

Communicating Processes

$c! 2 \parallel (c?. x := c)$

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$= \mathcal{M}_w = 2 \wedge (w:=w+1) \parallel (r:=r+1. x:=\mathcal{M}_{r-1})$

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$$= \mathcal{M}_w = 2 \wedge (w:=w+1) \parallel (r:=r+1. x:=\mathcal{M}_{r-1})$$

$$= \mathcal{M}_w = 2 \wedge w' = w+1 \wedge r' = r+1 \wedge x' = \mathcal{M}_r$$

Communicating Processes

$c! 2 \parallel (c?. x:=c)$

$= \mathcal{M}_w = 2 \wedge (w:=w+1) \parallel (r:=r+1. x:=\mathcal{M}_{r-1})$

$= \mathcal{M}_w = 2 \wedge w' = w+1 \wedge r' = r+1 \wedge x' = \mathcal{M}_r$

$c! 1. (c! 2 \parallel (c?. x:=c)). c?$

Communicating Processes

$$\begin{aligned} & c! 2 \parallel (c?. x:=c) \\ = & \mathcal{M}_w = 2 \wedge (w:=w+1) \parallel (r:=r+1. x:=\mathcal{M}_{r-1}) \\ = & \mathcal{M}_w = 2 \wedge w' = w+1 \wedge r' = r+1 \wedge x' = \mathcal{M}_r \end{aligned}$$

$$c! 1. (c! 2 \parallel (c?. x:=c)). c?$$

channel declaration

Communicating Processes

$$\begin{aligned} & c! 2 \parallel (c?. x:=c) \\ = & \mathcal{M}_w = 2 \wedge (w:=w+1) \parallel (r:=r+1. x:=\mathcal{M}_{r-1}) \\ = & \mathcal{M}_w = 2 \wedge w' = w+1 \wedge r' = r+1 \wedge x' = \mathcal{M}_r \end{aligned}$$

$$c! 1. (c! 2 \parallel (c?. x:=c)). c?$$

channel declaration

chan $c: T \cdot P$

Communicating Processes

$$\begin{aligned} & c! 2 \parallel (c?. x:= c) \\ = & \mathcal{M}_w = 2 \wedge (w:= w+1) \parallel (r:= r+1. x:= \mathcal{M}_{r-1}) \\ = & \mathcal{M}_w = 2 \wedge w' = w+1 \wedge r' = r+1 \wedge x' = \mathcal{M}_r \end{aligned}$$

$$c! 1. (c! 2 \parallel (c?. x:= c)). c?$$

channel declaration

$$\begin{aligned} & \text{chan } c: T \cdot P \\ = & \exists \mathcal{M}c: \infty^* T. \exists \mathcal{J}c: \infty^* \text{xnat}. \text{var } rc, wc: \text{xnat} := 0. P \end{aligned}$$

ignoring time

```
chan c: int · c! 2 || (c? · x:= c)
```


ignoring time

chan $c: int \cdot c! 2 \parallel (c?. x:= c)$

= $\exists \mathcal{M}: \infty * int \cdot \mathbf{var} \mathbf{r}, \mathbf{w}: xnat := 0 \cdot$

$x' = \mathcal{M}_{\mathbf{r}} \wedge \mathcal{M}_{\mathbf{w}} = 2 \wedge \mathbf{r}' = \mathbf{r}+1 \wedge \mathbf{w}' = \mathbf{w}+1 \wedge$ (other variables unchanged)

ignoring time

chan $c: int \cdot c! 2 \parallel (c?. x:= c)$

= $\exists \mathcal{M}: \infty * int \cdot \mathbf{var} \mathbf{r}, \mathbf{w}: xnat := 0 \cdot$

$x' = \mathcal{M}_{\mathbf{r}} \wedge \mathcal{M}_{\mathbf{w}} = 2 \wedge \mathbf{r}' = \mathbf{r} + 1 \wedge \mathbf{w}' = \mathbf{w} + 1 \wedge$ (other variables unchanged)

= $\exists \mathcal{M}: \infty * int \cdot \mathbf{var} \mathbf{r}, \mathbf{w}: xnat \cdot$

$x' = \mathcal{M}_0 \wedge \mathcal{M}_0 = 2 \wedge \mathbf{r}' = 1 \wedge \mathbf{w}' = 1 \wedge$ (other variables unchanged)

ignoring time

chan $c: int \cdot c! 2 \parallel (c?. x:= c)$

= $\exists \mathcal{M}: \infty * int \cdot \mathbf{var} \mathbf{r}, \mathbf{w}: xnat := 0 \cdot$

$x' = \mathcal{M}_{\mathbf{r}} \wedge \mathcal{M}_{\mathbf{w}} = 2 \wedge \mathbf{r}' = \mathbf{r} + 1 \wedge \mathbf{w}' = \mathbf{w} + 1 \wedge$ (other variables unchanged)

= $\exists \mathcal{M}: \infty * int \cdot \mathbf{var} \mathbf{r}, \mathbf{w}: xnat \cdot$

$x' = \mathcal{M}_0 \wedge \mathcal{M}_0 = 2 \wedge \mathbf{r}' = 1 \wedge \mathbf{w}' = 1 \wedge$ (other variables unchanged)

= $x' = 2 \wedge$ (other variables unchanged)

ignoring time

chan $c: int \cdot c! 2 \parallel (c?. x:= c)$

= $\exists \mathcal{M}: \infty * int \cdot \mathbf{var} \mathbf{r}, \mathbf{w}: xnat := 0 \cdot$

$x' = \mathcal{M}_{\mathbf{r}} \wedge \mathcal{M}_{\mathbf{w}} = 2 \wedge \mathbf{r}' = \mathbf{r} + 1 \wedge \mathbf{w}' = \mathbf{w} + 1 \wedge$ (other variables unchanged)

= $\exists \mathcal{M}: \infty * int \cdot \mathbf{var} \mathbf{r}, \mathbf{w}: xnat \cdot$

$x' = \mathcal{M}_0 \wedge \mathcal{M}_0 = 2 \wedge \mathbf{r}' = 1 \wedge \mathbf{w}' = 1 \wedge$ (other variables unchanged)

= $x' = 2 \wedge$ (other variables unchanged)

= $x := 2$

ignoring time

chan $c: int \cdot c! 2 \parallel (c?. x:= c)$

= $\exists \mathcal{M}: \infty * int \cdot \mathbf{var} \mathbf{r}, \mathbf{w}: xnat := 0 \cdot$

$x' = \mathcal{M}_{\mathbf{r}} \wedge \mathcal{M}_{\mathbf{w}} = 2 \wedge \mathbf{r}' = \mathbf{r} + 1 \wedge \mathbf{w}' = \mathbf{w} + 1 \wedge$ (other variables unchanged)

= $\exists \mathcal{M}: \infty * int \cdot \mathbf{var} \mathbf{r}, \mathbf{w}: xnat \cdot$

$x' = \mathcal{M}_0 \wedge \mathcal{M}_0 = 2 \wedge \mathbf{r}' = 1 \wedge \mathbf{w}' = 1 \wedge$ (other variables unchanged)

= $x' = 2 \wedge$ (other variables unchanged)

= $x := 2$

including time

chan $c: int \cdot c! 2 \parallel (t := t \uparrow (\mathcal{J}_{\mathbf{r}} + 1). c?. x := c)$

= $x' = 2 \wedge t' = t + 1 \wedge$ (other variables unchanged)

Deadlock

Deadlock

chan $c: int$ $t := t \uparrow (\mathcal{J}_r + 1). c?. c! 5$

Deadlock

chan $c: int$. $t := t \uparrow (\mathcal{J}_r + 1)$. $c?$. $c! 5$

=

$\exists \mathcal{M}: \infty * int$. $\exists \mathcal{J}: \infty * xnat$. **var** $r, w: xnat := 0$.

$t := t \uparrow (\mathcal{J}_r + 1)$. $r := r + 1$. $\mathcal{M}_w = 5 \wedge \mathcal{J}_w = t \wedge (w := w + 1)$

Deadlock

chan $c: int$. $t := t \uparrow (\mathcal{J}_r + 1)$. $c?$. $c! 5$

= $\exists \mathcal{M}: \infty^* int$. $\exists \mathcal{J}: \infty^* xnat$. **var** $r, w: xnat := 0$.

$t := t \uparrow (\mathcal{J}_r + 1)$. $r := r + 1$. $\mathcal{M}_w = 5 \wedge \mathcal{J}_w = t \wedge (w := w + 1)$

= $\exists \mathcal{M}: \infty^* int$. $\exists \mathcal{J}: \infty^* xnat$. $\exists r, r', w, w': xnat$.

$r := 0$. $w := 0$. $t := t \uparrow (\mathcal{J}_r + 1)$. $r := r + 1$.

$\mathcal{M}_w = 5 \wedge \mathcal{J}_w = t \wedge r' = r \wedge w' = w + 1 \wedge t' = t$

Deadlock

chan $c: int$. $t := t \uparrow (\mathcal{J}_r + 1)$. $c?$. $c! 5$

= $\exists \mathcal{M}: \infty * int$. $\exists \mathcal{J}: \infty * xnat$. **var** $r, w: xnat := 0$.

$t := t \uparrow (\mathcal{J}_r + 1)$. $r := r + 1$. $\mathcal{M}_w = 5 \wedge \mathcal{J}_w = t \wedge (w := w + 1)$

= $\exists \mathcal{M}: \infty * int$. $\exists \mathcal{J}: \infty * xnat$. $\exists r, r', w, w': xnat$.

$r := 0$. $w := 0$. $t := t \uparrow (\mathcal{J}_r + 1)$. $r := r + 1$.

$\mathcal{M}_w = 5 \wedge \mathcal{J}_w = t \wedge r' = r \wedge w' = w + 1 \wedge t' = t$

= $\exists \mathcal{M}: \infty * int$. $\exists \mathcal{J}: \infty * xnat$. $\exists r, r', w, w': xnat$.

$\mathcal{M}_0 = 5 \wedge \mathcal{J}_0 = t \uparrow (\mathcal{J}_0 + 1) \wedge r' = 1 \wedge w' = 1 \wedge t' = t \uparrow (\mathcal{J}_0 + 1)$

Deadlock

chan $c: int$. $t := t \uparrow (\mathcal{J}_r + 1)$. $c?$. $c! 5$

= $\exists \mathcal{M}: \infty * int$. $\exists \mathcal{J}: \infty * xnat$. **var** $r, w: xnat := 0$.

$t := t \uparrow (\mathcal{J}_r + 1)$. $r := r + 1$. $\mathcal{M}_w = 5 \wedge \mathcal{J}_w = t \wedge (w := w + 1)$

= $\exists \mathcal{M}: \infty * int$. $\exists \mathcal{J}: \infty * xnat$. $\exists r, r', w, w': xnat$.

$r := 0$. $w := 0$. $t := t \uparrow (\mathcal{J}_r + 1)$. $r := r + 1$.

$\mathcal{M}_w = 5 \wedge \mathcal{J}_w = t \wedge r' = r \wedge w' = w + 1 \wedge t' = t$

= $\exists \mathcal{M}: \infty * int$. $\exists \mathcal{J}: \infty * xnat$. $\exists r, r', w, w': xnat$.

$\mathcal{M}_0 = 5 \wedge \mathcal{J}_0 = t \uparrow (\mathcal{J}_0 + 1) \wedge r' = 1 \wedge w' = 1 \wedge t' = t \uparrow (\mathcal{J}_0 + 1)$



Deadlock

chan $c: int$. $t := t \uparrow (\mathcal{J}_r + 1)$. $c?$. $c! 5$

= $\exists \mathcal{M}: \infty * int$. $\exists \mathcal{J}: \infty * xnat$. **var** $r, w: xnat := 0$.

$t := t \uparrow (\mathcal{J}_r + 1)$. $r := r + 1$. $\mathcal{M}_w = 5 \wedge \mathcal{J}_w = t \wedge (w := w + 1)$

= $\exists \mathcal{M}: \infty * int$. $\exists \mathcal{J}: \infty * xnat$. $\exists r, r', w, w': xnat$.

$r := 0$. $w := 0$. $t := t \uparrow (\mathcal{J}_r + 1)$. $r := r + 1$.

$\mathcal{M}_w = 5 \wedge \mathcal{J}_w = t \wedge r' = r \wedge w' = w + 1 \wedge t' = t$

= $\exists \mathcal{M}: \infty * int$. $\exists \mathcal{J}: \infty * xnat$. $\exists r, r', w, w': xnat$.

$\mathcal{M}_0 = 5 \wedge \mathcal{J}_0 = t \uparrow (\mathcal{J}_0 + 1) \wedge r' = 1 \wedge w' = 1 \wedge t' = t \uparrow (\mathcal{J}_0 + 1)$



Deadlock

chan $c: \text{int}$ $t := t \uparrow (\mathcal{J}_r + 1)$. $c?$. $c! 5$

$$= \exists \mathcal{M}: \infty^* \text{int} \cdot \exists \mathcal{J}: \infty^* \text{xnat} \cdot \text{var } \mathbf{r}, \mathbf{w}: \text{xnat} := 0 \cdot \\ t := t \uparrow (\mathcal{J}_r + 1) \cdot \mathbf{r} := \mathbf{r} + 1 \cdot \mathcal{M}_w = 5 \wedge \mathcal{J}_w = t \wedge (\mathbf{w} := \mathbf{w} + 1)$$

$$= \exists \mathcal{M}: \infty^* \text{int} \cdot \exists \mathcal{J}: \infty^* \text{xnat} \cdot \exists \mathbf{r}, \mathbf{r}', \mathbf{w}, \mathbf{w}': \text{xnat} \cdot \\ \mathbf{r} := 0 \cdot \mathbf{w} := 0 \cdot t := t \uparrow (\mathcal{J}_r + 1) \cdot \mathbf{r} := \mathbf{r} + 1 \cdot \\ \mathcal{M}_w = 5 \wedge \mathcal{J}_w = t \wedge \mathbf{r}' = \mathbf{r} \wedge \mathbf{w}' = \mathbf{w} + 1 \wedge t' = t$$

$$= \exists \mathcal{M}: \infty^* \text{int} \cdot \exists \mathcal{J}: \infty^* \text{xnat} \cdot \exists \mathbf{r}, \mathbf{r}', \mathbf{w}, \mathbf{w}': \text{xnat} \cdot \\ \mathcal{M}_0 = 5 \wedge \mathcal{J}_0 = t \uparrow (\mathcal{J}_0 + 1) \wedge \mathbf{r}' = 1 \wedge \mathbf{w}' = 1 \wedge t' = t \uparrow (\mathcal{J}_0 + 1)$$

$$= t' = \infty$$

Deadlock

```
chan c, d: int (c?. d! 6) || (d?. c! 7)
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Deadlock

chan $c, d: int$ ($c?. d! 6$) \parallel ($d?. c! 7$)

chan $c, d: int$ ($t := t \uparrow (\mathcal{J}c_{rc} + 1). c?. d! 6$) \parallel ($t := t \uparrow (\mathcal{J}d_{rd} + 1). d?. c! 7$)

Deadlock

chan $c, d: \text{int}$ $(c?. d! 6) \parallel (d?. c! 7)$

chan $c, d: \text{int}$ $(t := t \uparrow (\mathcal{J}c_{rc} + 1). c?. d! 6) \parallel (t := t \uparrow (\mathcal{J}d_{rd} + 1). d?. c! 7)$

= $\exists \mathcal{M}c, \mathcal{M}d: \infty^* \text{int} \cdot \exists \mathcal{J}c, \mathcal{J}d: \infty^* \text{xnat} \cdot \exists rc, rc', wc, wc', rd, rd', wd, wd': \text{xnat} \cdot$

$\mathcal{M}d_0 = 6 \wedge \mathcal{M}c_0 = 7 \wedge rc' = wc' = rd' = wd' = 1$

$\wedge \mathcal{J}c_0 = t \uparrow (\mathcal{J}d_0 + 1) \wedge \mathcal{J}d_0 = t \uparrow (\mathcal{J}c_0 + 1)$

$\wedge t' = t \uparrow (\mathcal{J}d_0 + 1) \uparrow (\mathcal{J}c_0 + 1)$

Deadlock

chan $c, d: \text{int}$ ($c?. d! 6$) \parallel ($d?. c! 7$)

chan $c, d: \text{int}$ ($t := t \uparrow (\mathcal{J}c_{rc} + 1). c?. d! 6$) \parallel ($t := t \uparrow (\mathcal{J}d_{rd} + 1). d?. c! 7$)

= $\exists \mathcal{M}c, \mathcal{M}d: \infty^* \text{int} \cdot \exists \mathcal{J}c, \mathcal{J}d: \infty^* \text{xnat} \cdot \exists rc, rc', wc, wc', rd, rd', wd, wd': \text{xnat} \cdot$

$\mathcal{M}d_0 = 6 \wedge \mathcal{M}c_0 = 7 \wedge rc' = wc' = rd' = wd' = 1$

$\wedge \mathcal{J}c_0 = t \uparrow (\mathcal{J}d_0 + 1) \wedge \mathcal{J}d_0 = t \uparrow (\mathcal{J}c_0 + 1) \leftarrow$

$\wedge t' = t \uparrow (\mathcal{J}d_0 + 1) \uparrow (\mathcal{J}c_0 + 1)$

Deadlock

chan $c, d: int$ ($c?. d! 6$) \parallel ($d?. c! 7$)

chan $c, d: int$ ($t := t \uparrow (\mathcal{J}c_{rc} + 1). c?. d! 6$) \parallel ($t := t \uparrow (\mathcal{J}d_{rd} + 1). d?. c! 7$)

= $\exists \mathcal{M}c, \mathcal{M}d: \infty * int. \exists \mathcal{J}c, \mathcal{J}d: \infty * xnat. \exists rc, rc', wc, wc', rd, rd', wd, wd': xnat.$

$\mathcal{M}d_0 = 6 \wedge \mathcal{M}c_0 = 7 \wedge rc' = wc' = rd' = wd' = 1$

$\wedge \mathcal{J}c_0 = t \uparrow (\mathcal{J}d_0 + 1) \wedge \mathcal{J}d_0 = t \uparrow (\mathcal{J}c_0 + 1)$

$\wedge t' = t \uparrow (\mathcal{J}d_0 + 1) \uparrow (\mathcal{J}c_0 + 1) \leftarrow$

Deadlock

chan $c, d: int$ ($c?. d! 6$) \parallel ($d?. c! 7$)

chan $c, d: int$ ($t := t \uparrow (\mathcal{J}c_{rc} + 1). c?. d! 6$) \parallel ($t := t \uparrow (\mathcal{J}d_{rd} + 1). d?. c! 7$)

= $\exists \mathcal{M}c, \mathcal{M}d: \infty * int. \exists \mathcal{J}c, \mathcal{J}d: \infty * xnat. \exists rc, rc', wc, wc', rd, rd', wd, wd': xnat.$

$\mathcal{M}d_0 = 6 \wedge \mathcal{M}c_0 = 7 \wedge rc' = wc' = rd' = wd' = 1$

$\wedge \mathcal{J}c_0 = t \uparrow (\mathcal{J}d_0 + 1) \wedge \mathcal{J}d_0 = t \uparrow (\mathcal{J}c_0 + 1)$

$\wedge t' = t \uparrow (\mathcal{J}d_0 + 1) \uparrow (\mathcal{J}c_0 + 1)$

= $t' = \infty$

Power Series Multiplication

Power Series Multiplication

Input on channel a : $a_0 a_1 a_2 \dots$

Power Series Multiplication

Input on channel a : $a_0 a_1 a_2 \dots$

$$A = a_0 + a_1 \times x + a_2 \times x^2 + \dots$$

Power Series Multiplication

Input on channel a : $a_0 a_1 a_2 \dots$

$$A = a_0 + a_1x + a_2x^2 + \dots$$

Input on channel b : $b_0 b_1 b_2 \dots$

Power Series Multiplication

Input on channel a : $a_0 a_1 a_2 \dots$

$$A = a_0 + a_1x + a_2x^2 + \dots$$

Input on channel b : $b_0 b_1 b_2 \dots$

$$B = b_0 + b_1x + b_2x^2 + \dots$$

Power Series Multiplication

Input on channel a : $a_0 a_1 a_2 \dots$

$$A = a_0 + a_1x + a_2x^2 + \dots$$

Input on channel b : $b_0 b_1 b_2 \dots$

$$B = b_0 + b_1x + b_2x^2 + \dots$$

Output on channel c : $c_0 c_1 c_2 \dots$

Power Series Multiplication

Input on channel a : $a_0 a_1 a_2 \dots$

$$A = a_0 + a_1x + a_2x^2 + \dots$$

Input on channel b : $b_0 b_1 b_2 \dots$

$$B = b_0 + b_1x + b_2x^2 + \dots$$

Output on channel c : $c_0 c_1 c_2 \dots$

$$C = c_0 + c_1x + c_2x^2 + \dots$$

Power Series Multiplication

Input on channel a : $a_0 a_1 a_2 \dots$

$$A = a_0 + a_1 \times x + a_2 \times x^2 + \dots$$

Input on channel b : $b_0 b_1 b_2 \dots$

$$B = b_0 + b_1 \times x + b_2 \times x^2 + \dots$$

Output on channel c : $c_0 c_1 c_2 \dots$

$$C = c_0 + c_1 \times x + c_2 \times x^2 + \dots$$

$$C = A \times B$$

Power Series Multiplication

Input on channel a : $a_0 a_1 a_2 \dots$

$$A = a_0 + a_1 \times x + a_2 \times x^2 + \dots$$

Input on channel b : $b_0 b_1 b_2 \dots$

$$B = b_0 + b_1 \times x + b_2 \times x^2 + \dots$$

Output on channel c : $c_0 c_1 c_2 \dots$

$$C = c_0 + c_1 \times x + c_2 \times x^2 + \dots$$

$$C = A \times B = a_0 \times b_0 + (a_0 \times b_1 + a_1 \times b_0)x + (a_0 \times b_2 + a_1 \times b_1 + a_2 \times b_0)x^2 + \dots$$

Power Series Multiplication

Input on channel a : $a_0 a_1 a_2 \dots$

$$A = a_0 + a_1 \times x + a_2 \times x^2 + \dots$$

Input on channel b : $b_0 b_1 b_2 \dots$

$$B = b_0 + b_1 \times x + b_2 \times x^2 + \dots$$

Output on channel c : $c_0 c_1 c_2 \dots$

$$C = c_0 + c_1 \times x + c_2 \times x^2 + \dots$$

$$A_1 = a_1 + a_2 \times x + a_3 \times x^2 + \dots$$

$$B_1 = b_1 + b_2 \times x + b_3 \times x^2 + \dots$$

$$A_2 = a_2 + a_3 \times x + a_4 \times x^2 + \dots$$

$$B_2 = b_2 + b_3 \times x + b_4 \times x^2 + \dots$$

$$C = A \times B = a_0 \times b_0 + (a_0 \times b_1 + a_1 \times b_0)x + (a_0 \times b_2 + a_1 \times b_1 + a_2 \times b_0)x^2 + \dots$$

Power Series Multiplication

Input on channel a : $a_0 a_1 a_2 \dots$

$$A = a_0 + a_1 \times x + a_2 \times x^2 + \dots$$

Input on channel b : $b_0 b_1 b_2 \dots$

$$B = b_0 + b_1 \times x + b_2 \times x^2 + \dots$$

Output on channel c : $c_0 c_1 c_2 \dots$

$$C = c_0 + c_1 \times x + c_2 \times x^2 + \dots$$

$$A_1 = a_1 + a_2 \times x + a_3 \times x^2 + \dots$$

$$B_1 = b_1 + b_2 \times x + b_3 \times x^2 + \dots$$

$$A_2 = a_2 + a_3 \times x + a_4 \times x^2 + \dots$$

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$$C = A \times B = a_0 \times b_0 + (a_0 \times b_1 + a_1 \times b_0)x + (a_0 \times b_2 + a_1 \times b_1 + a_2 \times b_0)x^2 + \dots$$



Power Series Multiplication

Input on channel a : $a_0 a_1 a_2 \dots$

$$A = a_0 + a_1 \times x + a_2 \times x^2 + \dots$$

Input on channel b : $b_0 b_1 b_2 \dots$

$$B = b_0 + b_1 \times x + b_2 \times x^2 + \dots$$

Output on channel c : $c_0 c_1 c_2 \dots$

$$C = c_0 + c_1 \times x + c_2 \times x^2 + \dots$$


$$A_1 = a_1 + a_2 \times x + a_3 \times x^2 + \dots$$

$$B_1 = b_1 + b_2 \times x + b_3 \times x^2 + \dots$$

$$A_2 = a_2 + a_3 \times x + a_4 \times x^2 + \dots$$

$$B_2 = b_2 + b_3 \times x + b_4 \times x^2 + \dots$$

$$C = A \times B = a_0 \times b_0 + (a_0 \times b_1 + a_1 \times b_0)x + (a_0 \times b_2 + a_1 \times b_1 + a_2 \times b_0)x^2 + \dots$$



Power Series Multiplication

Input on channel a : $a_0 a_1 a_2 \dots$

$$A = a_0 + a_1 \times x + a_2 \times x^2 + \dots$$

Input on channel b : $b_0 b_1 b_2 \dots$

$$B = b_0 + b_1 \times x + b_2 \times x^2 + \dots$$

Output on channel c : $c_0 c_1 c_2 \dots$

$$C = c_0 + c_1 \times x + c_2 \times x^2 + \dots$$

$$A_1 = a_1 + a_2 \times x + a_3 \times x^2 + \dots$$

$$B_1 = b_1 + b_2 \times x + b_3 \times x^2 + \dots$$

$$A_2 = a_2 + a_3 \times x + a_4 \times x^2 + \dots$$

$$B_2 = b_2 + b_3 \times x + b_4 \times x^2 + \dots$$

$$C = A \times B = a_0 \times b_0 + (a_0 \times b_1 + a_1 \times b_0)x + (a_0 \times b_2 + A_1 \times B_1 + A_2 \times b_0) \times x^2$$

Power Series Multiplication

Input on channel a : $a_0 a_1 a_2 \dots$

$$A = a_0 + a_1 \times x + a_2 \times x^2 + \dots$$

Input on channel b : $b_0 b_1 b_2 \dots$

$$B = b_0 + b_1 \times x + b_2 \times x^2 + \dots$$

Output on channel c : $c_0 c_1 c_2 \dots$


$$C = c_0 + c_1 \times x + c_2 \times x^2 + \dots$$

$$A_1 = a_1 + a_2 \times x + a_3 \times x^2 + \dots$$

$$B_1 = b_1 + b_2 \times x + b_3 \times x^2 + \dots$$

$$A_2 = a_2 + a_3 \times x + a_4 \times x^2 + \dots$$

$$B_2 = b_2 + b_3 \times x + b_4 \times x^2 + \dots$$

$$C = A \times B = a_0 \times b_0 + (a_0 \times b_1 + a_1 \times b_0)x + (a_0 \times B_2 + A_1 \times B_1 + A_2 \times b_0) \times x^2$$


Power Series Multiplication

Input on channel a : $a_0 a_1 a_2 \dots$

$$A = a_0 + a_1 \times x + a_2 \times x^2 + \dots$$

Input on channel b : $b_0 b_1 b_2 \dots$

$$B = b_0 + b_1 \times x + b_2 \times x^2 + \dots$$

Output on channel c : $c_0 c_1 c_2 \dots$

$$C = c_0 + c_1 \times x + c_2 \times x^2 + \dots$$

$$A_1 = a_1 + a_2 \times x + a_3 \times x^2 + \dots$$

$$B_1 = b_1 + b_2 \times x + b_3 \times x^2 + \dots$$

$$A_2 = a_2 + a_3 \times x + a_4 \times x^2 + \dots$$

$$B_2 = b_2 + b_3 \times x + b_4 \times x^2 + \dots$$

$$C = A \times B = a_0 \times b_0 + (a_0 \times b_1 + a_1 \times b_0)x + (a_0 \times b_2 + A_1 \times B_1 + A_2 \times b_0) \times x^2$$



Power Series Multiplication

Input on channel a : $a_0 a_1 a_2 \dots$

$$A = a_0 + a_1 \times x + a_2 \times x^2 + \dots$$

Input on channel b : $b_0 b_1 b_2 \dots$

$$B = b_0 + b_1 \times x + b_2 \times x^2 + \dots$$

Output on channel c : $c_0 c_1 c_2 \dots$

$$C = c_0 + c_1 \times x + c_2 \times x^2 + \dots$$

$$A_1 = a_1 + a_2 \times x + a_3 \times x^2 + \dots$$

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$\langle \mathbf{chan} \ c: \mathit{rat} \cdot C = A \times B \rangle$

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$$\mathbf{var} \ a0: \mathit{rat} := a \cdot \mathbf{var} \ b0: \mathit{rat} := b \cdot \mathbf{chan} \ d: \mathit{rat}$$

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Power Series Multiplication

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