satisfiable for domain element x : ¢ $S\,x \geq 1$

Specification S is deterministic for domain element x : ¢ $S\,x \leq 1$

Function Refinement

Specification S is unsatisfiable for domain element x : $¢\, S\, x < 1$

Specification S is satisfiable for domain element x : $¢\, S\, x \geq 1$

Specification S is deterministic for domain element x : $¢\, S\, x \leq 1$

Specification S is nondeterministic for domain element x : $¢\, S\, x > 1$

Function Refinement

Specification S is unsatisfiable for domain element x : $¢\, S\, x < 1$

Specification S is satisfiable for domain element x : $¢\, S\, x \geq 1$

Specification S is deterministic for domain element x : $¢\, S\, x \leq 1$

Specification S is nondeterministic for domain element x : $¢\, S\, x > 1$

Specification S is satisfiable for domain element x : $\exists y \cdot\ y: S\, x$

Function Refinement

Specification S is unsatisfiable for domain element x : $¢\, S\, x < 1$

Specification S is satisfiable for domain element x : $¢\, S\, x \geq 1$

Specification S is deterministic for domain element x : $¢\, S\, x \leq 1$

Specification S is nondeterministic for domain element x : $¢\, S\, x > 1$

Specification S is satisfiable for domain element x : $\exists y \cdot\ y{:}\ S\, x$

Specification S is implementable: $\forall x \cdot\ \exists y \cdot\ y{:}\ S\, x$

Function Refinement

Specification S is unsatisfiable for domain element x : ¢ $S\,x < 1$

Specification S is satisfiable for domain element x : ¢ $S\,x \geq 1$

Specification S is deterministic for domain element x : ¢ $S\,x \leq 1$

Specification S is nondeterministic for domain element x : ¢ $S\,x > 1$

Specification S is satisfiable for domain element x : $\exists y \cdot\ y\!: S\,x$

Specification S is implementable: $\forall x \cdot\ \exists y \cdot\ y\!: S\,x$

$\forall x \cdot\ S\,x \neq null$

Function Refinement

Specification S is unsatisfiable for domain element x : $\quad ¢\, S\, x < 1$

Specification S is satisfiable for domain element x : $\quad ¢\, S\, x \geq 1$

Specification S is deterministic for domain element x : $\quad ¢\, S\, x \leq 1$

Specification S is nondeterministic for domain element x : $\quad ¢\, S\, x > 1$

Specification S is satisfiable for domain element x : $\quad \exists y \cdot\ y{:}\ S\, x$

Specification S is implementable: $\quad \forall x \cdot\ \exists y \cdot\ y{:}\ S\, x$

$\forall x \cdot\ S\, x \neq null$

S is refined by P

Function Refinement

Specification S is unsatisfiable for domain element x : $\quad ¢\, S\, x < 1$

Specification S is satisfiable for domain element x : $\quad ¢\, S\, x \ge 1$

Specification S is deterministic for domain element x : $\quad ¢\, S\, x \le 1$

Specification S is nondeterministic for domain element x : $\quad ¢\, S\, x > 1$

Specification S is satisfiable for domain element x : $\quad \exists y \cdot\ y : S\, x$

Specification S is implementable: $\quad \forall x \cdot\ \exists y \cdot\ y : S\, x$

$\forall x \cdot\ S\, x \ne null$

S is refined by P $\quad P : S$

Function Refinement

Specification S is unsatisfiable for domain element x : $\quad ¢\, S\, x < 1$

Specification S is satisfiable for domain element x : $\quad ¢\, S\, x \geq 1$

Specification S is deterministic for domain element x : $\quad ¢\, S\, x \leq 1$

Specification S is nondeterministic for domain element x : $\quad ¢\, S\, x > 1$

Specification S is satisfiable for domain element x : $\quad \exists y \cdot\ y: S\, x$

Specification S is implementable: $\quad \forall x \cdot\ \exists y \cdot\ y: S\, x$

$\forall x \cdot\ S\, x \neq null$

S is refined by P $\quad P: S$

$S:: P$

example search for an item in a list

example search for an item in a list

$\langle L\text{: } [*int]\cdot \langle x\text{: } int\cdot \S n\text{: } 0,..\#L\cdot\ L\ n = x\ \rangle\rangle$

example search for an item in a list

$\langle L\text{: } [{*}int]\cdot\ \langle x\text{: } int\cdot\ \S n\text{: } 0,..\#L\cdot\ L\ n = x\ \rangle\rangle$ unimplementable

example search for an item in a list

$\langle L: [*int]\cdot \langle x: int\cdot \S n: 0,..\#L\cdot L\ n = x \rangle\rangle$ unimplementable

$\langle L: [*int]\cdot \langle x: int\cdot$ **if** $x: L\ (0,..\#L)$ **then** $\S n: 0,..\#L\cdot L\ n = x$ **else** $\#L,..\infty$ **fi**$\rangle\rangle$

example search for an item in a list

$\langle L: [*int]\cdot \langle x: int\cdot \S n: 0,..\#L\cdot L\ n = x \rangle\rangle$ unimplementable

$\langle L: [*int]\cdot \langle x: int\cdot$ **if** $x: L\ (0,..\#L)$ **then** $\S n: 0,..\#L\cdot L\ n = x$ **else** $\#L,..\infty$ **fi**$\rangle\rangle$

if $x: L\ (0,..\#L)$ **then** $\S n: 0,..\#L\cdot L\ n = x$ **else** $\#L,..\infty$ **fi** ::

example search for an item in a list

$\langle L: [*int]\cdot \langle x: int\cdot \S n: 0,..\#L\cdot L\ n = x \rangle\rangle$ unimplementable

$\langle L: [*int]\cdot \langle x: int\cdot$ **if** $x: L\ (0,..\#L)$ **then** $\S n: 0,..\#L\cdot L\ n = x$ **else** $\#L,..\infty$ **fi**$\rangle\rangle$

if $x: L\ (0,..\#L)$ **then** $\S n: 0,..\#L\cdot L\ n = x$ **else** $\#L,..\infty$ **fi** $::$

$\langle i: nat\cdot$ **if** $x: L\ (i,..\#L)$ **then** $\S n: i,..\#L\cdot L\ n = x$ **else** $\#L,..\infty$ **fi**$\rangle\ 0$

example search for an item in a list

$\langle L: [*int]\cdot \langle x: int\cdot \S n: 0,..\#L\cdot L\ n = x\ \rangle\rangle$ unimplementable

$\langle L: [*int]\cdot \langle x: int\cdot \textbf{if } x: L\ (0,..\#L) \textbf{ then } \S n: 0,..\#L\cdot L\ n = x \textbf{ else } \#L,..\infty \textbf{ fi}\rangle\rangle$

$\textbf{if } x: L\ (0,..\#L) \textbf{ then } \S n: 0,..\#L\cdot L\ n = x \textbf{ else } \#L,..\infty \textbf{ fi} ::$

$\quad\quad \langle i: nat\cdot \textbf{if } x: L\ (i,..\#L) \textbf{ then } \S n: i,..\#L\cdot L\ n = x \textbf{ else } \#L,..\infty \textbf{ fi}\rangle\ 0$

$\textbf{if } x: L\ (i,..\#L) \textbf{ then } \S n: i,..\#L\cdot L\ n = x \textbf{ else } \#L,..\infty \textbf{ fi} ::$

example search for an item in a list

$\langle L: [*int]\cdot \langle x: int\cdot \S n: 0,..\#L\cdot L\,n = x \rangle\rangle$ unimplementable

$\langle L: [*int]\cdot \langle x: int\cdot$ **if** $x: L\,(0,..\#L)$ **then** $\S n: 0,..\#L\cdot L\,n = x$ **else** $\#L,..\infty$ **fi**$\rangle\rangle$

if $x: L\,(0,..\#L)$ **then** $\S n: 0,..\#L\cdot L\,n = x$ **else** $\#L,..\infty$ **fi** ::

$\langle i: nat\cdot$ **if** $x: L\,(i,..\#L)$ **then** $\S n: i,..\#L\cdot L\,n = x$ **else** $\#L,..\infty$ **fi**$\rangle\ 0$

if $x: L\,(i,..\#L)$ **then** $\S n: i,..\#L\cdot L\,n = x$ **else** $\#L,..\infty$ **fi** ::

if $i = \#L$ **then** $\#L$

example search for an item in a list

$\langle L\text{: } [*int]\cdot \langle x\text{: } int\cdot\ \S n\text{: } 0,..\#L\cdot\ L\ n = x\ \rangle\rangle$ unimplementable

$\langle L\text{: } [*int]\cdot \langle x\text{: } int\cdot\ \mathbf{if}\ x\text{: } L\ (0,..\#L)\ \mathbf{then}\ \S n\text{: } 0,..\#L\cdot\ L\ n = x\ \mathbf{else}\ \#L,..\infty\ \mathbf{fi}\rangle\rangle$

$\mathbf{if}\ x\text{: } L\ (0,..\#L)\ \mathbf{then}\ \S n\text{: } 0,..\#L\cdot\ L\ n = x\ \mathbf{else}\ \#L,..\infty\ \mathbf{fi}\ ::$

$\qquad \langle i\text{: } nat\cdot\ \mathbf{if}\ x\text{: } L\ (i,..\#L)\ \mathbf{then}\ \S n\text{: } i,..\#L\cdot\ L\ n = x\ \mathbf{else}\ \#L,..\infty\ \mathbf{fi}\rangle\ 0$

$\mathbf{if}\ x\text{: } L\ (i,..\#L)\ \mathbf{then}\ \S n\text{: } i,..\#L\cdot\ L\ n = x\ \mathbf{else}\ \#L,..\infty\ \mathbf{fi}\ ::$

$\qquad \mathbf{if}\ i = \#L\ \mathbf{then}\ \#L$

$\qquad \mathbf{else\ if}\ x = L\ i\ \mathbf{then}\ i$

example search for an item in a list

$\langle L: [*int]\cdot \langle x: int\cdot \S n: 0,..\#L\cdot L\ n = x\ \rangle\rangle$ unimplementable

$\langle L: [*int]\cdot \langle x: int\cdot \mathbf{if}\ x: L\ (0,..\#L)\ \mathbf{then}\ \S n: 0,..\#L\cdot L\ n = x\ \mathbf{else}\ \#L,..\infty\ \mathbf{fi}\rangle\rangle$

$\mathbf{if}\ x: L\ (0,..\#L)\ \mathbf{then}\ \S n: 0,..\#L\cdot L\ n = x\ \mathbf{else}\ \#L,..\infty\ \mathbf{fi}\ ::$

$\quad\quad \langle i: nat\cdot \mathbf{if}\ x: L\ (i,..\#L)\ \mathbf{then}\ \S n: i,..\#L\cdot L\ n = x\ \mathbf{else}\ \#L,..\infty\ \mathbf{fi}\rangle\ 0$

$\mathbf{if}\ x: L\ (i,..\#L)\ \mathbf{then}\ \S n: i,..\#L\cdot L\ n = x\ \mathbf{else}\ \#L,..\infty\ \mathbf{fi}\ ::$

$\quad\quad \mathbf{if}\ i = \#L\ \mathbf{then}\ \#L$

$\quad\quad \mathbf{else\ if}\ x = L\ i\ \mathbf{then}\ i$

$\quad\quad\quad \mathbf{else}\ \langle i: nat\cdot \mathbf{if}\ x: L\ (i,..\#L)\ \mathbf{then}\ \S n: i,..\#L\cdot L\ n = x\ \mathbf{else}\ \#L,..\infty\ \mathbf{fi}\rangle\ (i{+}1)\ \mathbf{fi}\ \mathbf{fi}$

recursive timing $\langle L\cdot \langle x\cdot 0,..\#L+1\rangle\rangle$

recursive timing $\langle L\cdot \langle x\cdot 0,..\#L+1\rangle\rangle$

$$0,..\#L+1 \;::\; \langle i\cdot 0,..\#L-i+1\rangle\, 0$$

recursive timing $\langle L\cdot \langle x\cdot 0,..\#L+1\rangle\rangle$

$0,..\#L+1 :: \langle i\cdot 0,..\#L-i+1\rangle\, 0$

$0,..\#L-i+1 ::$

recursive timing $\langle L\cdot \langle x\cdot 0,..\#L+1\rangle\rangle$

$0,..\#L+1 ::\ \ \langle i\cdot 0,..\#L{-}i+1\rangle\, 0$

$0,..\#L{-}i+1 ::$ **if** $i = \#L$ **then** 0

recursive timing $\langle L\cdot \langle x\cdot 0,..\#L+1\rangle\rangle$

$0,..\#L+1$ :: $\langle i\cdot 0,..\#L\text{–}i+1\rangle\ 0$

$0,..\#L\text{–}i+1$ :: **if** $i = \#L$ **then** 0

else if $x = L\ i$ **then** 0

recursive timing $\langle L\cdot \langle x\cdot 0,..\#L+1\rangle\rangle$

$0,..\#L+1 \;::\; \langle i\cdot 0,..\#L-i+1\rangle\, 0$

$0,..\#L-i+1 \;::\;$ **if** $i = \#L$ **then** 0

else if $x = L\ i$ **then** 0

else $1 + \langle i\cdot 0,..\#L-i+1\rangle\,(i+1)$ **fi fi**

recursive timing $\langle L\cdot \langle x\cdot 0,..\#L+1\rangle\rangle$

$0,..\#L+1 \;::\; \langle i\cdot 0,..\#L-i+1\rangle\, 0$

$0,..\#L-i+1 \;::\;$ **if** $i = \#L$ **then** 0

else if $x = L\ i$ **then** 0

else $1 + \langle i\cdot 0,..\#L-i+1\rangle\, (i+1)$ **fi fi**

$1 + \langle i\cdot 0,..\#L-i+1\rangle\, (i+1)$

recursive timing $\langle L\cdot \langle x\cdot 0,..\#L+1\rangle\rangle$

$0,..\#L+1 \;::\; \langle i\cdot 0,..\#L-i+1\rangle\, 0$

$0,..\#L-i+1 \;::\;$ **if** $i = \#L$ **then** 0

else if $x = L\, i$ **then** 0

else $1 + \langle i\cdot 0,..\#L-i+1\rangle\, (i+1)$ **fi fi**

$1 + \langle i\cdot 0,..\#L-i+1\rangle\, (i+1)$

$= \quad 1 + (0,..\#L-(i+1)+1)$

recursive timing $\langle L\cdot \langle x\cdot 0,..\#L{+}1\rangle\rangle$

$0,..\#L{+}1 \;::\; \langle i\cdot 0,..\#L{-}i{+}1\rangle\, 0$

$0,..\#L{-}i{+}1 \;::\;$ **if** $i = \#L$ **then** 0

else if $x = L\ i$ **then** 0

else $1 + \langle i\cdot 0,..\#L{-}i{+}1\rangle\,(i{+}1)$ **fi fi**

$1 + \langle i\cdot 0,..\#L{-}i{+}1\rangle\,(i{+}1)$

$= \quad 1 + (0,..\#L{-}(i{+}1){+}1)$

$= \quad 1 + (0,..\#L{-}i)$

recursive timing $\langle L\cdot \langle x\cdot 0,..\#L+1\rangle\rangle$

$0,..\#L+1 :: \quad \langle i\cdot 0,..\#L-i+1\rangle\ 0$

$0,..\#L-i+1 :: \quad$ **if** $i = \#L$ **then** 0

else if $x = L\ i$ **then** 0

else $1 + \langle i\cdot 0,..\#L-i+1\rangle\ (i+1)$ **fi fi**

$$1 + \langle i\cdot 0,..\#L-i+1\rangle\ (i+1)$$

$$= \quad 1 + (0,..\#L-(i+1)+1)$$

$$= \quad 1 + (0,..\#L-i)$$

$$= \quad 1,..\#L-i+1$$

recursive timing $\langle L\cdot \langle x\cdot 0,..\#L+1\rangle\rangle$

$0,..\#L+1 :: \quad \langle i\cdot 0,..\#L-i+1\rangle\, 0$

$0,..\#L-i+1 :: \quad$ **if** $i = \#L$ **then** 0
else if $x = L\ i$ **then** 0
else $1 + \langle i\cdot 0,..\#L-i+1\rangle\ (i+1)$ **fi fi**

$$1 + \langle i\cdot 0,..\#L-i+1\rangle\ (i+1)$$
$$= \quad 1 + (0,..\#L-(i+1)+1)$$
$$= \quad 1 + (0,..\#L-i)$$
$$= \quad 1,..\#L-i+1$$
$$: \quad 0,..\#L-i+1$$

functional versus imperative

functional versus imperative

same programming steps, different notation

functional versus imperative

same programming steps, different notation

functional programming has Application Axiom

$$\langle v\colon D\cdot\ b\rangle\ x \ = \ (\text{for } v \text{ substitute } x \text{ in } b)$$

functional versus imperative

same programming steps, different notation

functional programming has Application Axiom

$$\langle v{:}\, D\cdot\, b\rangle\, x \;=\; (\text{for } v \text{ substitute } x \text{ in } b)$$

imperative programming has Substitution Law

$$x{:=}\, e.\, P \;=\; (\text{for } x \text{ substitute } e \text{ in } P)$$