Sequential to Concurrent Transformation

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$x := y.\ \ x := x+1.\ \ z := y$

Sequential to Concurrent Transformation

$x := y.\ \ x := x + 1.\ \ z := y$

$\text{start} \longrightarrow x := y \longrightarrow x := x + 1 \longrightarrow z := y \longrightarrow \text{finish}$

Sequential to Concurrent Transformation

$$x := y.\ \ x := x+1.\ \ z := y$$

$$= \quad x := y.\ \ (x := x+1 \ \| \ z := y)$$

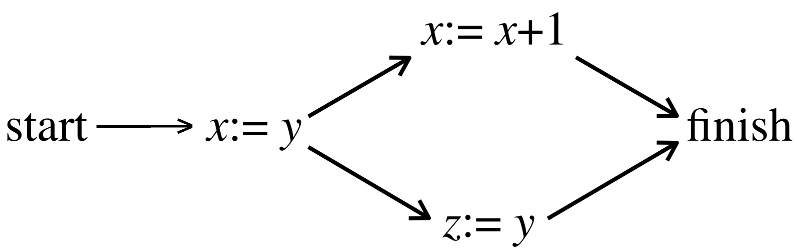
$$\text{start} \longrightarrow x := y \longrightarrow x := x+1 \longrightarrow z := y \longrightarrow \text{finish}$$

Sequential to Concurrent Transformation

$$x := y.\ \ x := x+1.\ \ z := y$$

$$=\qquad x := y.\ \ (x := x+1 \parallel z := y)$$

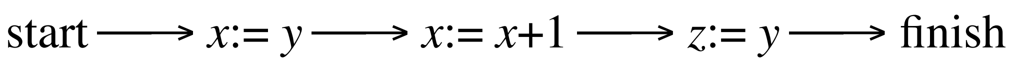
start $\longrightarrow$ $x := y$ $\longrightarrow$ $x := x+1$ $\longrightarrow$ $z := y$ $\longrightarrow$ finish

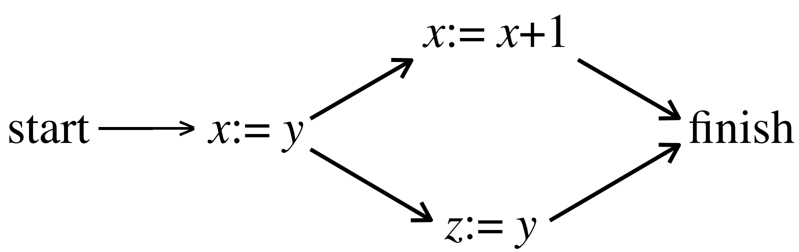


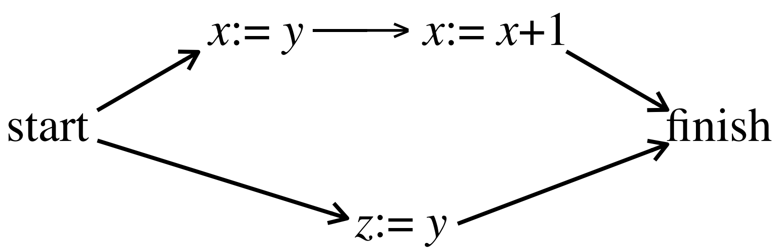
Sequential to Concurrent Transformation

$$x := y.\ x := x+1.\ z := y$$

$$= \quad x := y.\ (x := x+1 \parallel z := y)$$





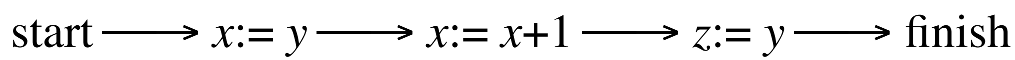


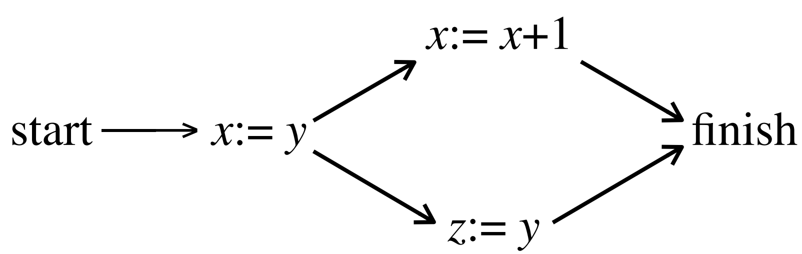
Sequential to Concurrent Transformation

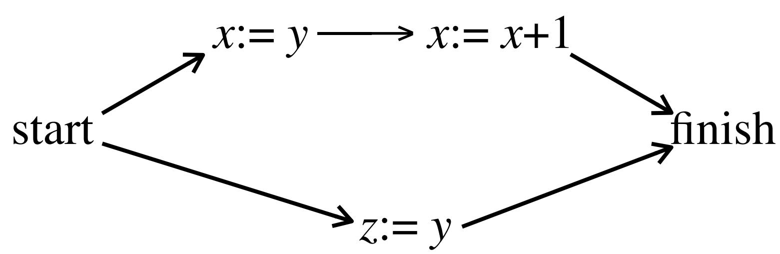
$$x := y.\ \ x := x+1.\ \ z := y$$

$$= \quad x := y.\ \ (x := x+1 \ \| \ z := y)$$

$$= \quad (x := y.\ \ x := x+1) \ \| \ z := y$$







Sequential to Concurrent Transformation

rules

Sequential to Concurrent Transformation

rules

Whenever two programs occur in sequence, and neither assigns to a variable appearing in the other, they can be placed in parallel.

example $x := z.\ y := z$ becomes $x := z \parallel y := z$

Sequential to Concurrent Transformation

rules

Whenever two programs occur in sequence, and neither assigns to a variable appearing in the other, they can be placed in parallel.

example $x:=z.\ y:=z$ becomes $x:=z \parallel y:=z$

Whenever two programs occur in sequence, and neither assigns to a variable assigned in the other, and no variable assigned in the first appears in the second, they can be placed in parallel.

example $x:=y.\ y:=z$ becomes $x:=y \parallel y:=z$

Sequential to Concurrent Transformation

rules

Whenever two programs occur in sequence, and neither assigns to a variable appearing in the other, they can be placed in parallel.

example $x:= z.\ y:= z$ becomes $x:= z \parallel y:= z$

Whenever two programs occur in sequence, and neither assigns to a variable assigned in the other, and no variable assigned in the first appears in the second, they can be placed in parallel; a copy must be made of the initial value of any variable appearing in the first and assigned in the second.

example $x:= y.\ y:= z$ becomes $c:= y.\ (x:= c \parallel y:= z)$

Buffer

$$produce = \cdots\cdots b := e \cdots\cdots$$

$$consume = \cdots\cdots x := b \cdots\cdots$$

Buffer

$$produce = \cdots\cdots\cdots b := e \cdots\cdots\cdots$$

$$consume = \cdots\cdots\cdots x := b \cdots\cdots\cdots$$

$$control = produce.\ consume.\ control$$

Buffer

$$produce = \cdots\cdots\cdots b := e \cdots\cdots\cdots$$

$$consume = \cdots\cdots\cdots x := b \cdots\cdots\cdots$$

$$control = produce.\ consume.\ control$$

$$P \longrightarrow C \longrightarrow P \longrightarrow C \longrightarrow P \longrightarrow C \longrightarrow P \longrightarrow C \longrightarrow$$

Buffer

$$produce = \cdots\cdots\cdots b := e \cdots\cdots\cdots$$

$$consume = \cdots\cdots\cdots x := b \cdots\cdots\cdots$$

$$control = produce\,.\; consume\,.\; control$$

Buffer

$$produce = \cdots\cdots\cdots b := e \cdots\cdots\cdots$$

$$consume = \cdots\cdots\cdots x := b \cdots\cdots\cdots$$

$$control = produce.\ newcontrol$$

$$newcontrol = consume.\ produce.\ newcontrol$$

Buffer

$$produce \;=\; \cdots\cdots\cdots b := e \cdots\cdots\cdots$$

$$consume \;=\; \cdots\cdots\cdots x := b \cdots\cdots\cdots$$

$$control \;=\; produce.\ newcontrol$$

$$newcontrol \;=\; (consume \parallel produce).\ newcontrol$$

Buffer

$$produce = \cdots\cdots b := e \cdots\cdots$$

$$consume = \cdots\cdots x := c \cdots\cdots$$

$$control = produce.\ newcontrol$$

$$newcontrol = c := b.\ (consume \parallel produce).\ newcontrol$$

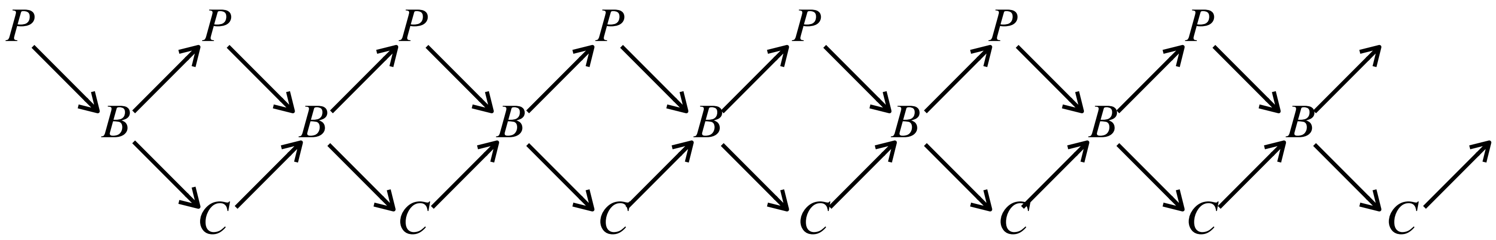
Buffer

$$produce = \cdots\cdots\cdots b := e \cdots\cdots\cdots$$

$$consume = \cdots\cdots\cdots x := c \cdots\cdots\cdots$$

$$control = produce.\ newcontrol$$

$$newcontrol = c := b.\ (consume \parallel produce).\ newcontrol$$



Buffer

$produce = \cdots\cdots b\,w := e.\ w := w+1 \cdots\cdots$

$consume = \cdots\cdots x := b\,r.\ r := r+1 \cdots\cdots$

$control = w := 0.\ r := 0.\ newcontrol$

$newcontrol = produce.\ consume.\ newcontrol$

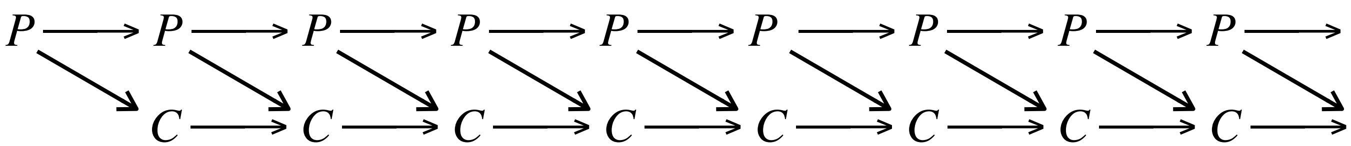
Buffer

$produce \;=\; \cdots\cdots\cdots b\,w := e.\;\; w := w+1 \cdots\cdots\cdots$

$consume \;=\; \cdots\cdots\cdots x := b\,r.\;\; r := r+1 \cdots\cdots\cdots$

$control \;=\; w := 0.\;\; r := 0.\;\; newcontrol$

$newcontrol \;=\; produce.\;\; consume.\;\; newcontrol$



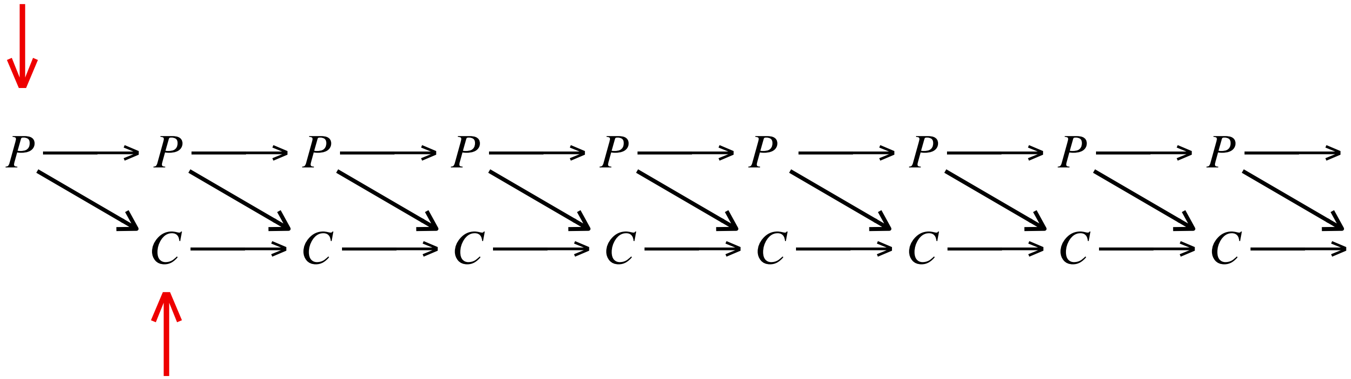
Buffer

$$produce \;=\; \cdots\cdots\cdots b\ w := e.\ \ w := w+1 \cdots\cdots\cdots$$

$$consume \;=\; \cdots\cdots\cdots x := b\ r.\ \ r := r+1 \cdots\cdots\cdots$$

$$control \;=\; w := 0.\ \ r := 0.\ \ newcontrol$$

$$newcontrol \;=\; produce.\ \ consume.\ \ newcontrol$$



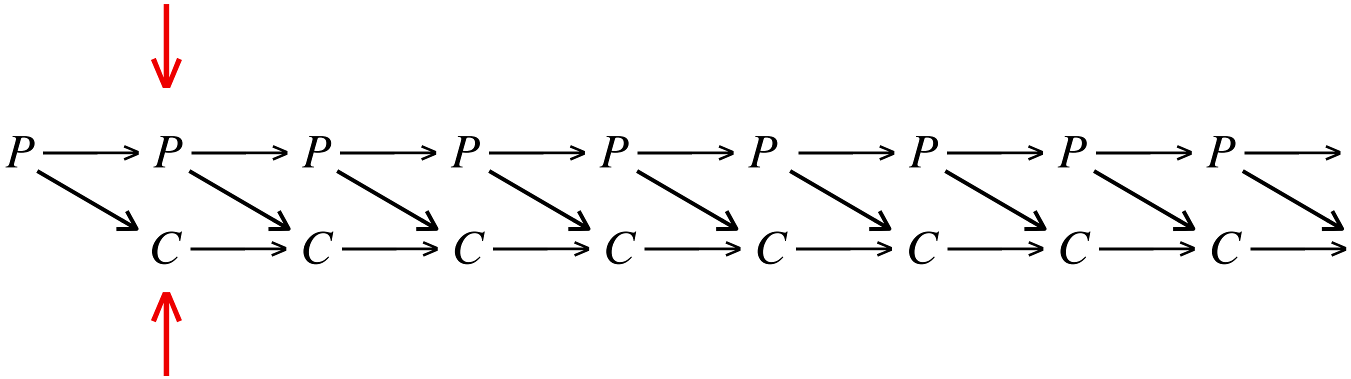
Buffer

$$produce \;=\; \cdots\cdots b\, w := e.\; w := w+1 \cdots\cdots$$

$$consume \;=\; \cdots\cdots x := b\, r.\; r := r+1 \cdots\cdots$$

$$control \;=\; w := 0.\; r := 0.\; newcontrol$$

$$newcontrol \;=\; produce.\; consume.\; newcontrol$$



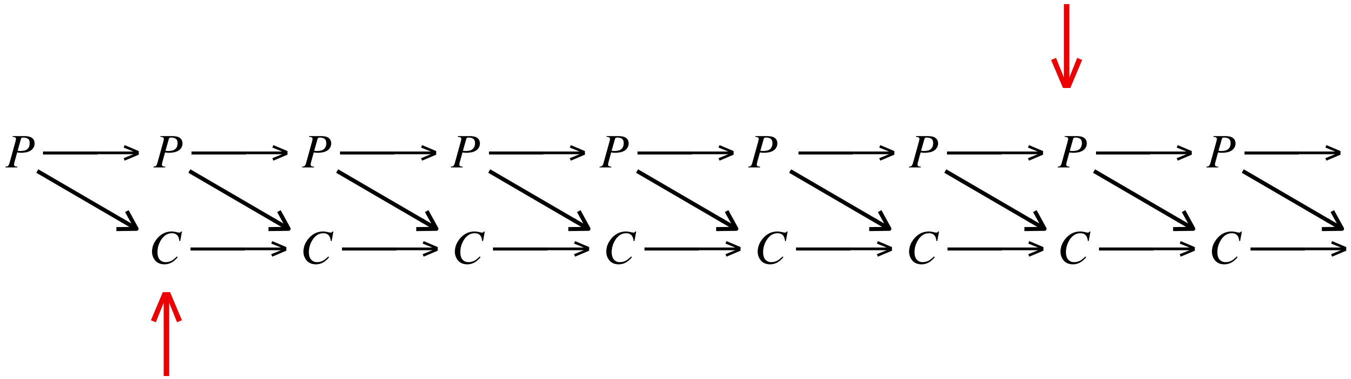
Buffer

$$produce \;=\; \cdots\cdots\cdots b\;w := e.\;\; w := w+1\cdots\cdots\cdots$$

$$consume \;=\; \cdots\cdots\cdots x := b\;r.\;\; r := r+1\cdots\cdots\cdots$$

$$control \;=\; w := 0.\;\; r := 0.\;\; newcontrol$$

$$newcontrol \;=\; produce.\;\; consume.\;\; newcontrol$$



Buffer

$$produce = \cdots\cdots b\ w := e.\ w := mod\ (w+1)\ n \cdots\cdots$$

$$consume = \cdots\cdots x := b\ r.\ r := mod\ (r+1)\ n \cdots\cdots$$

$$control = w := 0.\ r := 0.\ newcontrol$$

$$newcontrol = produce.\ consume.\ newcontrol$$

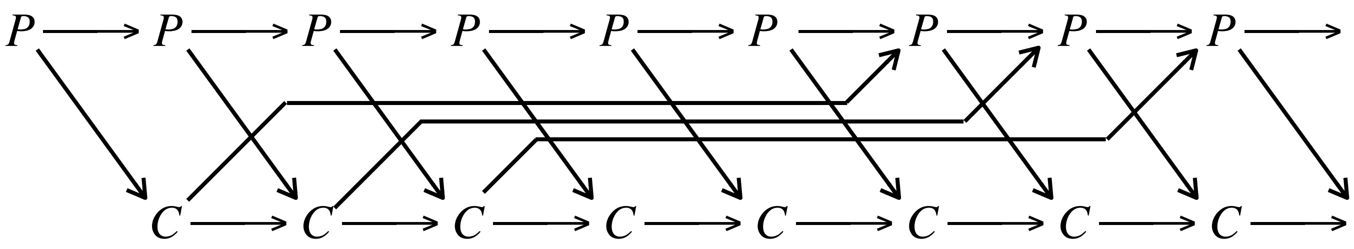
Buffer

$$produce \;=\; \cdots\cdots\, b\; w{:=}\; e.\;\; w{:=}\; mod\;(w{+}1)\; n\cdots\cdots$$

$$consume \;=\; \cdots\cdots\, x{:=}\; b\; r.\;\; r{:=}\; mod\;(r{+}1)\; n\cdots\cdots$$

$$control \;=\; w{:=}\,0.\;\; r{:=}\,0.\;\; newcontrol$$

$$newcontrol \;=\; produce.\;\; consume.\;\; newcontrol$$



Insertion Sort

define

$$sort \;=\; \langle n\cdot\ \forall i, j: 0,..n\cdot\ i{\le}j \Rightarrow L\,i \le L\,j\rangle$$

Insertion Sort

define

$$sort \;=\; \langle n \cdot\; \forall i, j: 0,..n \cdot\; i \le j \Rightarrow L\, i \le L\, j \rangle$$

$$sort'\,(\#L)$$

Insertion Sort

define

$$sort \;=\; \langle n \cdot\; \forall i, j{:}\, 0,..n \cdot\; i \le j \Rightarrow L\, i \le L\, j \rangle$$

$$sort' \,(\#L) \quad \Leftarrow \quad sort\, 0 \Rightarrow sort' \,(\#L)$$

Insertion Sort

define

$$sort \;=\; \langle n\cdot\; \forall i, j: 0,..n\cdot\; i{\le}j \Rightarrow L\,i \le L\,j\rangle$$

$$sort'\;(\#L) \quad\Leftarrow\quad sort\;0 \Rightarrow sort'\;(\#L)$$

$$sort\;0 \Rightarrow sort'\;(\#L) \quad\Leftarrow\quad \textbf{for}\; n{:=}\,0;..\#L\; \textbf{do}\; sort\;n \Rightarrow sort'\;(n{+}1)\;\textbf{od}$$

Insertion Sort

define

$$sort \;=\; \langle n\cdot\; \forall i, j\!: 0,..n\cdot\; i{\le}j \Rightarrow L\,i \le L\,j\rangle$$

$$sort'\,(\#L) \quad\Leftarrow\quad sort\,0 \Rightarrow sort'\,(\#L)$$

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$$sort\,n \Rightarrow sort'\,(n{+}1) \quad\Leftarrow$$

Insertion Sort

define

$$sort \;=\; \langle n\cdot\ \forall i, j{:}\ 0,..n\cdot\ i{\le}j \Rightarrow L\,i \le L\,j\rangle$$

$$sort'\ (\#L) \quad\Leftarrow\quad sort\ 0 \Rightarrow sort'\ (\#L)$$

$$sort\ 0 \Rightarrow sort'\ (\#L) \quad\Leftarrow\quad \textbf{for}\ n{:=}\ 0;..\#L\ \textbf{do}\ sort\ n \Rightarrow sort'\ (n{+}1)\ \textbf{od}$$

$$sort\ n \Rightarrow sort'\ (n{+}1) \quad\Leftarrow$$

$$[\underset{0}{}\ L\,0\ \underset{1}{;}\ L\,1\ \underset{2}{;}\ L\,2\ \underset{3}{;}\ L\,3\ \underset{4}{;}\ L\,4\ \underset{5}{]}$$

Insertion Sort

define

$sort \;=\; \langle n\cdot\ \forall i, j: 0,..n\cdot\ i{\le}j \Rightarrow L\,i \le L\,j\rangle$

$sort'\,(\#L) \quad \Leftarrow \quad sort\,0 \Rightarrow sort'\,(\#L)$

$sort\,0 \Rightarrow sort'\,(\#L) \quad \Leftarrow \quad$ **for** $n{:=}\,0;..\#L$ **do** $sort\,n \Rightarrow sort'\,(n{+}1)$ **od**

$sort\,n \Rightarrow sort'\,(n{+}1) \quad \Leftarrow$

if $n{=}0$ **then**

$[\,L\,0 \;;L\,1 \;;L\,2 \;;L\,3 \;;L\,4\;]$

0 1 2 3 4 5

Insertion Sort

define

$$sort \;=\; \langle n\cdot\ \forall i, j\colon 0,..n\cdot\ i{\le}j \Rightarrow L\,i \le L\,j\rangle$$

$$sort'\,(\#L) \quad\Leftarrow\quad sort\,0 \Rightarrow sort'\,(\#L)$$

$$sort\,0 \Rightarrow sort'\,(\#L) \quad\Leftarrow\quad \textbf{for}\ n{:=}\,0;..\#L\ \textbf{do}\ sort\,n \Rightarrow sort'\,(n{+}1)\ \textbf{od}$$

$$sort\,n \Rightarrow sort'\,(n{+}1) \quad\Leftarrow$$

$$\textbf{if}\ n{=}0\ \textbf{then}$$

$$\underset{0}{[}\ L\,0\ \underset{1}{;}\ L\,1\ \underset{2}{;}\ L\,2\ \underset{3}{;}\ L\,3\ \underset{4}{;}\ L\,4\ \underset{5}{]}$$

Insertion Sort

define

$$sort \;=\; \langle n\cdot\ \forall i, j{:}\ 0,..n\cdot\ i{\le}j \Rightarrow L\,i \le L\,j\rangle$$

$$sort'\ (\#L) \quad \Leftarrow \quad sort\ 0 \Rightarrow sort'\ (\#L)$$

$$sort\ 0 \Rightarrow sort'\ (\#L) \quad \Leftarrow \quad \textbf{for}\ n{:=}\ 0;..\#L\ \textbf{do}\ sort\ n \Rightarrow sort'\ (n{+}1)\ \textbf{od}$$

$$sort\ n \Rightarrow sort'\ (n{+}1) \quad \Leftarrow$$

if $n{=}0$ **then**

↓

$$[\ L\,0\ \ ;L\,1\ \ ;L\,2\ \ ;L\,3\ \ ;L\,4\ \ ]$$

0 1 2 3 4 5

Insertion Sort

define

$$sort \;=\; \langle n\cdot\ \forall i, j: 0,..n\cdot\ i{\le}j \Rightarrow L\,i \le L\,j\rangle$$

$$sort'\,(\#L) \quad\Leftarrow\quad sort\,0 \Rightarrow sort'\,(\#L)$$

$$sort\,0 \Rightarrow sort'\,(\#L) \quad\Leftarrow\quad \textbf{for}\ n{:=}\,0;..\#L\ \textbf{do}\ sort\,n \Rightarrow sort'\,(n{+}1)\ \textbf{od}$$

$$sort\,n \Rightarrow sort'\,(n{+}1) \quad\Leftarrow$$

$$\qquad\qquad \textbf{if}\ n{=}0\ \textbf{then}\ ok$$

$$[\ L\,0\ ;\ L\,1\ ;\ L\,2\ ;\ L\,3\ ;\ L\,4\ ]$$

$$0 \qquad 1 \qquad 2 \qquad 3 \qquad 4 \qquad 5$$

Insertion Sort

define

$$sort \;=\; \langle n\cdot\ \forall i,j{:}\ 0,..n\cdot\ i{\le}j \Rightarrow L\,i \le L\,j\rangle$$

$$sort'\,(\#L) \quad\Leftarrow\quad sort\,0 \Rightarrow sort'\,(\#L)$$

$$sort\,0 \Rightarrow sort'\,(\#L) \quad\Leftarrow\quad \textbf{for}\ n{:=}\ 0;..\#L\ \textbf{do}\ sort\,n \Rightarrow sort'\,(n{+}1)\ \textbf{od}$$

$$sort\,n \Rightarrow sort'\,(n{+}1) \quad\Leftarrow$$

if $n{=}0$ **then** ok

else

$$[\,L\,0\ \ ;L\,1\ \ ;L\,2\ \ ;L\,3\ \ ;L\,4\ \ ]$$

0 1 2 3 4 5

Insertion Sort

define

$$sort \;=\; \langle n\cdot\; \forall i, j\colon 0,..n\cdot\; i{\le}j \Rightarrow L\,i \le L\,j \rangle$$

$$sort'\,(\#L) \quad\Leftarrow\quad sort\,0 \Rightarrow sort'\,(\#L)$$

$$sort\,0 \Rightarrow sort'\,(\#L) \quad\Leftarrow\quad \textbf{for}\ n{:=}\ 0;..\#L\ \textbf{do}\ sort\,n \Rightarrow sort'\,(n{+}1)\ \textbf{od}$$

$$sort\,n \Rightarrow sort'\,(n{+}1) \quad\Leftarrow$$

$$\textbf{if}\ n{=}0\ \textbf{then}\ ok$$

$$\textbf{else if}\ L\,(n{-}1) \le L\,n\ \textbf{then}\ ok$$

$$\underset{0}{[}\ L\,0\ \underset{1}{;}\ L\,1\ \underset{2}{;}\ L\,2\ \underset{3}{;}\ L\,3\ \underset{4}{;}\ L\,4\ \underset{5}{]}$$

Insertion Sort

define

$$sort \;=\; \langle n\cdot\ \forall i, j: 0,..n\cdot\ i{\le}j \Rightarrow L\,i \le L\,j\rangle$$

$$sort'\,(\#L) \quad \Leftarrow \quad sort\,0 \Rightarrow sort'\,(\#L)$$

$$sort\,0 \Rightarrow sort'\,(\#L) \quad \Leftarrow \quad \textbf{for } n{:=}\,0;..\#L \textbf{ do } sort\,n \Rightarrow sort'\,(n{+}1) \textbf{ od}$$

$$sort\,n \Rightarrow sort'\,(n{+}1) \quad \Leftarrow$$

if $n{=}0$ **then** *ok*

else if $L\,(n{-}1) \le L\,n$ **then** *ok*

else

↓

$$\underset{0}{[}\ L\,0\ \underset{1}{;}\ L\,1\ \underset{2}{;}\ L\,2\ \underset{3}{;}\ L\,3\ \underset{4}{;}\ L\,4\ \underset{5}{]}$$

Insertion Sort

define

$$sort \;=\; \langle n\cdot\; \forall i, j{:}\, 0,..n\cdot\; i{\le}j \Rightarrow L\,i \le L\,j\rangle$$

$$sort'\,(\#L) \;\Leftarrow\; sort\,0 \Rightarrow sort'\,(\#L)$$

$$sort\,0 \Rightarrow sort'\,(\#L) \;\Leftarrow\; \textbf{for } n{:=}\,0;..\#L \textbf{ do } sort\,n \Rightarrow sort'\,(n{+}1) \textbf{ od}$$

$$sort\,n \Rightarrow sort'\,(n{+}1) \;\Leftarrow$$

$$\textbf{if } n{=}0 \textbf{ then } ok$$

$$\textbf{else if } L\,(n{-}1) \le L\,n \textbf{ then } ok$$

$$\textbf{else } swap\,(n{-}1)\,n.$$

$$[\,L\,0\;;L\,1\;;L\,2\;;L\,3\;;L\,4\,]$$

0 1 2 3 4 5

Insertion Sort

define

$$sort \;=\; \langle n\cdot\ \forall i,j{:}\ 0,..n\cdot\ i{\le}j \Rightarrow L\,i \le L\,j\rangle$$

$$swap \;=\; \langle i,j{:}\ 0,..\#L\cdot\ \ L\,i{:=}\ L\,j \parallel L\,j{:=}\ L\,i\rangle$$

$$sort'\ (\#L) \quad\Leftarrow\quad sort\ 0 \Rightarrow sort'\ (\#L)$$

$$sort\ 0 \Rightarrow sort'\ (\#L) \quad\Leftarrow\quad \mathbf{for}\ n{:=}\ 0;..\#L\ \mathbf{do}\ sort\ n \Rightarrow sort'\ (n+1)\ \mathbf{od}$$

$$sort\ n \Rightarrow sort'\ (n+1) \quad\Leftarrow$$

$$\mathbf{if}\ n{=}0\ \mathbf{then}\ ok$$

$$\mathbf{else\ if}\ L\,(n-1) \le L\,n\ \mathbf{then}\ ok$$

$$\mathbf{else}\ swap\ (n-1)\ n.$$

$$\underset{0}{[}\ L\,0\ \ \underset{1}{;}\ L\,1\ \ \underset{2}{;}\ L\,2\ \ \underset{3}{;}\ L\,3\ \ \underset{4}{;}\ L\,4\ \ \underset{5}{]}$$

Insertion Sort

define

$$sort \;=\; \langle n\cdot\ \forall i,j\colon 0,..n\cdot\ i{\le}j \Rightarrow L\,i \le L\,j\rangle$$

$$swap \;=\; \langle i,j\colon 0,..\#L\cdot\ L\,i{:=}\,L\,j \parallel L\,j{:=}\,L\,i\rangle$$

$$sort'\,(\#L) \quad\Leftarrow\quad sort\,0 \Rightarrow sort'\,(\#L)$$

$$sort\,0 \Rightarrow sort'\,(\#L) \quad\Leftarrow\quad \textbf{for } n{:=}\,0;..\#L \textbf{ do } sort\,n \Rightarrow sort'\,(n{+}1) \textbf{ od}$$

$$sort\,n \Rightarrow sort'\,(n{+}1) \quad\Leftarrow$$

$$\textbf{if } n{=}0 \textbf{ then } ok$$

$$\textbf{else if } L\,(n{-}1) \le L\,n \textbf{ then } ok$$

$$\textbf{else } swap\,(n{-}1)\,n.\ \ sort\,(n{-}1) \Rightarrow sort'\,n \textbf{ fi fi}$$

[L 0 ; L 1 ; L 2 ; L 3 ; L 4]

0 1 2 3 4 5

Insertion Sort

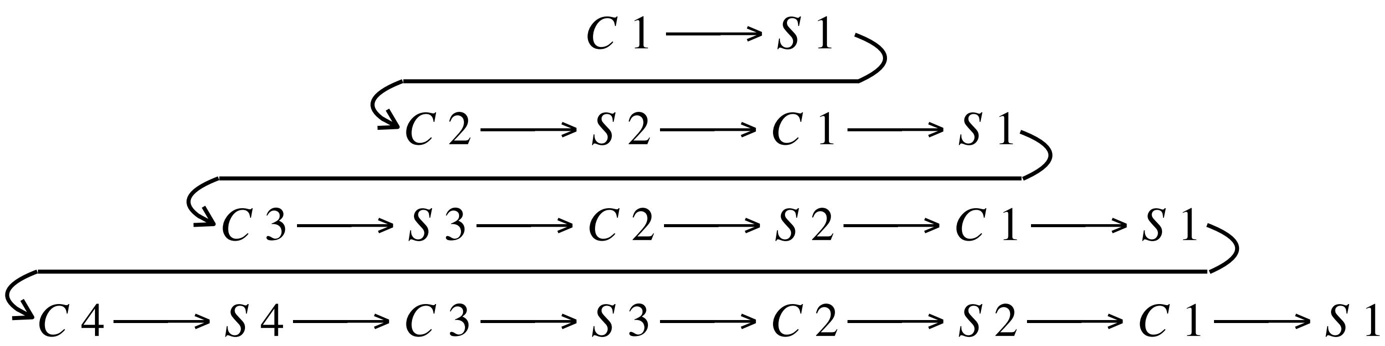
$$C\,1 \longrightarrow S\,1$$

$$C\,2 \longrightarrow S\,2 \longrightarrow C\,1 \longrightarrow S\,1$$

$$C\,3 \longrightarrow S\,3 \longrightarrow C\,2 \longrightarrow S\,2 \longrightarrow C\,1 \longrightarrow S\,1$$

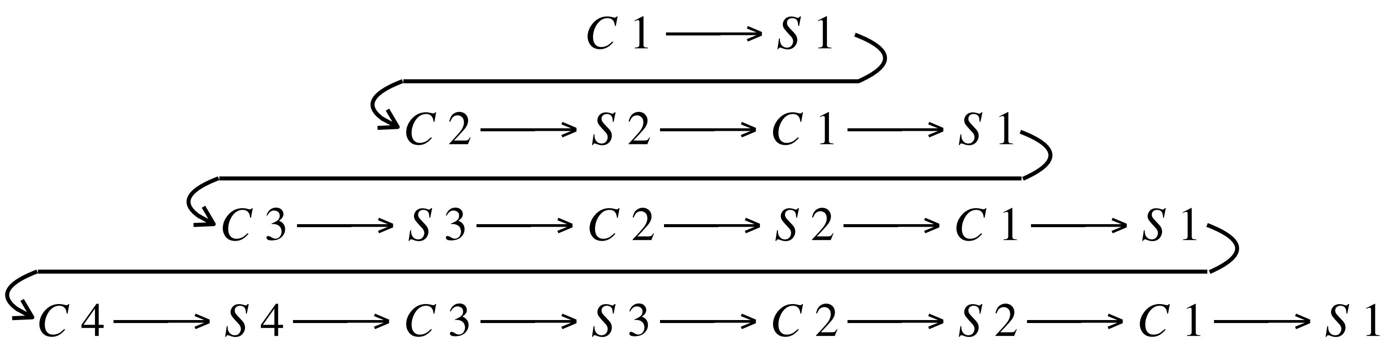
$$C\,4 \longrightarrow S\,4 \longrightarrow C\,3 \longrightarrow S\,3 \longrightarrow C\,2 \longrightarrow S\,2 \longrightarrow C\,1 \longrightarrow S\,1$$

Insertion Sort



If *abs* ($i-j$) > 1 then $S\,i$ and $S\,j$ in parallel

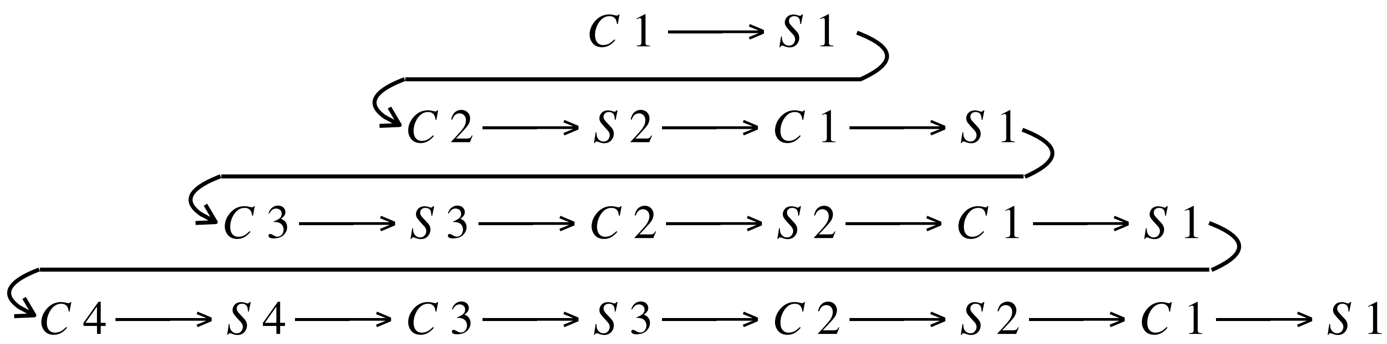
Insertion Sort



If *abs* $(i–j) > 1$ then $S\,i$ and $S\,j$ in parallel

If *abs* $(i–j) > 1$ then $S\,i$ and $C\,j$ in parallel

Insertion Sort

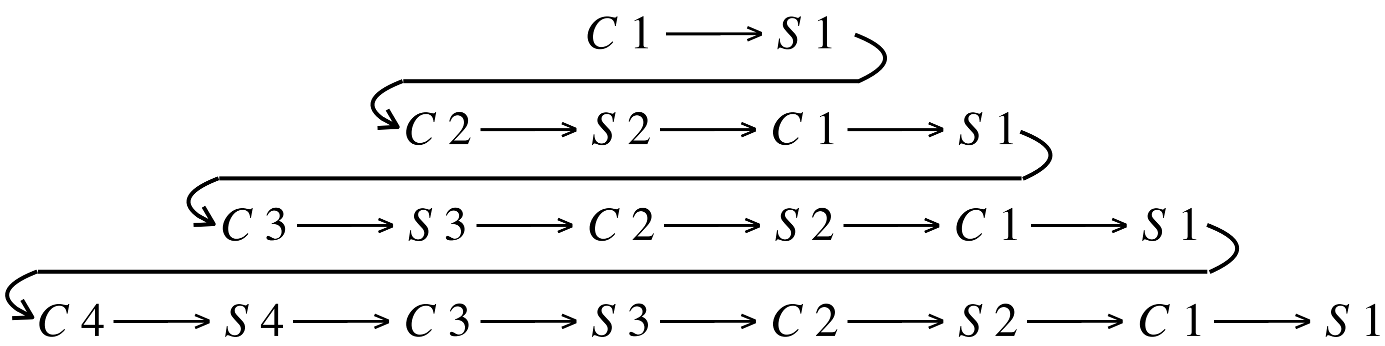


If *abs* $(i-j) > 1$ then $S\ i$ and $S\ j$ in parallel

If *abs* $(i-j) > 1$ then $S\ i$ and $C\ j$ in parallel

$C\ i$ and $C\ j$ in parallel

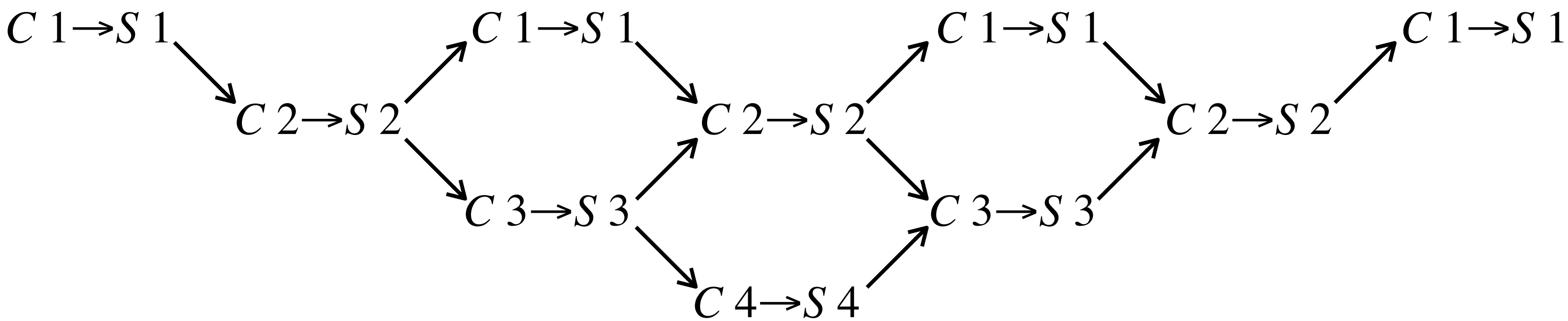
Insertion Sort



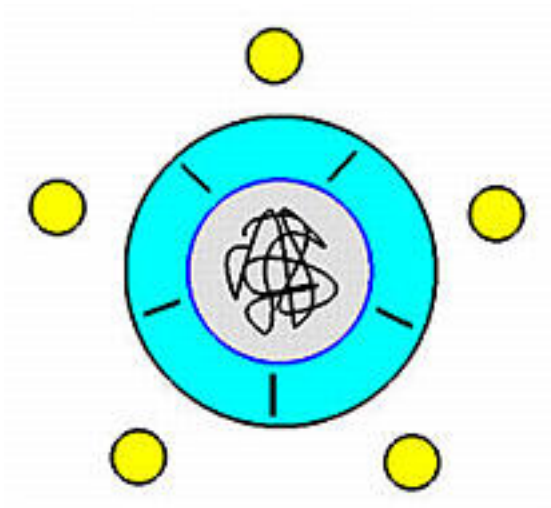
If *abs* (*i*–*j*) > 1 then *S i* and *S j* in parallel

If *abs* (*i*–*j*) > 1 then *S i* and *C j* in parallel

C i and *C j* in parallel



Dining Philosophers



Dining Philosophers

$$
\begin{aligned}
\mathit{life} \quad &= \quad (P\,0 \vee P\,1 \vee P\,2 \vee P\,3 \vee P\,4).\ \mathit{life} \\
P\,i \quad &= \quad \mathit{up}\,i.\ \mathit{up}(i{+}1).\ \mathit{eat}\,i.\ \mathit{down}\,i.\ \mathit{down}(i{+}1) \\
\mathit{up}\,i \quad &= \quad \mathit{chopstick}\,i := \top \\
\mathit{down}\,i \quad &= \quad \mathit{chopstick}\,i := \bot \\
\mathit{eat}\,i \quad &= \quad \cdots\cdots \mathit{chopstick}\,i \cdots\cdots \mathit{chopstick}(i{+}1) \cdots\cdots
\end{aligned}
$$

Dining Philosophers

$$life \;=\; (P\,0 \lor P\,1 \lor P\,2 \lor P\,3 \lor P\,4).\; life \quad \longleftarrow$$

$$P\,i \;=\; up\,i.\; up(i{+}1).\; eat\,i.\; down\,i.\; down(i{+}1)$$

$$up\,i \;=\; chopstick\,i := \top$$

$$down\,i \;=\; chopstick\,i := \bot$$

$$eat\,i \;=\; \cdots\cdots chopstick\,i \cdots\cdots chopstick(i{+}1)\cdots\cdots$$

Dining Philosophers

$$life \quad = \quad (P\,0 \vee P\,1 \vee P\,2 \vee P\,3 \vee P\,4).\ life$$

$$P\,i \quad = \quad up\ i.\ up(i{+}1).\ eat\ i.\ down\ i.\ down(i{+}1) \quad \longleftarrow$$

$$up\ i \quad = \quad chopstick\ i := \top$$

$$down\ i \quad = \quad chopstick\ i := \bot$$

$$eat\ i \quad = \quad \cdots\cdots chopstick\ i \cdots\cdots chopstick(i{+}1) \cdots\cdots$$

Dining Philosophers

$$life \quad = \quad (P\,0 \vee P\,1 \vee P\,2 \vee P\,3 \vee P\,4).\ life$$

$$P\,i \quad = \quad up\ i.\ up(i{+}1).\ eat\ i.\ down\ i.\ down(i{+}1)$$

$$up\ i \quad = \quad chopstick\ i := \top \quad \longleftarrow$$

$$down\ i \quad = \quad chopstick\ i := \bot \quad \longleftarrow$$

$$eat\ i \quad = \quad \cdots\cdots chopstick\ i \cdots\cdots chopstick(i{+}1)\cdots\cdots$$

Dining Philosophers

$$
\begin{aligned}
\mathit{life} &= (P\,0 \vee P\,1 \vee P\,2 \vee P\,3 \vee P\,4).\ \mathit{life}\\
P\,i &= \mathit{up}\ i.\ \mathit{up}(i{+}1).\ \mathit{eat}\ i.\ \mathit{down}\ i.\ \mathit{down}(i{+}1)\\
\mathit{up}\ i &= \mathit{chopstick}\ i := \top\\
\mathit{down}\ i &= \mathit{chopstick}\ i := \bot\\
\mathit{eat}\ i &= \cdots\cdots \mathit{chopstick}\ i \cdots\cdots \mathit{chopstick}(i{+}1) \cdots\cdots \quad \longleftarrow
\end{aligned}
$$

Dining Philosophers

$$life \quad = \quad (P\,0 \lor P\,1 \lor P\,2 \lor P\,3 \lor P\,4).\ life$$

$$P\,i \quad = \quad up\ i.\ up(i{+}1).\ eat\ i.\ down\ i.\ down(i{+}1)$$

$$up\ i \quad = \quad chopstick\ i := \top$$

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$$eat\ i \quad = \quad \cdots\cdots chopstick\ i \cdots\cdots chopstick(i{+}1)\cdots\cdots$$

If $i \neq j$, $(up\ i.\ up\ j)$ becomes $(up\ i \parallel up\ j)$.
If $i \neq j$, $(up\ i.\ down\ j)$ becomes $(up\ i \parallel down\ j)$.
If $i \neq j$, $(down\ i.\ up\ j)$ becomes $(down\ i \parallel up\ j)$.
If $i \neq j$, $(down\ i.\ down\ j)$ becomes $(down\ i \parallel down\ j)$.
If $i \neq j \land i{+}1 \neq j$, $(eat\ i.\ up\ j)$ becomes $(eat\ i \parallel up\ j)$.
If $i \neq j \land i \neq j{+}1$, $(up\ i.\ eat\ j)$ becomes $(up\ i \parallel eat\ j)$.
If $i \neq j \land i{+}1 \neq j$, $(eat\ i.\ down\ j)$ becomes $(eat\ i \parallel down\ j)$.
If $i \neq j \land i \neq j{+}1$, $(down\ i.\ eat\ j)$ becomes $(down\ i \parallel eat\ j)$.
If $i \neq j \land i{+}1 \neq j \land i \neq j{+}1$, $(eat\ i.\ eat\ j)$ becomes $(eat\ i \parallel eat\ j)$.

Dining Philosophers

$$life = (P\,0 \vee P\,1 \vee P\,2 \vee P\,3 \vee P\,4).\ life$$

$$P\,i = up\ i.\ up(i+1).\ eat\ i.\ down\ i.\ down(i+1)$$

$$up\ i = chopstick\ i := \top$$

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$$eat\ i = \cdots\cdots chopstick\ i \cdots\cdots chopstick(i+1) \cdots\cdots$$

If $i \neq j$, $(up\ i.\ up\ j)$ becomes $(up\ i \,\|\, up\ j)$. ←

If $i \neq j$, $(up\ i.\ down\ j)$ becomes $(up\ i \,\|\, down\ j)$.

If $i \neq j$, $(down\ i.\ up\ j)$ becomes $(down\ i \,\|\, up\ j)$.

If $i \neq j$, $(down\ i.\ down\ j)$ becomes $(down\ i \,\|\, down\ j)$.

If $i \neq j \wedge i+1 \neq j$, $(eat\ i.\ up\ j)$ becomes $(eat\ i \,\|\, up\ j)$.

If $i \neq j \wedge i \neq j+1$, $(up\ i.\ eat\ j)$ becomes $(up\ i \,\|\, eat\ j)$.

If $i \neq j \wedge i+1 \neq j$, $(eat\ i.\ down\ j)$ becomes $(eat\ i \,\|\, down\ j)$.

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Dining Philosophers

$life \;=\; (P\,0 \vee P\,1 \vee P\,2 \vee P\,3 \vee P\,4).\; life$

$P\,i \;=\; up\;i.\; up(i{+}1).\; eat\;i.\; down\;i.\; down(i{+}1)$

$up\;i \;=\; chopstick\;i := \top$

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If $i{\neq}j$, $(up\;i.\,up\;j)$ becomes $(up\;i \,\|\, up\;j)$.

If $i{\neq}j$, $(up\;i.\,down\;j)$ becomes $(up\;i \,\|\, down\;j)$. ←

If $i{\neq}j$, $(down\;i.\,up\;j)$ becomes $(down\;i \,\|\, up\;j)$.

If $i{\neq}j$, $(down\;i.\,down\;j)$ becomes $(down\;i \,\|\, down\;j)$.

If $i{\neq}j \wedge i{+}1{\neq}j$, $(eat\;i.\,up\;j)$ becomes $(eat\;i \,\|\, up\;j)$.

If $i{\neq}j \wedge i{\neq}j{+}1$, $(up\;i.\,eat\;j)$ becomes $(up\;i \,\|\, eat\;j)$.

If $i{\neq}j \wedge i{+}1{\neq}j$, $(eat\;i.\,down\;j)$ becomes $(eat\;i \,\|\, down\;j)$.

If $i{\neq}j \wedge i{\neq}j{+}1$, $(down\;i.\,eat\;j)$ becomes $(down\;i \,\|\, eat\;j)$.

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Dining Philosophers

$$life = (P\,0 \vee P\,1 \vee P\,2 \vee P\,3 \vee P\,4).\ life$$

$$P\,i = up\ i.\ up(i+1).\ eat\ i.\ down\ i.\ down(i+1)$$

$$up\ i = chopstick\ i := \top$$

$$down\ i = chopstick\ i := \bot$$

$$eat\ i = \cdots\cdots chopstick\ i \cdots\cdots chopstick(i+1)\cdots\cdots$$

If $i \neq j$, $(up\ i.\ up\ j)$ becomes $(up\ i \,\|\, up\ j)$.

If $i \neq j$, $(up\ i.\ down\ j)$ becomes $(up\ i \,\|\, down\ j)$.

If $i \neq j$, $(down\ i.\ up\ j)$ becomes $(down\ i \,\|\, up\ j)$. ←

If $i \neq j$, $(down\ i.\ down\ j)$ becomes $(down\ i \,\|\, down\ j)$.

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Dining Philosophers

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If $i \neq j$, $(up\,i.\,up\,j)$ becomes $(up\,i \,\|\, up\,j)$.
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If $i \neq j$, $(down\,i.\,up\,j)$ becomes $(down\,i \,\|\, up\,j)$.
If $i \neq j$, $(down\,i.\,down\,j)$ becomes $(down\,i \,\|\, down\,j)$. ←
If $i \neq j \wedge i{+}1 \neq j$, $(eat\,i.\,up\,j)$ becomes $(eat\,i \,\|\, up\,j)$.
If $i \neq j \wedge i \neq j{+}1$, $(up\,i.\,eat\,j)$ becomes $(up\,i \,\|\, eat\,j)$.
If $i \neq j \wedge i{+}1 \neq j$, $(eat\,i.\,down\,j)$ becomes $(eat\,i \,\|\, down\,j)$.
If $i \neq j \wedge i \neq j{+}1$, $(down\,i.\,eat\,j)$ becomes $(down\,i \,\|\, eat\,j)$.
If $i \neq j \wedge i{+}1 \neq j \wedge i \neq j{+}1$, $(eat\,i.\,eat\,j)$ becomes $(eat\,i \,\|\, eat\,j)$.

Dining Philosophers

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If $i \ne j \land i{+}1 \ne j$, $(eat\;i.\,up\;j)$ becomes $(eat\;i \,\|\, up\;j)$. ⟵

If $i \ne j \land i \ne j{+}1$, $(up\;i.\,eat\;j)$ becomes $(up\;i \,\|\, eat\;j)$. ⟵

If $i \ne j \land i{+}1 \ne j$, $(eat\;i.\,down\;j)$ becomes $(eat\;i \,\|\, down\;j)$. ⟵

If $i \ne j \land i \ne j{+}1$, $(down\;i.\,eat\;j)$ becomes $(down\;i \,\|\, eat\;j)$. ⟵

If $i \ne j \land i{+}1 \ne j \land i \ne j{+}1$, $(eat\;i.\,eat\;j)$ becomes $(eat\;i \,\|\, eat\;j)$.

Dining Philosophers

$$life \;=\; (P\,0 \lor P\,1 \lor P\,2 \lor P\,3 \lor P\,4).\; life$$

$$P\,i \;=\; up\;i.\; up(i+1).\; eat\;i.\; down\;i.\; down(i+1)$$

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Dining Philosophers

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$$life = P\,0 \parallel P\,1 \parallel P\,2 \parallel P\,3 \parallel P\,4$$

$$P\,i = (up\ i \parallel up(i{+}1)).\ eat\ i.\ (down\ i \parallel down(i{+}1)).\ P\,i$$

Dining Philosophers

$$
\begin{aligned}
life &= (P\,0 \lor P\,1 \lor P\,2 \lor P\,3 \lor P\,4).\ life \\
P\,i &= up\ i.\ up(i{+}1).\ eat\ i.\ down\ i.\ down(i{+}1) \\
up\ i &= chopstick\ i := \top \\
down\ i &= chopstick\ i := \bot \\
eat\ i &= \cdots\cdots chopstick\ i \cdots\cdots chopstick(i{+}1)\cdots\cdots
\end{aligned}
$$

$$
\begin{aligned}
life &= P\,0 \parallel P\,1 \parallel P\,2 \parallel P\,3 \parallel P\,4 \quad ✗ \\
P\,i &= (up\ i \parallel up(i{+}1)).\ eat\ i.\ (down\ i \parallel down(i{+}1)).\ P\,i
\end{aligned}
$$