Communication Channels

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Channel c is described by

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read cursor	$\boldsymbol{r}c$	extended natural variable
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$$\begin{array}{lcccccccccccc} \mathcal{M} & = & 6 & ;\ 4 & ;\ 7 & ;\ 1 & ;\ 0 & ;\ 3 & ;\ 8 & ;\ 9 & ;\ 2 & ;\ 5 & ;\ \ldots \\ \mathcal{T} & = & 3 & ;\ 5 & ;\ 5 & ;20 & ;25 & ;28 & ;31 & ;31 & ;45 & ;48 & ;\ \ldots \\ & & & & & \uparrow & & \uparrow & & & & & \\ & & & & & \boldsymbol{r} & & \boldsymbol{w} & & & & & \end{array}$$

Input and Output

$$c!\,e \;=\; \mathcal{M}_{\boldsymbol{w}} = e \;\wedge\; \mathcal{T}_{\boldsymbol{w}} = t \;\wedge\; (\boldsymbol{w}{:=}\,\boldsymbol{w}+1)$$

$$c? \;=\; \boldsymbol{r}{:=}\,\boldsymbol{r}+1$$

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if $\surd key$
then    $key?$.
        if $key$="y"
        then $screen$! "If you wish."
        else $screen$! "Not if you don't want." fi
else $screen$! "Well?" fi
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↑

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$$
\begin{aligned}
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= \quad & S
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$$

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input $c?$ becomes $t:= t\uparrow(\mathcal{T}c_{\boldsymbol{r}c} + 1).\ c?$

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real time need to know implementation

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input $c?$ becomes $t:= t\uparrow(\mathcal{T}c_{\boldsymbol{r}c}+1).\ c?$

check $\surd c$ becomes $\mathcal{T}c_{\boldsymbol{r}c}+1 \le t$

Communication Timing

$$
\begin{aligned}
W &= t := t \uparrow (\mathcal{T}_{\boldsymbol{r}} + 1).\ \ c? \\
&= \text{wait (if necessary) for input and then read it}
\end{aligned}
$$

Communication Timing

$W \quad = \quad t := t \uparrow (\mathcal{T}_r + 1).\ c?$

$\quad\quad = \quad$ wait (if necessary) for input and then read it

$W \quad \Leftarrow \quad \textbf{if } \surd c \textbf{ then } c? \textbf{ else } t := t+1.\ W \textbf{ fi}$

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$W \;=\; t:= t\uparrow(\mathcal{T}_r + 1).\;\; c?$

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$W \;\Leftarrow\;$ **if** $\surd c$ **then** $c?$ **else** $t:= t+1.\;\; W$ **fi**

proof

if $\surd c$ **then** $c?$ **else** $t:= t+1.\;\; W$ **fi**

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if $\surd c$ **then** $c?$ **else** $t{:=}\, t{+}1.\;\; W$ **fi**

$=\;$ **if** $\mathcal{T}_{\boldsymbol{r}} + 1 \le t$ **then** $c?$ **else** $t{:=}\, t{+}1.\;\; t{:=}\, t\uparrow(\mathcal{T}_{\boldsymbol{r}} + 1).\;\; c?$ **fi**

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if $\surd c$ **then** $c?$ **else** $t := t+1.\ W$ **fi**

$=$ **if** $\mathcal{T}_r + 1 \le t$ **then** $c?$ **else** $t := t+1.\ t := t \uparrow (\mathcal{T}_r + 1).\ c?$ **fi**

$=$ **if** $\mathcal{T}_r + 1 \le t$ **then** $t := t.\ c?$

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$= \quad \textbf{if } \mathcal{T}_r + 1 \le t \textbf{ then } c? \textbf{ else } t{:=}\ t{+}1.\ \ t{:=}\ t\uparrow(\mathcal{T}_r + 1).\ \ c? \textbf{ fi}$

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proof

$$\textbf{if } \surd c \textbf{ then } c? \textbf{ else } t{:=}\ t{+}1.\ \ W \textbf{ fi}$$

$$= \quad \textbf{if } \mathcal{T}_r + 1 \le t \textbf{ then } c? \textbf{ else } t{:=}\ t{+}1.\ \ t{:=}\ t\uparrow(\mathcal{T}_r + 1).\ \ c? \textbf{ fi}$$

$$= \quad \textbf{if } \mathcal{T}_r + 1 \le t \textbf{ then } t{:=}\ t.\ \ c? \textbf{ else } t{:=}\ (t{+}1)\uparrow(\mathcal{T}_r + 1).\ \ c? \textbf{ fi}$$

$$= \quad \textbf{if } \mathcal{T}_r + 1 \le t \textbf{ then } t{:=}\ t\uparrow(\mathcal{T}_r + 1).\ \ c? \textbf{ else } t{:=}\ t\uparrow(\mathcal{T}_r + 1).\ \ c? \textbf{ fi}$$

Communication Timing

$$
\begin{aligned}
W \quad &= \quad t{:=}\, t\uparrow(\mathcal{T}_{\boldsymbol{r}} + 1).\ \ c?\\
&= \quad \text{wait (if necessary) for input and then read it}
\end{aligned}
$$

$$W \quad \Leftarrow \quad \textbf{if } \surd c \textbf{ then } c? \textbf{ else } t{:=}\, t{+}1.\ \ W \textbf{ fi}$$

proof

$$
\begin{aligned}
& \textbf{if } \surd c \textbf{ then } c? \textbf{ else } t{:=}\, t{+}1.\ \ W \textbf{ fi}\\
= \quad & \textbf{if } \mathcal{T}_{\boldsymbol{r}} + 1 \le t \textbf{ then } c? \textbf{ else } t{:=}\, t{+}1.\ \ t{:=}\, t\uparrow(\mathcal{T}_{\boldsymbol{r}} + 1).\ \ c? \textbf{ fi}\\
= \quad & \textbf{if } \mathcal{T}_{\boldsymbol{r}} + 1 \le t \textbf{ then } t{:=}\, t.\ \ c? \textbf{ else } t{:=}\, (t{+}1)\uparrow(\mathcal{T}_{\boldsymbol{r}} + 1).\ \ c? \textbf{ fi}\\
= \quad & \textbf{if } \mathcal{T}_{\boldsymbol{r}} + 1 \le t \textbf{ then } t{:=}\, t\uparrow(\mathcal{T}_{\boldsymbol{r}} + 1).\ \ c? \textbf{ else } t{:=}\, t\uparrow(\mathcal{T}_{\boldsymbol{r}} + 1).\ \ c? \textbf{ fi}\\
= \quad & W
\end{aligned}
$$

Recursive Communication

$dbl \;\;=\;\; c?.\;\; d!\, 2{\times}c.\;\; t{:=}\, t{+}1.\;\; dbl$

Recursive Communication

$dbl \;=\; c?.\; d!\, 2{\times}c.\; t{:=}\, t{+}1.\; dbl$

weakest solution

$\forall n{:}\, nat\cdot\ \mathcal{M}d_{\,\boldsymbol{w}d+n} = 2\times\mathcal{M}c_{\,\boldsymbol{r}c+n}\ \wedge\ \mathcal{T}d_{\,\boldsymbol{w}d+n} = t{+}n$

Recursive Communication

$$dbl \;=\; c?.\ d!\ 2{\times}c.\ t{:=}\ t{+}1.\ dbl$$

weakest solution

$$\forall n{:}\ nat\cdot\ \mathcal{M}d_{\boldsymbol{w}d+n} = 2 \times \mathcal{M}c_{\boldsymbol{r}c+n} \;\wedge\; \mathcal{T}d_{\boldsymbol{w}d+n} = t+n$$

strongest implementable solution

$$(\forall n{:}\ nat\cdot\ \mathcal{M}d_{\boldsymbol{w}d+n} = 2 \times \mathcal{M}c_{\boldsymbol{r}c+n} \;\wedge\; \mathcal{T}d_{\boldsymbol{w}d+n} = t+n)$$

$$\wedge\;\; \boldsymbol{r}c'{=}\boldsymbol{w}d'{=}t'{=}\infty \;\wedge\; \boldsymbol{w}c'{=}\boldsymbol{w}c \;\wedge\; \boldsymbol{r}d'{=}\boldsymbol{r}d$$

Recursive Communication

$$dbl \;=\; c?.\; d!\,2{\times}c.\; t{:=}\,t{+}1.\; dbl$$

weakest solution

$$\forall n{:}\, nat\cdot\, \mathcal{M}d_{\boldsymbol{w}d+n} = 2\times\mathcal{M}c_{\boldsymbol{r}c+n} \;\land\; \mathcal{T}d_{\boldsymbol{w}d+n} = t+n$$

strongest implementable solution

$$(\forall n{:}\, nat\cdot\, \mathcal{M}d_{\boldsymbol{w}d+n} = 2\times\mathcal{M}c_{\boldsymbol{r}c+n} \;\land\; \mathcal{T}d_{\boldsymbol{w}d+n} = t+n)$$

$$\land\;\; \boldsymbol{r}c'{=}\boldsymbol{w}d'{=}t'{=}\infty \;\land\; \boldsymbol{w}c'{=}\boldsymbol{w}c \;\land\; \boldsymbol{r}d'{=}\boldsymbol{r}d$$

strongest solution

$$\bot$$

Recursive Communication

$dbl \;=\; c?.\; d!\,2{\times}c.\; t{:=}\,t{+}1.\; dbl$

weakest solution

$\forall n{:}\, nat\cdot \mathcal{M}d_{\boldsymbol{w}d+n} = 2 \times \mathcal{M}c_{\boldsymbol{r}c+n} \;\wedge\; \mathcal{T}d_{\boldsymbol{w}d+n} = t+n$

strongest implementable solution

$(\forall n{:}\, nat\cdot \mathcal{M}d_{\boldsymbol{w}d+n} = 2 \times \mathcal{M}c_{\boldsymbol{r}c+n} \;\wedge\; \mathcal{T}d_{\boldsymbol{w}d+n} = t+n)$

$\wedge\;\; \boldsymbol{r}c'{=}\boldsymbol{w}d'{=}t'{=}\infty \;\wedge\; \boldsymbol{w}c'{=}\boldsymbol{w}c \;\wedge\; \boldsymbol{r}d'{=}\boldsymbol{r}d$

strongest solution

$\bot$

$\forall n{:}\, nat\cdot \mathcal{M}d_{\boldsymbol{w}d+n} = 2 \times \mathcal{M}c_{\boldsymbol{r}c+n} \;\wedge\; \mathcal{T}d_{\boldsymbol{w}d+n} = t+n \;\;\Leftarrow\;\; dbl$

Recursive Communication

$dbl \;=\; c?.\; d!\, 2{\times}c.\; t{:=}\, t{+}1.\; dbl$

weakest solution

$\forall n{:}\, nat\cdot \mathcal{M}d_{\boldsymbol{w}d+n} = 2\times \mathcal{M}c_{\boldsymbol{r}c+n} \;\land\; \mathcal{T}d_{\boldsymbol{w}d+n} = t{+}n$

strongest implementable solution

$(\forall n{:}\, nat\cdot \mathcal{M}d_{\boldsymbol{w}d+n} = 2\times \mathcal{M}c_{\boldsymbol{r}c+n} \;\land\; \mathcal{T}d_{\boldsymbol{w}d+n} = t{+}n)$

$\land\;\; \boldsymbol{r}c'{=}\boldsymbol{w}d'{=}t'{=}\infty \;\land\; \boldsymbol{w}c'{=}\boldsymbol{w}c \;\land\; \boldsymbol{r}d'{=}\boldsymbol{r}d$

strongest solution

$\bot$

$\forall n{:}\, nat\cdot \mathcal{M}d_{\boldsymbol{w}d+n} = 2\times \mathcal{M}c_{\boldsymbol{r}c+n} \;\land\; \mathcal{T}d_{\boldsymbol{w}d+n}{=}\, t{+}n \;\;\Leftarrow\;\; dbl$

$dbl \;\;\Leftarrow\;\; c?.\; d!\, 2{\times}c.\; t{:=}\, t{+}1.\; dbl$

Recursive Construction

Recursive Construction

$$dbl_0 \quad = \quad \top$$

Recursive Construction

$$dbl_0 \;=\; \top$$

$$\begin{aligned}
dbl_1 &= c?.\;\; d!\;2{\times}c.\; t{:=}\; t{+}1.\;\; dbl_0 \\
&= \boldsymbol{r}c{:=}\;\boldsymbol{r}c{+}1.\;\;\; \boldsymbol{\mathcal{M}}d_{\,\boldsymbol{w}d} = 2 \times \boldsymbol{\mathcal{M}}c_{\,\boldsymbol{r}c-1} \land\; \boldsymbol{\mathcal{T}}d_{\,\boldsymbol{w}d} = t \;\land (\boldsymbol{w}d{:=}\;\boldsymbol{w}d{+}1).\;\;\; t{:=}\; t{+}1.\;\; \top \\
&= \boldsymbol{\mathcal{M}}d_{\,\boldsymbol{w}d} = 2 \times \boldsymbol{\mathcal{M}}c_{\,\boldsymbol{r}c} \;\land\; \boldsymbol{\mathcal{T}}d_{\,\boldsymbol{w}d} = t
\end{aligned}$$

Recursive Construction

$$dbl_0 \;=\; \top$$

$$dbl_1 \;=\; c?.\; d!\, 2{\times}c.\; t{:=}\, t{+}1.\; dbl_0$$

$$=\; \boldsymbol{r}c{:=}\, \boldsymbol{r}c{+}1.\;\; \mathcal{M}d_{\,\boldsymbol{w}d} = 2 \times \mathcal{M}c_{\,\boldsymbol{r}c-1} \wedge\; \mathcal{T}d_{\,\boldsymbol{w}d} = t \;\wedge (\boldsymbol{w}d{:=}\, \boldsymbol{w}d{+}1).\;\; t{:=}\, t{+}1.\; \top$$

$$=\; \mathcal{M}d_{\,\boldsymbol{w}d} = 2 \times \mathcal{M}c_{\,\boldsymbol{r}c} \;\wedge\; \mathcal{T}d_{\,\boldsymbol{w}d} = t$$

$$dbl_2 \;=\; c?.\; d!\, 2{\times}c.\;\; t{:=}\, t{+}1.\; dbl_1$$

$$=\; \boldsymbol{r}c{:=}\, \boldsymbol{r}c{+}1.\;\; \mathcal{M}d_{\,\boldsymbol{w}d} = 2 \times \mathcal{M}c_{\,\boldsymbol{r}c-1} \wedge\; \mathcal{T}d_{\,\boldsymbol{w}d} = t \;\wedge (\boldsymbol{w}d{:=}\, \boldsymbol{w}d{+}1).\;\; t{:=}\, t{+}1.$$

$$\mathcal{M}d_{\,\boldsymbol{w}d} = 2 \times \mathcal{M}c_{\,\boldsymbol{r}c} \;\wedge\; \mathcal{T}d_{\,\boldsymbol{w}d} = t$$

$$=\; \mathcal{M}d_{\,\boldsymbol{w}d} = 2 \times \mathcal{M}c_{\,\boldsymbol{r}c} \;\wedge\; \mathcal{T}d_{\,\boldsymbol{w}d} = t \;\wedge\; \mathcal{M}d_{\,\boldsymbol{w}d+1} = 2{\times}\mathcal{M}c_{\,\boldsymbol{r}c+1} \wedge \mathcal{T}d_{\,\boldsymbol{w}d+1} = t{+}1$$

Recursive Construction

$$dbl_0 \;=\; \top$$

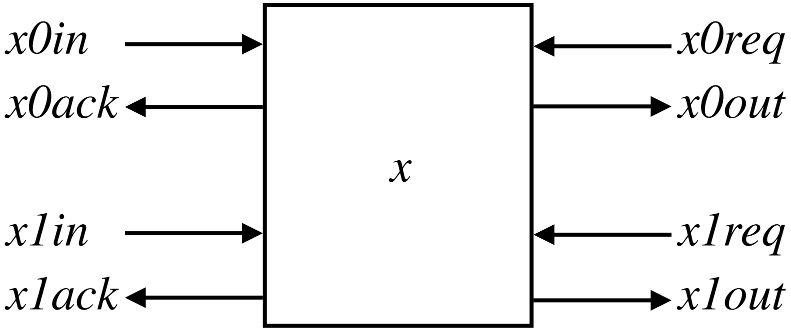
$$\begin{aligned}
dbl_1 &= c?.\ d!\ 2{\times}c.\ t{:=}\ t{+}1.\ dbl_0 \\
&= \boldsymbol{r}c{:=}\ \boldsymbol{r}c{+}1.\ \ \mathcal{M}d_{\boldsymbol{w}d} = 2 \times \mathcal{M}c_{\boldsymbol{r}c-1} \wedge \mathcal{T}d_{\boldsymbol{w}d} = t\ \wedge (\boldsymbol{w}d{:=}\ \boldsymbol{w}d{+}1).\ \ t{:=}\ t{+}1.\ \top \\
&= \mathcal{M}d_{\boldsymbol{w}d} = 2 \times \mathcal{M}c_{\boldsymbol{r}c} \ \wedge\ \mathcal{T}d_{\boldsymbol{w}d} = t
\end{aligned}$$

$$\begin{aligned}
dbl_2 &= c?.\ d!\ 2{\times}c.\ t{:=}\ t{+}1.\ dbl_1 \\
&= \boldsymbol{r}c{:=}\ \boldsymbol{r}c{+}1.\ \ \mathcal{M}d_{\boldsymbol{w}d} = 2 \times \mathcal{M}c_{\boldsymbol{r}c-1} \wedge \mathcal{T}d_{\boldsymbol{w}d} = t\ \wedge (\boldsymbol{w}d{:=}\ \boldsymbol{w}d{+}1).\ \ t{:=}\ t{+}1. \\
&\ \mathcal{M}d_{\boldsymbol{w}d} = 2 \times \mathcal{M}c_{\boldsymbol{r}c} \ \wedge\ \mathcal{T}d_{\boldsymbol{w}d} = t \\
&= \mathcal{M}d_{\boldsymbol{w}d} = 2 \times \mathcal{M}c_{\boldsymbol{r}c} \ \wedge\ \mathcal{T}d_{\boldsymbol{w}d} = t\ \wedge\ \mathcal{M}d_{\boldsymbol{w}d+1} = 2{\times}\mathcal{M}c_{\boldsymbol{r}c+1} \wedge \mathcal{T}d_{\boldsymbol{w}d+1} = t{+}1
\end{aligned}$$

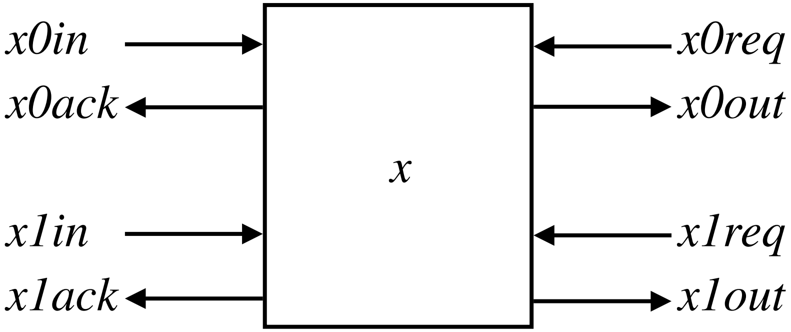
$$dbl_\infty \;=\; \forall n{:}\ nat\cdot\ \mathcal{M}d_{\boldsymbol{w}d+n} = 2 \times \mathcal{M}c_{\boldsymbol{r}c+n} \ \wedge\ \mathcal{T}d_{\boldsymbol{w}d+n} = t{+}n$$

Monitor

Monitor

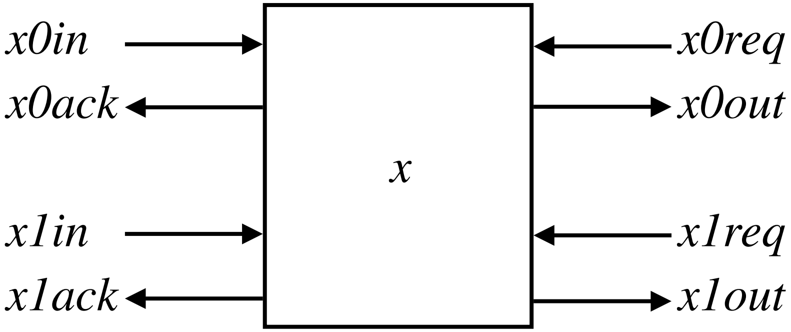


Monitor



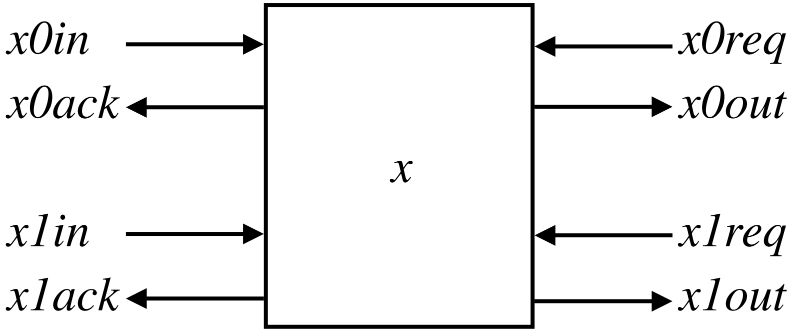
$$
\begin{aligned}
\mathit{monitor} \;=\; & \quad (\surd x0in \vee \mathcal{T}x0in_{\boldsymbol{r}x0in} = m) \wedge (x0in?.\; x{:=}\, x0in.\; x0ack!\, \top) \\
& \vee\; (\surd x1in \vee \mathcal{T}x1in_{\boldsymbol{r}x1in} = m) \wedge (x1in?.\; x{:=}\, x1in.\; x1ack!\, \top) \\
& \vee\; (\surd x0req \vee \mathcal{T}x0req_{\boldsymbol{r}x0req} = m) \wedge (x0req?.\; x0out!\, x) \\
& \vee\; (\surd x1req \vee \mathcal{T}x1req_{\boldsymbol{r}x1req} = m) \wedge (x1req?.\; x1out!\, x). \\
& \mathit{monitor}
\end{aligned}
$$

Monitor



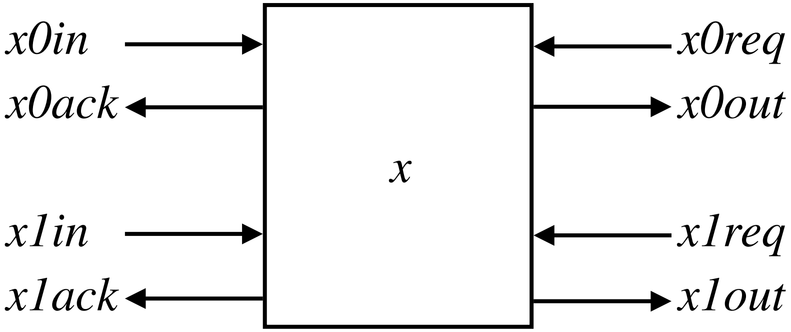
$$
\begin{aligned}
monitor \;\; = \;\; & (\surd x0in \vee \mathcal{T}x0in\ \boldsymbol{r}_{x0in} = m) \wedge (x0in?.\ x{:=}\,x0in.\ x0ack!\ \top) \longleftarrow \\
\vee\; & (\surd x1in \vee \mathcal{T}x1in\ \boldsymbol{r}_{x1in} = m) \wedge (x1in?.\ x{:=}\,x1in.\ x1ack!\ \top) \longleftarrow \\
\vee\; & (\surd x0req \vee \mathcal{T}x0req\ \boldsymbol{r}_{x0req} = m) \wedge (x0req?.\ x0out!\ x) \qquad \longleftarrow \\
\vee\; & (\surd x1req \vee \mathcal{T}x1req\ \boldsymbol{r}_{x1req} = m) \wedge (x1req?.\ x1out!\ x). \qquad \longleftarrow \\
& monitor
\end{aligned}
$$

Monitor



$$
\begin{aligned}
monitor \;\;=\;\; & (\surd x0in \vee \mathcal{T}x0in_{\boldsymbol{r}x0in} = m) \wedge (x0in?.\;\; x{:=}\, x0in.\;\; x0ack!\; \top) \leftarrow \\
\vee\;\; & (\surd x1in \vee \mathcal{T}x1in_{\boldsymbol{r}x1in} = m) \wedge (x1in?.\;\; x{:=}\, x1in.\;\; x1ack!\; \top) \\
\vee\;\; & (\surd x0req \vee \mathcal{T}x0req_{\boldsymbol{r}x0req} = m) \wedge (x0req?.\;\; x0out!\; x) \\
\vee\;\; & (\surd x1req \vee \mathcal{T}x1req_{\boldsymbol{r}x1req} = m) \wedge (x1req?.\;\; x1out!\; x). \\
& monitor
\end{aligned}
$$

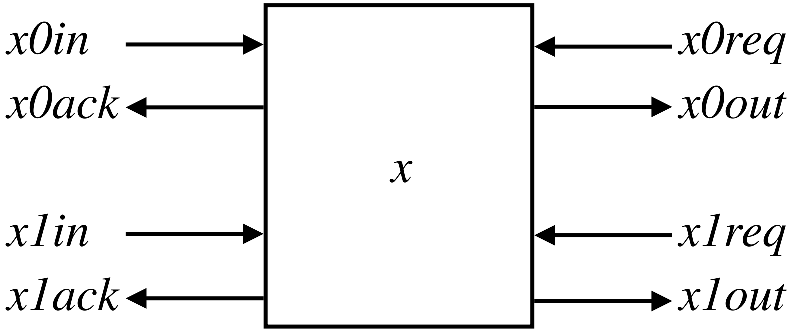
Monitor



↓

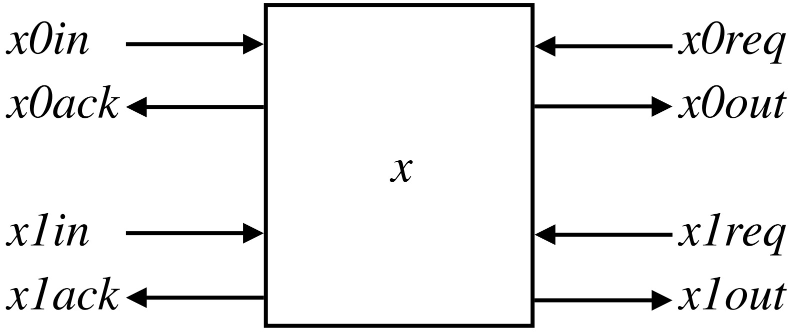
$$\begin{aligned}
monitor \;=\; & \quad (\surd x0in \lor \mathcal{T} x0in_{\boldsymbol{r} x0in} = m) \land (x0in?.\ x{:=}\, x0in.\ x0ack!\ \top) \leftarrow \\
& \lor\ (\surd x1in \lor \mathcal{T} x1in_{\boldsymbol{r} x1in} = m) \land (x1in?.\ x{:=}\, x1in.\ x1ack!\ \top) \\
& \lor\ (\surd x0req \lor \mathcal{T} x0req_{\boldsymbol{r} x0req} = m) \land (x0req?.\ x0out!\ x) \\
& \lor\ (\surd x1req \lor \mathcal{T} x1req_{\boldsymbol{r} x1req} = m) \land (x1req?.\ x1out!\ x). \\
& monitor
\end{aligned}$$

Monitor



$$
\begin{aligned}
\mathit{monitor} \;=\; & \;\; (\surd \mathit{x0in} \vee \mathcal{T}\mathit{x0in}\ \boldsymbol{r}_{\mathit{x0in}} = m) \wedge (\mathit{x0in}?.\ x{:=}\ \overset{\downarrow}{\mathit{x0in}}.\ \mathit{x0ack}!\ \top) \longleftarrow \\
& \vee\;\; (\surd \mathit{x1in} \vee \mathcal{T}\mathit{x1in}\ \boldsymbol{r}_{\mathit{x1in}} = m) \wedge (\mathit{x1in}?.\ x{:=}\ \mathit{x1in}.\ \mathit{x1ack}!\ \top) \\
& \vee\;\; (\surd \mathit{x0req} \vee \mathcal{T}\mathit{x0req}\ \boldsymbol{r}_{\mathit{x0req}} = m) \wedge (\mathit{x0req}?.\ \mathit{x0out}!\ x) \\
& \vee\;\; (\surd \mathit{x1req} \vee \mathcal{T}\mathit{x1req}\ \boldsymbol{r}_{\mathit{x1req}} = m) \wedge (\mathit{x1req}?.\ \mathit{x1out}!\ x). \\
& \mathit{monitor}
\end{aligned}
$$

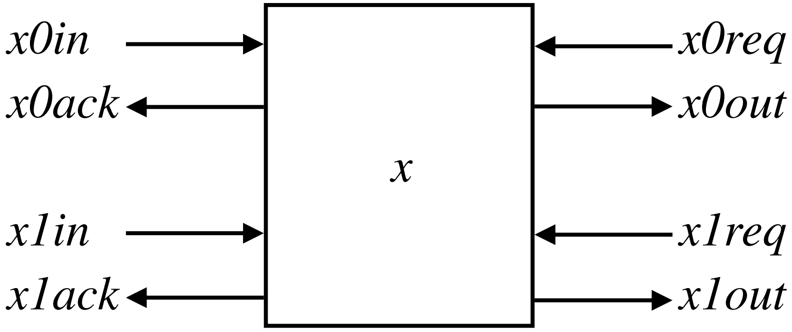
Monitor



$$\downarrow$$

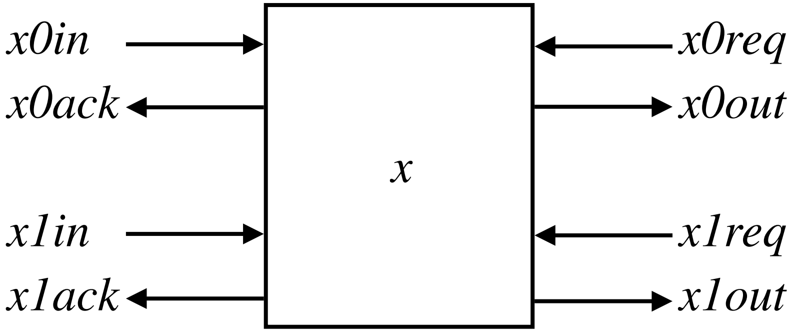
$$
\begin{aligned}
monitor \;=\;\; & (\surd x0in \lor \mathcal{T}x0in_{\boldsymbol{r}x0in} = m) \land (x0in?.\; x{:=}\, x0in.\; x0ack!\, \top) \leftarrow \\
\lor\;\; & (\surd x1in \lor \mathcal{T}x1in_{\boldsymbol{r}x1in} = m) \land (x1in?.\; x{:=}\, x1in.\; x1ack!\, \top) \\
\lor\;\; & (\surd x0req \lor \mathcal{T}x0req_{\boldsymbol{r}x0req} = m) \land (x0req?.\; x0out!\, x) \\
\lor\;\; & (\surd x1req \lor \mathcal{T}x1req_{\boldsymbol{r}x1req} = m) \land (x1req?.\; x1out!\, x). \\
& monitor
\end{aligned}
$$

Monitor



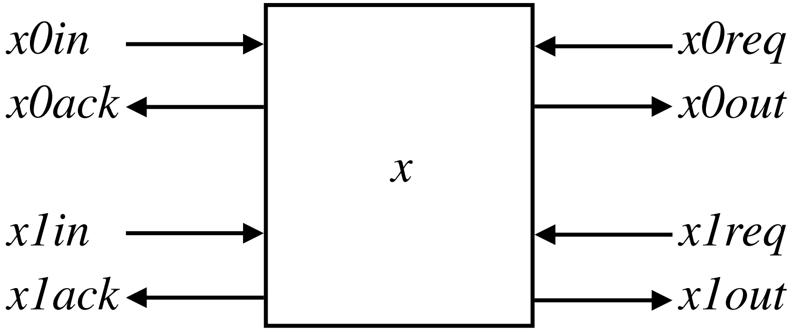
$$\begin{aligned}
monitor \;\;=\;\;& (\surd x0in \vee \mathcal{T}x0in_{\boldsymbol{r}x0in} = m) \wedge (x0in?.\; x{:=}\, x0in.\; x0ack!\; \top)\\
\vee\;& (\surd x1in \vee \mathcal{T}x1in_{\boldsymbol{r}x1in} = m) \wedge (x1in?.\; x{:=}\, x1in.\; x1ack!\; \top)\\
\vee\;& (\surd x0req \vee \mathcal{T}x0req_{\boldsymbol{r}x0req} = m) \wedge (x0req?.\; x0out!\; x)\\
\vee\;& (\surd x1req \vee \mathcal{T}x1req_{\boldsymbol{r}x1req} = m) \wedge (x1req?.\; x1out!\; x). \longleftarrow\\
& monitor
\end{aligned}$$

Monitor



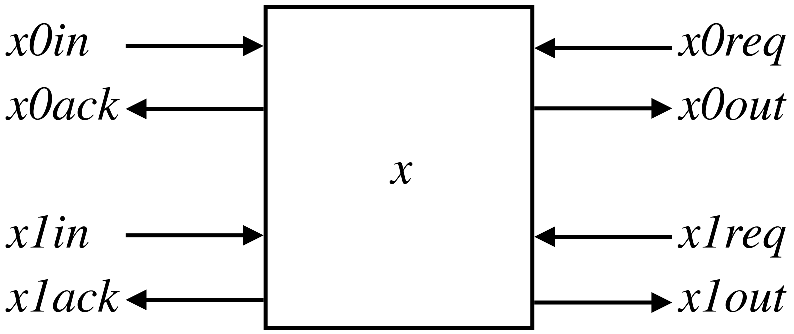
$$\begin{aligned}
monitor \;=\;\; & (\surd x0in \vee \mathcal{T}x0in_{\boldsymbol{r}x0in} = m) \wedge (x0in?.\; x{:=}\, x0in.\; x0ack!\, \top) \\
\vee\; & (\surd x1in \vee \mathcal{T}x1in_{\boldsymbol{r}x1in} = m) \wedge (x1in?.\; x{:=}\, x1in.\; x1ack!\, \top) \\
\vee\; & (\surd x0req \vee \mathcal{T}x0req_{\boldsymbol{r}x0req} = m) \wedge (x0req?.\; x0out!\, x) \\
\vee\; & (\surd x1req \vee \mathcal{T}x1req_{\boldsymbol{r}x1req} = m) \wedge (\underset{\uparrow}{x1req?}.\; x1out!\, x). \longleftarrow \\
& monitor
\end{aligned}$$

Monitor



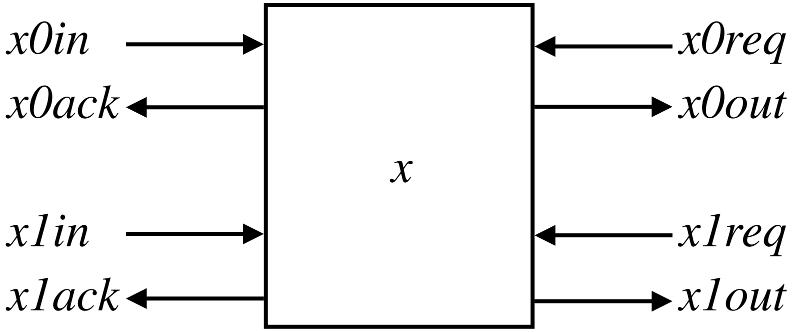
$$\begin{aligned} monitor \;=\;\; & (\sqrt{x0in} \lor \mathcal{T}x0in_{\boldsymbol{r}x0in} = m) \land (x0in?.\;\; x{:=}\, x0in.\;\; x0ack!\ \top) \\ \lor\; & (\sqrt{x1in} \lor \mathcal{T}x1in_{\boldsymbol{r}x1in} = m) \land (x1in?.\;\; x{:=}\, x1in.\;\; x1ack!\ \top) \\ \lor\; & (\sqrt{x0req} \lor \mathcal{T}x0req_{\boldsymbol{r}x0req} = m) \land (x0req?.\;\; x0out!\ x) \\ \lor\; & (\sqrt{x1req} \lor \mathcal{T}x1req_{\boldsymbol{r}x1req} = m) \land (x1req?.\;\; x1out!\ x). \leftarrow \\ & monitor \qquad\qquad\qquad\qquad\qquad\qquad\qquad \uparrow \end{aligned}$$

Monitor



$$\begin{aligned}
monitor \;=\;\; & (\surd x0in \lor \mathcal{T}x0in_{\mathbf{r}x0in} = m) \land (x0in?.\;\; x{:=}\, x0in.\;\; x0ack!\; \top) \\
\lor\;\; & (\surd x1in \lor \mathcal{T}x1in_{\mathbf{r}x1in} = m) \land (x1in?.\;\; x{:=}\, x1in.\;\; x1ack!\; \top) \\
\lor\;\; & (\surd x0req \lor \mathcal{T}x0req_{\mathbf{r}x0req} = m) \land (x0req?.\;\; x0out!\; x) \\
\lor\;\; & (\surd x1req \lor \mathcal{T}x1req_{\mathbf{r}x1req} = m) \land (x1req?.\;\; x1out!\; x). \\
& monitor
\end{aligned}$$

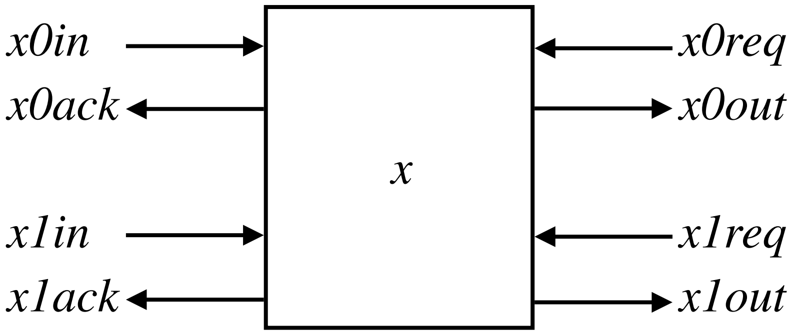
Monitor



$$\downarrow$$

$$\begin{array}{lll} monitor & = & \quad (\surd x0in \lor \mathcal{T}x0in_{\mathbf{r}x0in} = m) \land (x0in?.\ \ x{:=}\ x0in.\ \ x0ack!\ \top) \\ & & \lor\ (\surd x1in \lor \mathcal{T}x1in_{\mathbf{r}x1in} = m) \land (x1in?.\ \ x{:=}\ x1in.\ \ x1ack!\ \top) \\ & & \lor\ (\surd x0req \lor \mathcal{T}x0req_{\mathbf{r}x0req} = m) \land (x0req?.\ \ x0out!\ x) \\ & & \lor\ (\surd x1req \lor \mathcal{T}x1req_{\mathbf{r}x1req} = m) \land (x1req?.\ \ x1out!\ x). \\ & & monitor \end{array}$$

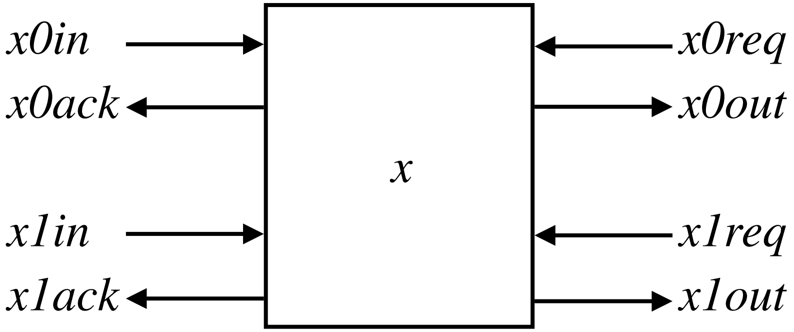
Monitor



↓

$$
\begin{aligned}
\mathit{monitor} \;=\; & (\surd x0in \vee \mathcal{T}x0in_{\boldsymbol{r}x0in} = m) \wedge (x0in?.\; x := x0in.\; x0ack!\, \top) \\
\vee\; & (\surd x1in \vee \mathcal{T}x1in_{\boldsymbol{r}x1in} = m) \wedge (x1in?.\; x := x1in.\; x1ack!\, \top) \\
\vee\; & (\surd x0req \vee \mathcal{T}x0req_{\boldsymbol{r}x0req} = m) \wedge (x0req?.\; x0out!\, x) \\
\vee\; & (\surd x1req \vee \mathcal{T}x1req_{\boldsymbol{r}x1req} = m) \wedge (x1req?.\; x1out!\, x). \\
& \mathit{monitor}
\end{aligned}
$$

Monitor



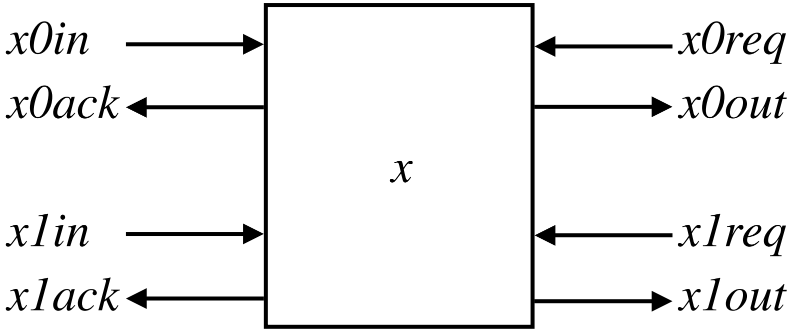
Let $m = \Downarrow[\mathcal{T}x0in_{\boldsymbol{r}x0in};\ \mathcal{T}x1in_{\boldsymbol{r}x1in};\ \mathcal{T}x0req_{\boldsymbol{r}x0req};\ \mathcal{T}x1req_{\boldsymbol{r}x1req}]$

$\downarrow$

$$\begin{aligned}
monitor \;=\;\; & (\surd x0in \vee \mathcal{T}x0in_{\boldsymbol{r}x0in} = m) \wedge (x0in?.\ x := x0in.\ x0ack!\ \top) \\
\vee\;\; & (\surd x1in \vee \mathcal{T}x1in_{\boldsymbol{r}x1in} = m) \wedge (x1in?.\ x := x1in.\ x1ack!\ \top) \\
\vee\;\; & (\surd x0req \vee \mathcal{T}x0req_{\boldsymbol{r}x0req} = m) \wedge (x0req?.\ x0out!\ x) \\
\vee\;\; & (\surd x1req \vee \mathcal{T}x1req_{\boldsymbol{r}x1req} = m) \wedge (x1req?.\ x1out!\ x).
\end{aligned}$$

$monitor$

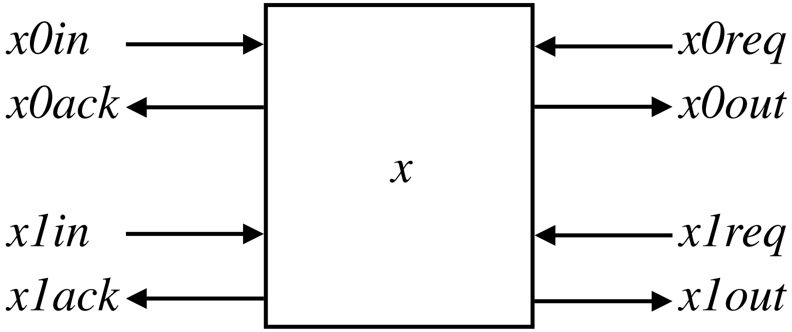
Monitor



Let $m = \Downarrow[\mathcal{T}x0in_{\boldsymbol{r}x0in};\ \mathcal{T}x1in_{\boldsymbol{r}x1in};\ \mathcal{T}x0req_{\boldsymbol{r}x0req};\ \mathcal{T}x1req_{\boldsymbol{r}x1req}]$

$$
\begin{aligned}
monitor \;=\; & \quad (\surd x0in \lor \mathcal{T}x0in_{\boldsymbol{r}x0in} = m) \land (x0in?.\ x := x0in.\ x0ack!\ \top) \\
& \lor\ (\surd x1in \lor \mathcal{T}x1in_{\boldsymbol{r}x1in} = m) \land (x1in?.\ x := x1in.\ x1ack!\ \top) \\
& \lor\ (\surd x0req \lor \mathcal{T}x0req_{\boldsymbol{r}x0req} = m) \land (x0req?.\ x0out!\ x) \\
& \lor\ (\surd x1req \lor \mathcal{T}x1req_{\boldsymbol{r}x1req} = m) \land (x1req?.\ x1out!\ x). \\
& monitor
\end{aligned}
$$

Monitor



$$
\begin{aligned}
monitor \quad \Leftarrow \quad & \textbf{if } \surd x0in \textbf{ then } x0in?.\ \ x{:=}\, x0in.\ \ x0ack!\ \top \textbf{ else } ok \textbf{ fi}. \\
& \textbf{if } \surd x1in \textbf{ then } x1in?.\ \ x{:=}\, x1in.\ \ x1ack!\ \top \textbf{ else } ok \textbf{ fi}. \\
& \textbf{if } \surd x0req \textbf{ then } x0req?.\ \ x0out!\ x \textbf{ else } ok \textbf{ fi}. \\
& \textbf{if } \surd x1req \textbf{ then } x1req?.\ \ x1out!\ x \textbf{ else } ok \textbf{ fi}. \\
& t{:=}\, t{+}1.\ \ monitor
\end{aligned}
$$