Specification

state space memory

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state memory contents

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state space	memory	*int*; (0,..20); *char*; *rat*
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state	memory contents	–2; 15; "A"; 3.14

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addresses		0 , 1 , 2 , 3

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prestate	initial state	$\sigma = \sigma_0; \sigma_1; \sigma_2; \sigma_3$
poststate	final state	$\sigma' = \sigma'_0; \sigma'_1; \sigma'_2; \sigma'_3$

addresses 0 , 1 , 2 , 3

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addresses	low level	0 , 1 , 2 , 3

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addresses	low level	0 , 1 , 2 , 3
state variables	high level	i , n , c , x

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state variables	high level	i , n , c , x
	initial values	i , n , c , x
	final values	i' , n' , c' , x'

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state variables	high level	i , n , c , x
	initial values	i , n , c , x
	final values	i' , n' , c' , x'

For now: prestate, poststate

Later: time (termination = finite time), space, interaction, communication

Specification

Specification

specification of computer behavior: a binary expression

in variables σ and σ'

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We provide a prestate as input.

A computation satisfies a specification by computing a satisfactory poststate as output.

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A computation satisfies a specification by computing a satisfactory poststate as output.

The given prestate and computed poststate must make the specification true.

Specification

specification of computer behavior: a binary expression

in the initial values x , y , ... and final values x' , y' , ... of some state variables

We provide initial values as input.

A computation satisfies a specification by computing satisfactory final values as output.

The given initial values and computed final values must make the specification true.

Specification

Specification *S* is **unsatisfiable** for prestate σ : ¢(§σ′· *S*) < 1

Specification *S* is **satisfiable** for prestate σ : ¢(§σ′· *S*) ≥ 1

Specification *S* is **deterministic** for prestate σ : ¢(§σ′· *S*) ≤ 1

Specification *S* is **nondeterministic** for prestate σ : ¢(§σ′· *S*) > 1

Specification

Specification S is **unsatisfiable** for prestate σ : $¢(§\sigma' \cdot S) < 1$ ←

Specification S is **satisfiable** for prestate σ : $¢(§\sigma' \cdot S) \geq 1$

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Specification

Specification S is **unsatisfiable** for prestate σ : ¢(§σ'· S) $<$ 1

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Specification S is **satisfiable** for prestate σ : $\exists \sigma' \cdot S$

Specification

Specification S is **unsatisfiable** for prestate σ : $¢(\S\sigma' \cdot S) < 1$

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Specification S is **nondeterministic** for prestate σ : $¢(\S\sigma' \cdot S) > 1$

Specification S is **satisfiable** for prestate σ : $\exists\sigma' \cdot S$

Specification S is **implementable**: $\forall\sigma \cdot \exists\sigma' \cdot S$

Specification

examples

$$x' = x+1 \;\; \wedge \;\; y' = y$$

Specification

examples

$x' = x+1 \;\land\; y' = y$ implementable, deterministic

Specification

examples

$x' = x+1 \land y' = y$ implementable, deterministic

$x' > x$

Specification

examples

$x' = x+1 \ \wedge \ y' = y$	implementable, deterministic
$x' > x$	implementable, nondeterministic

Specification

examples

$x' = x+1 \;\wedge\; y' = y$	implementable, deterministic
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$\top$	

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Specification

examples

$x' = x+1 \wedge y' = y$	implementable, deterministic
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examples

$x' = x+1 \land y' = y$	implementable, deterministic
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$x \ge 0 \land y'=0$	

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$x' = x+1 \ \land \ y' = y$	implementable, deterministic
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ok

Specification

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$x' = x+1 \;\wedge\; y' = y$	implementable, deterministic
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$$ok \quad = \quad \sigma'{=}\sigma$$

Specification

examples

$x' = x+1 \ \land \ y' = y$	implementable, deterministic
$x' > x$	implementable, nondeterministic
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$$ok \quad = \quad \sigma'{=}\sigma \quad = \quad x'{=}x \land y'{=}y \land \ldots$$

Specification

examples

$x' = x+1 \;\wedge\; y' = y$	implementable, deterministic
$x' > x$	implementable, nondeterministic
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$ok \quad = \quad \sigma'{=}\sigma \quad = \quad x'{=}x \wedge y'{=}y \wedge \ldots$

$x{:=}\ e$

Specification

examples

$x' = x+1 \ \wedge \ y' = y$	implementable, deterministic
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$x{:=}\ e$

$x{:=}\ x{+}1$

Specification

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$ok \quad = \quad \sigma'{=}\sigma \quad = \quad x'{=}x \wedge y'{=}y \wedge \ldots$

$x{:=}\ e$

$x{:=}\ x{+}1$ **NOT** $x = x{+}1$

Specification

examples

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$ok \quad = \quad \sigma'=\sigma \quad = \quad x'=x \wedge y'=y \wedge \ldots$

$x:= e$

$x:= x+1$

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$$ok \quad = \quad \sigma'=\sigma \quad = \quad x'=x \land y'=y \land \ldots$$

$$x{:=}\ e \quad = \quad \sigma' \ = \ \sigma \triangleleft \textit{address} \text{ “}x\text{” } \triangleright e$$

$$x{:=}\ x+1$$

Specification

examples

$x' = x+1 \ \wedge \ y' = y$	implementable, deterministic
$x' > x$	implementable, nondeterministic
$\top$	implementable, extremely nondeterministic
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$x \ge 0 \ \wedge \ y'=0$	unimplementable, overly deterministic
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$$ok \quad = \quad \sigma'=\sigma \quad = \quad x'=x \wedge y'=y \wedge \ldots$$

$$x := e \quad = \quad \sigma' = \sigma \triangleleft \mathit{address}\ \text{“}x\text{”} \triangleright e \quad = \quad x'=e \wedge y'=y \wedge \ldots$$

$x := x+1$

Specification

examples

$x' = x+1 \ \land \ y' = y$	implementable, deterministic
$x' > x$	implementable, nondeterministic
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ok	=	$\sigma'{=}\sigma$	=	$x'{=}x \land y'{=}y \land \ldots$
$x{:=}\ e$	=	$\sigma' \ = \ \sigma \triangleleft \mathit{address}$ "x" $\triangleright\ e$	=	$x'{=}e \land y'{=}y \land \ldots$
$x{:=}\ x{+}1$			=	$x'{=}x{+}1 \land y'{=}y$

Specification

examples

$x' = x+1 \land y' = y$	implementable, deterministic
$x' > x$	implementable, nondeterministic
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$x \ge 0 \land y'=0$	unimplementable, overly deterministic
$x \ge 0 \Rightarrow y'=0$	implementable, nondeterministic

ok	$=$	$\sigma'=\sigma$	$=$	$x'=x \land y'=y \land ...$
$x:= e$	$=$	$\sigma' = \sigma \triangleleft address$ “x” $\triangleright e$	$=$	$x'=e \land y'=y \land ...$
$x:= x+1$			$=$	$x'=x+1 \land y'=y$

if $x=y$ **then** $x:= x+1$ **else** $x'+y' = 3$ **fi**

sequential composition

$S.R$

sequential composition

$$S.R \quad = \quad \exists x'', y'', ... \cdot \quad \text{(substitute } x'', y'', ... \text{ for } x', y', ... \text{ in } S \text{)}$$
$$\wedge \quad \text{(substitute } x'', y'', ... \text{ for } x, y, ... \text{ in } R \text{)}$$

sequential composition

$$S. R \quad = \quad \exists x'', y'', ...\cdot \quad (\text{substitute } x'', y'', ... \text{ for } x', y', ... \text{ in } S)$$

$$\land \quad (\text{substitute } x'', y'', ... \text{ for } x, y, ... \text{ in } R)$$

In integer variable x

$$x'=x \lor x'=x+1 \;.\; x'=x \lor x'=x+1$$

sequential composition

$$S.R \;=\; \exists x'', y'', ...\cdot \quad (\text{substitute } x'', y'', ... \text{ for } x', y', ... \text{ in } S)$$

$$\wedge \quad (\text{substitute } x'', y'', ... \text{ for } x, y, ... \text{ in } R)$$

In integer variable x

$$x'=x \vee x'=x+1 \;.\; x'=x \vee x'=x+1$$

$$= \quad \exists x''\cdot\ (x''=x \vee x''=x+1) \wedge (x'=x'' \vee x'=x''+1)$$

sequential composition

$$S.R \quad = \quad \exists x'', y'', ...\cdot \quad (\text{substitute } x'', y'', ... \text{ for } x', y', ... \text{ in } S)$$
$$\land \quad (\text{substitute } x'', y'', ... \text{ for } x, y, ... \text{ in } R)$$

In integer variable x

$$x'=x \lor x'=x+1 \;.\; x'=x \lor x'=x+1$$

$$= \quad \exists x''\cdot\; (x''=x \lor x''=x+1) \land (x'=x'' \lor x'=x''+1) \qquad \text{distribute } \land \text{ over } \lor$$

sequential composition

$$S.R \quad = \quad \exists x'', y'', ...\cdot \quad \text{(substitute } x'', y'', ... \text{ for } x', y', ... \text{ in } S\text{)}$$

$$\land \quad \text{(substitute } x'', y'', ... \text{ for } x, y, ... \text{ in } R\text{)}$$

In integer variable x

$$x'=x \lor x'=x+1 \ . \ x'=x \lor x'=x+1$$

$$= \quad \exists x''\cdot \ (x''=x \lor x''=x+1) \land (x'=x'' \lor x'=x''+1) \qquad \text{distribute } \land \text{ over } \lor$$

$$\uparrow \qquad\qquad \uparrow$$

sequential composition

$$S.R \quad = \quad \exists x'', y'', ...\cdot \quad (\text{substitute } x'', y'', ... \text{ for } x', y', ... \text{ in } S)$$
$$\land \quad (\text{substitute } x'', y'', ... \text{ for } x, y, ... \text{ in } R)$$

In integer variable x

$$x'{=}x \lor x'{=}x{+}1 \;.\; x'{=}x \lor x'{=}x{+}1$$

$$= \quad \exists x''\cdot\ (x''{=}x \lor x''{=}x{+}1) \land (x'{=}x'' \lor x'{=}x''{+}1) \qquad \text{distribute } \land \text{ over } \lor$$

sequential composition

$$S.R \quad = \quad \exists x'', y'', ...\cdot \quad (\text{substitute } x'', y'', ... \text{ for } x', y', ... \text{ in } S)$$

$$\land \quad (\text{substitute } x'', y'', ... \text{ for } x, y, ... \text{ in } R)$$

In integer variable x

$$x'=x \lor x'=x+1 \ . \ x'=x \lor x'=x+1$$

$$= \quad \exists x''\cdot \ (x''=x \lor x''=x+1) \land (x'=x'' \lor x'=x''+1) \qquad \text{distribute } \land \text{ over } \lor$$

$$\uparrow \qquad\qquad\qquad \uparrow$$

sequential composition

$$S . R \quad = \quad \exists x'', y'', \ldots \cdot \quad (\text{substitute } x'', y'', \ldots \text{ for } x', y', \ldots \text{ in } S\,)$$

$$\land \quad (\text{substitute } x'', y'', \ldots \text{ for } x, y, \ldots \text{ in } R\,)$$

In integer variable x

$$x'{=}x \lor x'{=}x{+}1 \ . \ x'{=}x \lor x'{=}x{+}1$$

$$= \quad \exists x'' \cdot \ (x''{=}x \lor x''{=}x{+}1) \ \land \ (x'{=}x'' \lor x'{=}x''{+}1) \qquad \text{distribute } \land \text{ over } \lor$$

sequential composition

$$S.R \quad = \quad \exists x'', y'', ...\cdot \quad (\text{substitute } x'', y'', ... \text{ for } x', y', ... \text{ in } S)$$

$$\land \quad (\text{substitute } x'', y'', ... \text{ for } x, y, ... \text{ in } R)$$

In integer variable x

$$x'=x \lor x'=x+1 \;.\; x'=x \lor x'=x+1$$

$$= \quad \exists x''\cdot\ (x''=x \lor x''=x+1) \land (x'=x'' \lor x'=x''+1) \qquad \text{distribute } \land \text{ over } \lor$$

$$= \quad \exists x''\cdot\ \quad x''=x \land x'=x'' \lor x''=x+1 \land x'=x''$$

$$\lor \quad x''=x \land x'=x''+1 \lor x''=x+1 \land x'=x''+1$$

sequential composition

$$S.\,R \quad = \quad \exists x'', y'', ...\cdot \quad (\text{substitute } x'', y'', ... \text{ for } x', y', ... \text{ in } S)$$
$$\land \quad (\text{substitute } x'', y'', ... \text{ for } x, y, ... \text{ in } R)$$

In integer variable x

$$x'=x \lor x'=x+1 \ . \ x'=x \lor x'=x+1$$

$$= \quad \exists x''\cdot\ (x''=x \lor x''=x+1) \land (x'=x'' \lor x'=x''+1) \qquad \text{distribute } \land \text{ over } \lor$$

$$= \quad \exists x''\cdot\ \ x''=x \land x'=x'' \lor x''=x+1 \land x'=x''$$
$$\lor \ x''=x \land x'=x''+1 \lor x''=x+1 \land x'=x''+1 \qquad \text{distribute } \exists \text{ over } \lor$$

$$= \quad (\exists x''\cdot\ x''=x \land x'=x'') \lor (\exists x''\cdot\ x''=x+1 \land x'=x'')$$
$$\lor\ (\exists x''\cdot\ x''=x \land x'=x''+1) \lor (\exists x''\cdot\ x''=x+1 \land x'=x''+1)$$

sequential composition

$$S.R \quad = \quad \exists x'', y'', \ldots \cdot \quad (\text{substitute } x'', y'', \ldots \text{ for } x', y', \ldots \text{ in } S)$$
$$\land \quad (\text{substitute } x'', y'', \ldots \text{ for } x, y, \ldots \text{ in } R)$$

In integer variable x

$$x'=x \lor x'=x+1 \,.\, x'=x \lor x'=x+1$$

$$= \quad \exists x''\cdot\ (x''=x \lor x''=x+1) \land (x'=x'' \lor x'=x''+1) \qquad \text{distribute } \land \text{ over } \lor$$

$$= \quad \exists x''\cdot\ \quad x''=x \land x'=x'' \lor x''=x+1 \land x'=x''$$
$$\lor \quad x''=x \land x'=x''+1 \lor x''=x+1 \land x'=x''+1 \qquad \text{distribute } \exists \text{ over } \lor$$

$$= \quad (\exists x''\cdot\ x''=x \land x'=x'') \lor (\exists x''\cdot\ x''=x+1 \land x'=x'')$$
$$\lor (\exists x''\cdot\ x''=x \land x'=x''+1) \lor (\exists x''\cdot\ x''=x+1 \land x'=x''+1) \qquad \text{One-Point Law 4 times}$$

sequential composition

$$S.R \quad = \quad \exists x'', y'', ...\cdot \quad (\text{substitute } x'', y'', ... \text{ for } x', y', ... \text{ in } S)$$

$$\land \quad (\text{substitute } x'', y'', ... \text{ for } x, y, ... \text{ in } R)$$

In integer variable x

$$x'=x \lor x'=x+1 \ . \ x'=x \lor x'=x+1$$

$$= \quad \exists x''\cdot \ (x''=x \lor x''=x+1) \land (x'=x'' \lor x'=x''+1) \qquad \text{distribute } \land \text{ over } \lor$$

$$= \quad \exists x''\cdot \quad x''=x \land x'=x'' \lor x''=x+1 \land x'=x''$$

$$\lor \quad x''=x \land x'=x''+1 \lor x''=x+1 \land x'=x''+1 \qquad \text{distribute } \exists \text{ over } \lor$$

$$= \quad (\exists x''\cdot \ x''=x \land x'=x'') \lor (\exists x''\cdot \ x''=x+1 \land x'=x'')$$

$$\lor \ (\exists x''\cdot \ x''=x \land x'=x''+1) \lor (\exists x''\cdot \ x''=x+1 \land x'=x''+1) \qquad \text{One-Point Law 4 times}$$

↓

$$\exists v\cdot \ v=e \land P \quad = \quad (\text{replace } v \text{ with } e \text{ in } P)$$

sequential composition

$$S.R \quad = \quad \exists x'', y'', ...\cdot \quad \text{(substitute } x'', y'', ... \text{ for } x', y', ... \text{ in } S\text{)}$$
$$\land \quad \text{(substitute } x'', y'', ... \text{ for } x, y, ... \text{ in } R\text{)}$$

In integer variable x

$$x'=x \lor x'=x+1 \ . \ x'=x \lor x'=x+1$$

$$= \quad \exists x''\cdot \ (x''=x \lor x''=x+1) \land (x'=x'' \lor x'=x''+1) \qquad \text{distribute } \land \text{ over } \lor$$

$$= \quad \exists x''\cdot \quad x''=x \land x'=x'' \lor x''=x+1 \land x'=x''$$
$$\lor \quad x''=x \land x'=x''+1 \lor x''=x+1 \land x'=x''+1 \qquad \text{distribute } \exists \text{ over } \lor$$

$$= \quad (\exists x''\cdot \ x''=x \land x'=x'') \lor (\exists x''\cdot \ x''=x+1 \land x'=x'')$$
$$\lor \ (\exists x''\cdot \ x''=x \land x'=x''+1) \lor (\exists x''\cdot \ x''=x+1 \land x'=x''+1) \qquad \text{One-Point Law 4 times}$$

↓

$$\exists v\cdot \ v=e \land P \quad = \quad \text{(replace } v \text{ with } e \text{ in } P\text{)}$$

sequential composition

$$S.R \quad = \quad \exists x'', y'', ...\cdot \quad \text{(substitute } x'', y'', ... \text{ for } x', y', ... \text{ in } S\text{)}$$
$$\land \quad \text{(substitute } x'', y'', ... \text{ for } x, y, ... \text{ in } R\text{)}$$

In integer variable x

$$x'=x \lor x'=x+1 \;.\; x'=x \lor x'=x+1$$

$= \quad \exists x''\cdot\ (x''=x \lor x''=x+1) \land (x'=x'' \lor x'=x''+1)$ — distribute $\land$ over $\lor$

$= \quad \exists x''\cdot\ \quad x''=x \land x'=x'' \lor x''=x+1 \land x'=x''$
$\quad \lor \quad x''=x \land x'=x''+1 \lor x''=x+1 \land x'=x''+1$ — distribute $\exists$ over $\lor$

$= \quad (\exists x''\cdot\ x''=x \land x'=x'') \lor (\exists x''\cdot\ x''=x+1 \land x'=x'')$
$\quad \lor (\exists x''\cdot\ x''=x \land x'=x''+1) \lor (\exists x''\cdot\ x''=x+1 \land x'=x''+1)$ — One-Point Law 4 times

↓

$$\exists v\cdot\ v=e \land P \quad = \quad \text{(replace } v \text{ with } e \text{ in } P\text{)}$$

sequential composition

$$S.\,R \quad = \quad \exists x'', y'', ...\cdot \quad (\text{substitute } x'', y'', ... \text{ for } x', y', ... \text{ in } S)$$

$$\land \quad (\text{substitute } x'', y'', ... \text{ for } x, y, ... \text{ in } R)$$

In integer variable x

$$x'=x \lor x'=x+1 \;.\; x'=x \lor x'=x+1$$

$$= \quad \exists x''\cdot\ (x''=x \lor x''=x+1) \land (x'=x'' \lor x'=x''+1) \qquad \text{distribute } \land \text{ over } \lor$$

$$= \quad \exists x''\cdot\ \quad x''=x \land x'=x'' \lor x''=x+1 \land x'=x''$$

$$\lor \quad x''=x \land x'=x''+1 \lor x''=x+1 \land x'=x''+1 \qquad \text{distribute } \exists \text{ over } \lor$$

$$= \quad (\exists x''\cdot\ x''=x \land x'=x'') \lor (\exists x''\cdot\ x''=x+1 \land x'=x'')$$

$$\lor\ (\exists x''\cdot\ x''=x \land x'=x''+1) \lor (\exists x''\cdot\ x''=x+1 \land x'=x''+1) \qquad \text{One-Point Law 4 times}$$

↓

$$\exists v\cdot\ v=e \land P \quad = \quad (\text{replace } v \text{ with } e \text{ in } P)$$

sequential composition

$$S.R \quad = \quad \exists x'', y'', ...\cdot \quad (\text{substitute } x'', y'', ... \text{ for } x', y', ... \text{ in } S)$$

$$\land \quad (\text{substitute } x'', y'', ... \text{ for } x, y, ... \text{ in } R)$$

In integer variable x

$$x'=x \lor x'=x+1 \;.\; x'=x \lor x'=x+1$$

$$= \quad \exists x''\cdot\ (x''=x \lor x''=x+1) \land (x'=x'' \lor x'=x''+1) \qquad \text{distribute } \land \text{ over } \lor$$

$$= \quad \exists x''\cdot\ \quad x''=x \land x'=x'' \lor x''=x+1 \land x'=x''$$

$$\lor \quad x''=x \land x'=x''+1 \lor x''=x+1 \land x'=x''+1 \qquad \text{distribute } \exists \text{ over } \lor$$

$$= \quad (\exists x''\cdot\ x''=x \land x'=x'') \lor (\exists x''\cdot\ x''=x+1 \land x'=x'')$$

$$\lor (\exists x''\cdot\ x''=x \land x'=x''+1) \lor (\exists x''\cdot\ x''=x+1 \land x'=x''+1) \qquad \text{One-Point Law 4 times}$$

sequential composition

$$S.R \quad = \quad \exists x'', y'', ...\cdot \quad (\text{substitute } x'', y'', ... \text{ for } x', y', ... \text{ in } S)$$

$$\land \quad (\text{substitute } x'', y'', ... \text{ for } x, y, ... \text{ in } R)$$

In integer variable x

$$x'=x \lor x'=x+1 \ . \ x'=x \lor x'=x+1$$

$$= \quad \exists x''\cdot \ (x''=x \lor x''=x+1) \ \land \ (x'=x'' \lor x'=x''+1) \qquad \text{distribute } \land \text{ over } \lor$$

$$= \quad \exists x''\cdot \quad x''=x \land x'=x'' \ \lor \ x''=x+1 \land x'=x''$$

$$\lor \quad x''=x \land x'=x''+1 \ \lor \ x''=x+1 \land x'=x''+1 \qquad \text{distribute } \exists \text{ over } \lor$$

$$= \quad (\exists x''\cdot \ x''=x \land x'=x'') \ \lor \ (\exists x''\cdot \ x''=x+1 \land x'=x'')$$

$$\lor \ (\exists x''\cdot \ x''=x \land x'=x''+1) \ \lor \ (\exists x''\cdot \ x''=x+1 \land x'=x''+1) \qquad \text{One-Point Law 4 times}$$

$$= \quad x'=x \lor$$

sequential composition

$$S.R \quad = \quad \exists x'', y'', ...\cdot \quad (\text{substitute } x'', y'', ... \text{ for } x', y', ... \text{ in } S)$$
$$\land \quad (\text{substitute } x'', y'', ... \text{ for } x, y, ... \text{ in } R)$$

In integer variable x

$$x'=x \lor x'=x+1 \;.\; x'=x \lor x'=x+1$$

$= \quad \exists x''\cdot\ (x''=x \lor x''=x+1) \land (x'=x'' \lor x'=x''+1)$ distribute $\land$ over $\lor$

$= \quad \exists x''\cdot\ x''=x \land x'=x'' \lor x''=x+1 \land x'=x''$
$\quad \lor\ x''=x \land x'=x''+1 \lor x''=x+1 \land x'=x''+1$ distribute $\exists$ over $\lor$

$= \quad (\exists x''\cdot\ x''=x \land x'=x'') \lor (\exists x''\cdot\ x''=x+1 \land x'=x'')$
$\quad \lor (\exists x''\cdot\ x''=x \land x'=x''+1) \lor (\exists x''\cdot\ x''=x+1 \land x'=x''+1)$ One-Point Law 4 times

$= \quad x'=x \lor x'=x+1 \lor$

sequential composition

$$S.R \quad = \quad \exists x'', y'', ...\cdot \quad (\text{substitute } x'', y'', ... \text{ for } x', y', ... \text{ in } S)$$
$$\land \quad (\text{substitute } x'', y'', ... \text{ for } x, y, ... \text{ in } R)$$

In integer variable x

$$x'=x \lor x'=x+1 \ . \ x'=x \lor x'=x+1$$

$$= \quad \exists x''\cdot\ (x''=x \lor x''=x+1) \land (x'=x'' \lor x'=x''+1) \qquad \text{distribute } \land \text{ over } \lor$$

$$= \quad \exists x''\cdot\ x''=x \land x'=x'' \lor x''=x+1 \land x'=x''$$
$$\lor\ x''=x \land x'=x''+1 \lor x''=x+1 \land x'=x''+1 \qquad \text{distribute } \exists \text{ over } \lor$$

$$= \quad (\exists x''\cdot\ x''=x \land x'=x'') \lor (\exists x''\cdot\ x''=x+1 \land x'=x'')$$
$$\lor\ (\exists x''\cdot\ x''=x \land x'=x''+1) \lor (\exists x''\cdot\ x''=x+1 \land x'=x''+1) \qquad \text{One-Point Law 4 times}$$

$$= \quad x'=x \lor x'=x+1 \lor x'=x+2$$

sequential composition

$$S.\,R \quad = \quad \exists x'', y'', ...\cdot \quad (\text{substitute } x'', y'', ... \text{ for } x', y', ... \text{ in } S)$$

$$\wedge \quad (\text{substitute } x'', y'', ... \text{ for } x, y, ... \text{ in } R)$$

In integer variable x

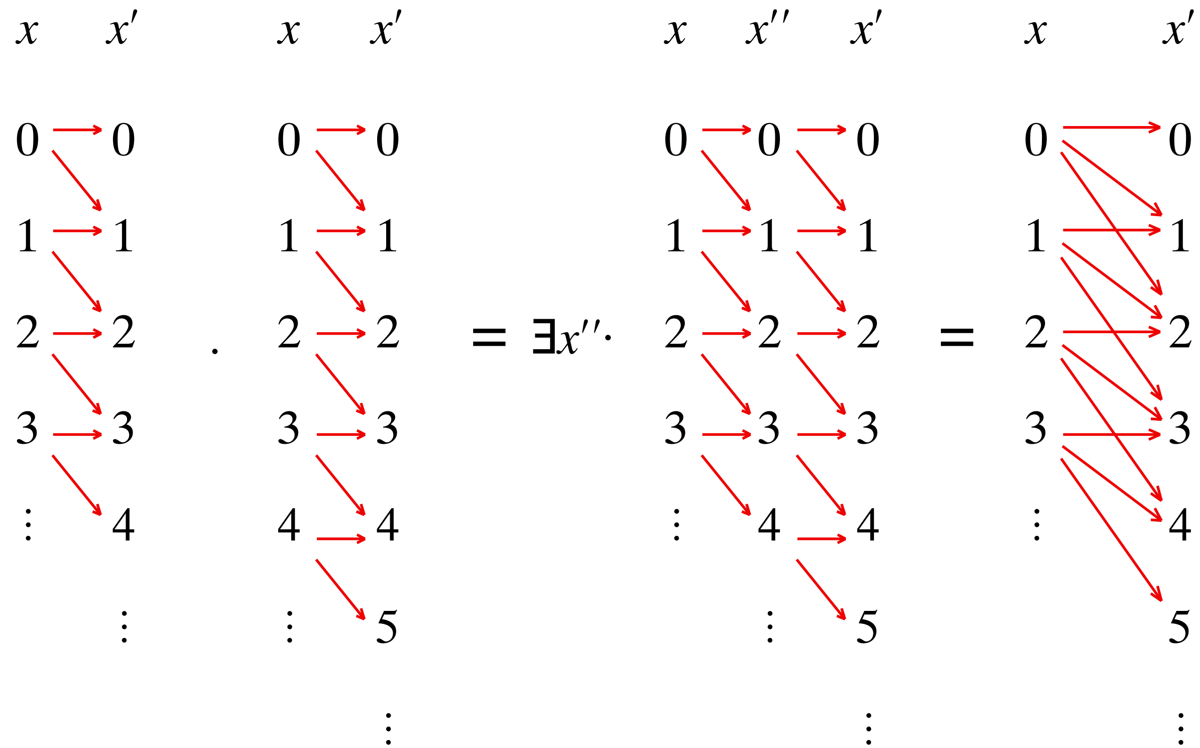
$$x'{=}x \vee x'{=}x{+}1 \ . \ x'{=}x \vee x'{=}x{+}1$$

sequential composition

$$S.R \quad = \quad \exists x'', y'', ... \cdot \quad (\text{substitute } x'', y'', ... \text{ for } x', y', ... \text{ in } S)$$
$$\wedge \quad (\text{substitute } x'', y'', ... \text{ for } x, y, ... \text{ in } R)$$

In integer variable x

$$x'=x \vee x'=x+1 \ . \ x'=x \vee x'=x+1$$

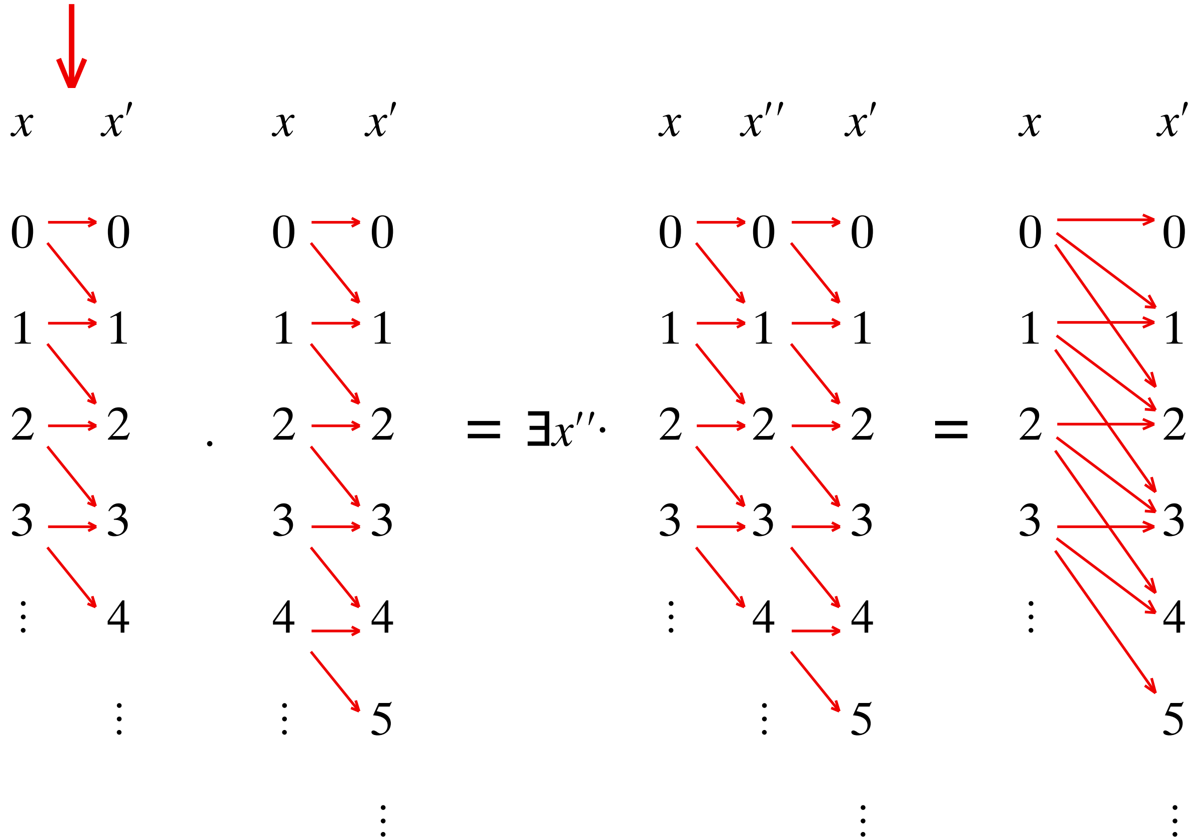


sequential composition

$$S.R \quad = \quad \exists x'', y'', ...\cdot \quad (\text{substitute } x'', y'', ... \text{ for } x', y', ... \text{ in } S)$$
$$\wedge \quad (\text{substitute } x'', y'', ... \text{ for } x, y, ... \text{ in } R)$$

In integer variable x

$$x'=x \vee x'=x+1 \;.\; x'=x \vee x'=x+1$$

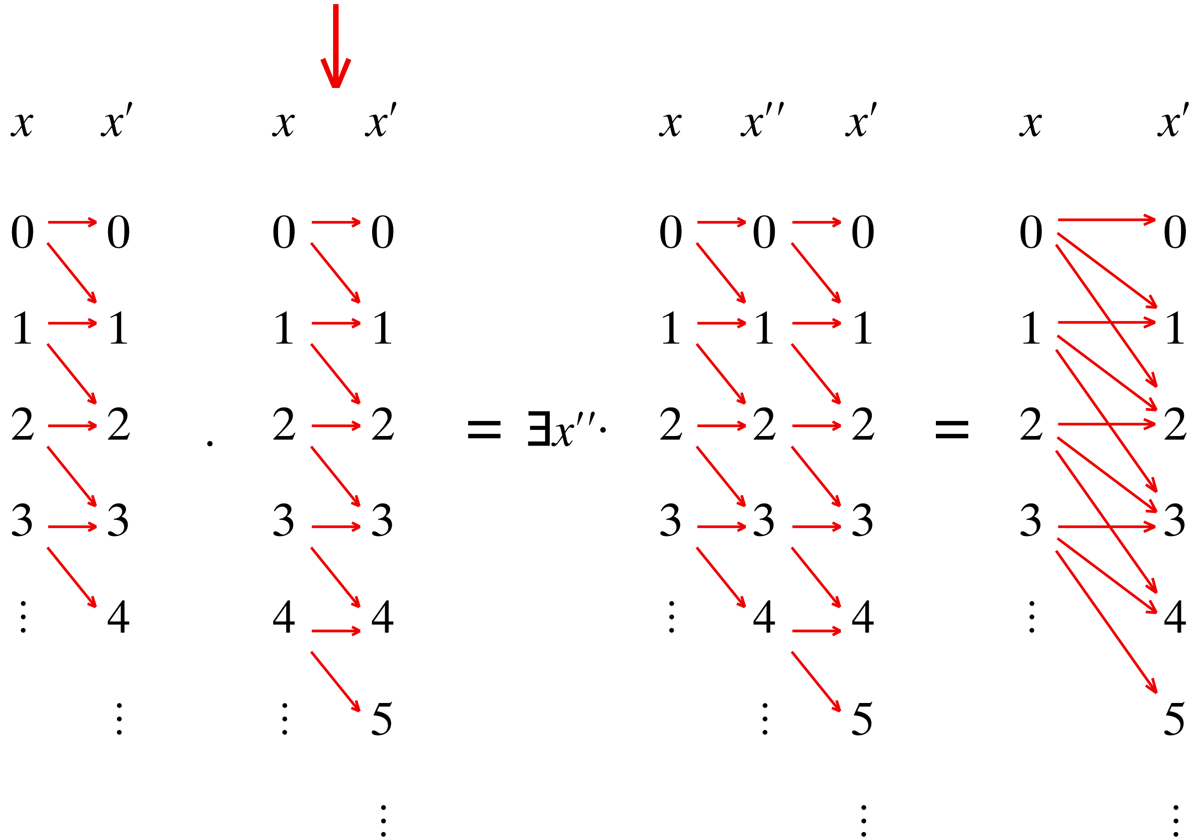


sequential composition

$$S.R \quad = \quad \exists x'', y'', ...\cdot \quad (\text{substitute } x'', y'', ... \text{ for } x', y', ... \text{ in } S)$$
$$\land \quad (\text{substitute } x'', y'', ... \text{ for } x, y, ... \text{ in } R)$$

In integer variable x

$$x'=x \lor x'=x+1 \;.\; x'=x \lor x'=x+1$$



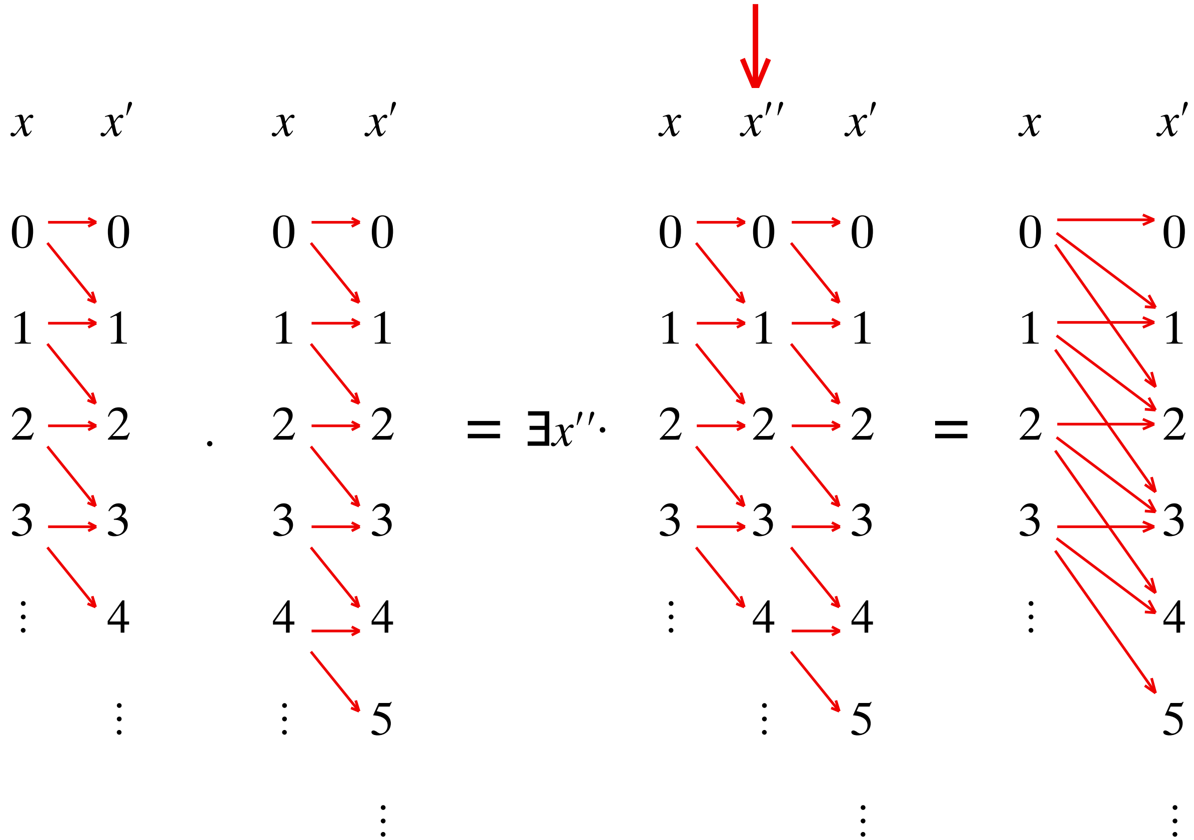
sequential composition

$$S.R \quad = \quad \exists x'', y'', ... \cdot \quad (\text{substitute } x'', y'', ... \text{ for } x', y', ... \text{ in } S)$$

$$\wedge \quad (\text{substitute } x'', y'', ... \text{ for } x, y, ... \text{ in } R)$$

In integer variable x

$$x'=x \vee x'=x+1 \;.\; x'=x \vee x'=x+1$$

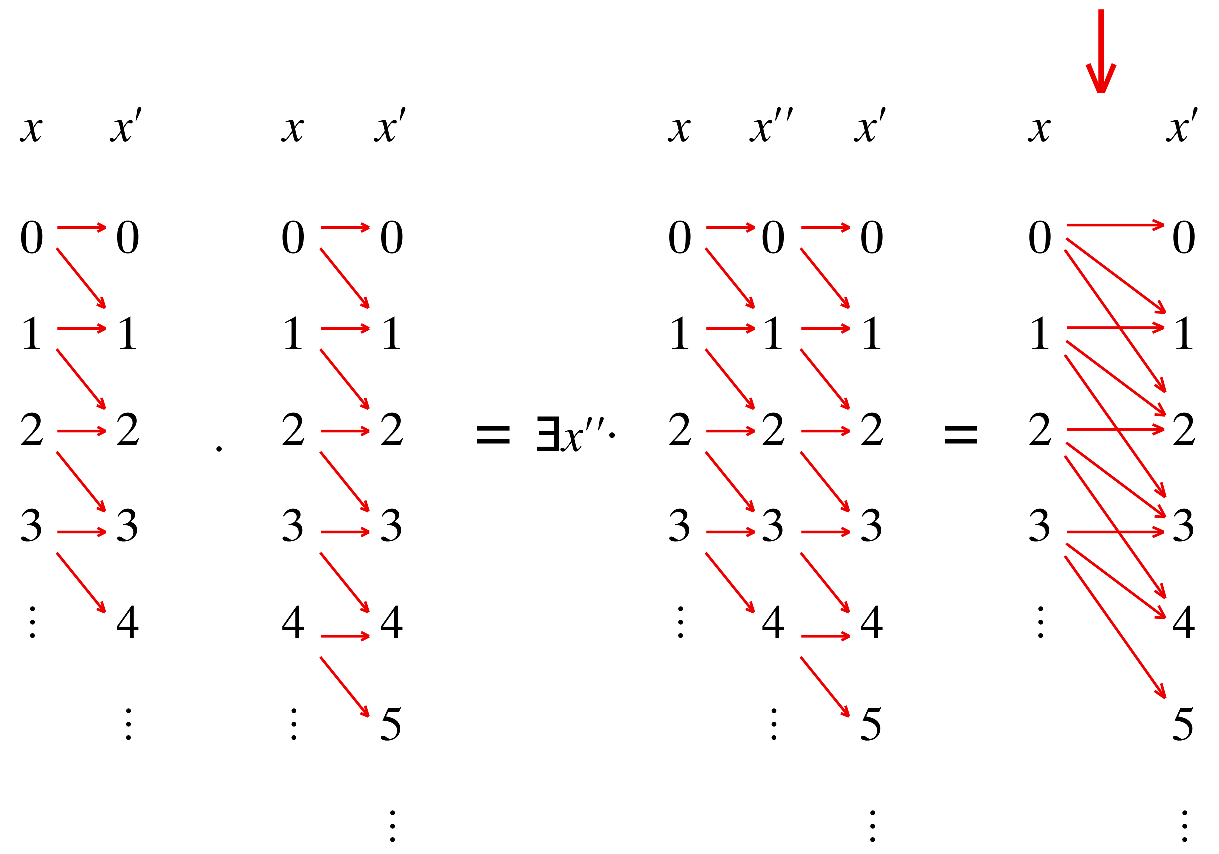


sequential composition

$$S.R \quad = \quad \exists x'', y'', ... \cdot \quad (\text{substitute } x'', y'', ... \text{ for } x', y', ... \text{ in } S)$$
$$\land \quad (\text{substitute } x'', y'', ... \text{ for } x, y, ... \text{ in } R)$$

In integer variable x

$$x'=x \lor x'=x+1 \ . \ x'=x \lor x'=x+1$$



sequential composition

$$S.\ R \quad = \quad \exists x'', y'', ...\cdot \quad \text{(substitute } x'', y'', ... \text{ for } x', y', ... \text{ in } S\text{)}$$

$$\wedge \quad \text{(substitute } x'', y'', ... \text{ for } x, y, ... \text{ in } R\text{)}$$

sequential composition

$$S.\,R \quad = \quad \exists x'', y'', ...\cdot \quad (\text{substitute } x'', y'', ... \text{ for } x', y', ... \text{ in } S)$$

$$\land \quad (\text{substitute } x'', y'', ... \text{ for } x, y, ... \text{ in } R)$$

In integer variables x and y

$$x{:=}\ 3.\ \ y{:=}\ x{+}y$$

sequential composition

$$S.\ R \quad = \quad \exists x'', y'', ...\cdot \quad (\text{substitute } x'', y'', ... \text{ for } x', y', ... \text{ in } S)$$

$$\land \quad (\text{substitute } x'', y'', ... \text{ for } x, y, ... \text{ in } R)$$

In integer variables x and y

$x{:=}\ 3.\ \ y{:=}\ x+y$ eliminate assignments first

$=$ $x'{=}3 \land y'{=}y.\ \ x'{=}x \land y'{=}x+y$

sequential composition

$$S.\,R \quad = \quad \exists x'', y'', ...\cdot \quad (\text{substitute } x'', y'', ... \text{ for } x', y', ... \text{ in } S)$$

$$\wedge \quad (\text{substitute } x'', y'', ... \text{ for } x, y, ... \text{ in } R)$$

In integer variables x and y

$x{:=}\ 3.\ \ y{:=}\ x{+}y$ — eliminate assignments first

$=$ $x'{=}3 \wedge y'{=}y.\ \ x'{=}x \wedge y'{=}x{+}y$ — then eliminate sequential composition

$=$ $\exists x'', y''{:}\ int\cdot\ x''{=}3 \wedge y''{=}y\ \wedge\ x'{=}x'' \wedge y'{=}x''{+}y''$

sequential composition

$$S.\,R \quad = \quad \exists x'', y'', ...\cdot \qquad (\text{substitute } x'', y'', ... \text{ for } x', y', ... \text{ in } S)$$

$$\land \quad (\text{substitute } x'', y'', ... \text{ for } x, y, ... \text{ in } R)$$

In integer variables x and y

	$x{:=}\ 3.\ \ y{:=}\ x{+}y$	eliminate assignments first
$=$	$x'{=}3 \land y'{=}y.\ \ x'{=}x \land y'{=}x{+}y$	then eliminate sequential composition
$=$	$\exists x'', y''{:}\ int\cdot\ x''{=}3 \land y''{=}y \ \land\ x'{=}x'' \land y'{=}x''{+}y''$	use One-Point Law twice
$=$	$x'{=}3 \ \land\ y' = 3{+}y$	

specification laws

$ok.\ P \;=\; P.\ ok \;=\; P$	Identity Law
$P.\ (Q.\ R) \;=\; (P.\ Q).\ R$	Associative Law
if b **then** P **else** P **fi** $\;=\; P$	Idempotent Law
if b **then** P **else** Q **fi** $\;=\;$ **if** $\neg b$ **then** Q **else** P **fi**	Case Reversal Law
$P \;=\;$ **if** b **then** $b \Rightarrow P$ **else** $\neg b \Rightarrow P$ **fi**	Case Creation Law
if b **then** S **else** R **fi** $\;=\; b\wedge S \vee \neg b\wedge R$	Case Analysis Law
if b **then** S **else** R **fi** $\;=\; (b\Rightarrow S) \wedge (\neg b\Rightarrow R)$	Case Analysis Law
$P\vee Q.\ R\vee S \;=\; (P.\ R) \vee (P.\ S) \vee (Q.\ R) \vee (Q.\ S)$	Distributive Law
if b **then** P **else** Q **fi**$\wedge R \;=\;$ **if** b **then** $P\wedge R$ **else** $Q\wedge R$ **fi**	Distributive Law
if b **then** P **else** Q **fi**$.\ R \;=\;$ **if** b **then** $P.\ R$ **else** $Q.\ R$ **fi**	Distributive Law
$x{:=}$ **if** b **then** e **else** f **fi** $\;=\;$ **if** b **then** $x{:=}\ e$ **else** $x{:=}\ f$ **fi**	Functional-Imperative Law
$x{:=}\ e.\ P \;=\;$ (for x substitute e in P)	Substitution Law

specification laws

$\longrightarrow$ $ok.\,P \;=\; P.\,ok \;=\; P$	Identity Law
$P.\,(Q.\,R) \;=\; (P.\,Q).\,R$	Associative Law
if b **then** P **else** P **fi** $\;=\; P$	Idempotent Law
if b **then** P **else** Q **fi** $\;=\;$ **if** $\neg b$ **then** Q **else** P **fi**	Case Reversal Law
$P \;=\;$ **if** b **then** $b \Rightarrow P$ **else** $\neg b \Rightarrow P$ **fi**	Case Creation Law
if b **then** S **else** R **fi** $\;=\; b{\wedge}S \vee \neg b{\wedge}R$	Case Analysis Law
if b **then** S **else** R **fi** $\;=\; (b{\Rightarrow}S) \wedge (\neg b{\Rightarrow}R)$	Case Analysis Law
$P{\vee}Q.\,R{\vee}S \;=\; (P.\,R) \vee (P.\,S) \vee (Q.\,R) \vee (Q.\,S)$	Distributive Law
if b **then** P **else** Q **fi**$\wedge R \;=\;$ **if** b **then** $P{\wedge}R$ **else** $Q{\wedge}R$ **fi**	Distributive Law
if b **then** P **else** Q **fi**$.\,R \;=\;$ **if** b **then** $P.\,R$ **else** $Q.\,R$ **fi**	Distributive Law
$x{:=}$ **if** b **then** e **else** f **fi** $\;=\;$ **if** b **then** $x{:=}\,e$ **else** $x{:=}\,f$ **fi**	Functional-Imperative Law
$x{:=}\,e.\,P \;=\;$ (for x substitute e in P)	Substitution Law

specification laws

$ok.\,P \;=\; P.\,ok \;=\; P$	Identity Law
→ $P.\,(Q.\,R) \;=\; (P.\,Q).\,R$	Associative Law
if b **then** P **else** P **fi** $\;=\; P$	Idempotent Law
if b **then** P **else** Q **fi** $\;=\;$ **if** $\neg b$ **then** Q **else** P **fi**	Case Reversal Law
$P \;=\;$ **if** b **then** $b \Rightarrow P$ **else** $\neg b \Rightarrow P$ **fi**	Case Creation Law
if b **then** S **else** R **fi** $\;=\; b \wedge S \vee \neg b \wedge R$	Case Analysis Law
if b **then** S **else** R **fi** $\;=\; (b \Rightarrow S) \wedge (\neg b \Rightarrow R)$	Case Analysis Law
$P \vee Q.\,R \vee S \;=\; (P.\,R) \vee (P.\,S) \vee (Q.\,R) \vee (Q.\,S)$	Distributive Law
if b **then** P **else** Q **fi**$\wedge R \;=\;$ **if** b **then** $P \wedge R$ **else** $Q \wedge R$ **fi**	Distributive Law
if b **then** P **else** Q **fi**$.\,R \;=\;$ **if** b **then** $P.\,R$ **else** $Q.\,R$ **fi**	Distributive Law
$x{:=}$ **if** b **then** e **else** f **fi** $\;=\;$ **if** b **then** $x{:=}\,e$ **else** $x{:=}\,f$ **fi**	Functional-Imperative Law
$x{:=}\,e.\,P \;=\;$ (for x substitute e in P)	Substitution Law

specification laws

	$ok.\, P \;=\; P.\, ok \;=\; P$	Identity Law
	$P.\,(Q.\, R) \;=\; (P.\, Q).\, R$	Associative Law
→	$\textbf{if}\ b\ \textbf{then}\ P\ \textbf{else}\ P\ \textbf{fi} \;=\; P$	Idempotent Law
→	$\textbf{if}\ b\ \textbf{then}\ P\ \textbf{else}\ Q\ \textbf{fi} \;=\; \textbf{if}\ \neg b\ \textbf{then}\ Q\ \textbf{else}\ P\ \textbf{fi}$	Case Reversal Law
→	$P \;=\; \textbf{if}\ b\ \textbf{then}\ b \Rightarrow P\ \textbf{else}\ \neg b \Rightarrow P\ \textbf{fi}$	Case Creation Law
→	$\textbf{if}\ b\ \textbf{then}\ S\ \textbf{else}\ R\ \textbf{fi} \;=\; b{\wedge}S \vee \neg b{\wedge}R$	Case Analysis Law
→	$\textbf{if}\ b\ \textbf{then}\ S\ \textbf{else}\ R\ \textbf{fi} \;=\; (b{\Rightarrow}S) \wedge (\neg b{\Rightarrow}R)$	Case Analysis Law
	$P{\vee}Q.\, R{\vee}S \;=\; (P.\, R) \vee (P.\, S) \vee (Q.\, R) \vee (Q.\, S)$	Distributive Law
	$\textbf{if}\ b\ \textbf{then}\ P\ \textbf{else}\ Q\ \textbf{fi}{\wedge}R \;=\; \textbf{if}\ b\ \textbf{then}\ P{\wedge}R\ \textbf{else}\ Q{\wedge}R\ \textbf{fi}$	Distributive Law
	$\textbf{if}\ b\ \textbf{then}\ P\ \textbf{else}\ Q\ \textbf{fi}.\, R \;=\; \textbf{if}\ b\ \textbf{then}\ P.\, R\ \textbf{else}\ Q.\, R\ \textbf{fi}$	Distributive Law
	$x{:=}\ \textbf{if}\ b\ \textbf{then}\ e\ \textbf{else}\ f\ \textbf{fi} \;=\; \textbf{if}\ b\ \textbf{then}\ x{:=}\ e\ \textbf{else}\ x{:=}\ f\ \textbf{fi}$	Functional-Imperative Law
	$x{:=}\ e.\, P \;=\; (\text{for } x \text{ substitute } e \text{ in } P\,)$	Substitution Law

specification laws

$ok.\,P \;=\; P.\,ok \;=\; P$	Identity Law
$P.\,(Q.\,R) \;=\; (P.\,Q).\,R$	Associative Law
if b **then** P **else** P **fi** $\;=\; P$	Idempotent Law
if b **then** P **else** Q **fi** $\;=\;$ **if** $\neg b$ **then** Q **else** P **fi**	Case Reversal Law
$P \;=\;$ **if** b **then** $b \Rightarrow P$ **else** $\neg b \Rightarrow P$ **fi**	Case Creation Law
if b **then** S **else** R **fi** $\;=\; b{\wedge}S \vee \neg b{\wedge}R$	Case Analysis Law
if b **then** S **else** R **fi** $\;=\; (b{\Rightarrow}S) \wedge (\neg b{\Rightarrow}R)$	Case Analysis Law
→ $P{\vee}Q.\,R{\vee}S \;=\; (P.\,R) \vee (P.\,S) \vee (Q.\,R) \vee (Q.\,S)$	Distributive Law
if b **then** P **else** Q **fi**$\wedge R \;=\;$ **if** b **then** $P{\wedge}R$ **else** $Q{\wedge}R$ **fi**	Distributive Law
if b **then** P **else** Q **fi**$.\,R \;=\;$ **if** b **then** $P.\,R$ **else** $Q.\,R$ **fi**	Distributive Law
$x{:=}$ **if** b **then** e **else** f **fi** $\;=\;$ **if** b **then** $x{:=}\,e$ **else** $x{:=}\,f$ **fi**	Functional-Imperative Law
$x{:=}\,e.\,P \;=\;$ (for x substitute e in P)	Substitution Law

specification laws

$ok.\,P \;=\; P.\,ok \;=\; P$	Identity Law
$P.\,(Q.\,R) \;=\; (P.\,Q).\,R$	Associative Law
if b **then** P **else** P **fi** $\;=\; P$	Idempotent Law
if b **then** P **else** Q **fi** $\;=\;$ **if** $\neg b$ **then** Q **else** P **fi**	Case Reversal Law
$P \;=\;$ **if** b **then** $b \Rightarrow P$ **else** $\neg b \Rightarrow P$ **fi**	Case Creation Law
if b **then** S **else** R **fi** $\;=\; b \wedge S \vee \neg b \wedge R$	Case Analysis Law
if b **then** S **else** R **fi** $\;=\; (b \Rightarrow S) \wedge (\neg b \Rightarrow R)$	Case Analysis Law
$P \vee Q.\,R \vee S \;=\; (P.\,R) \vee (P.\,S) \vee (Q.\,R) \vee (Q.\,S)$	Distributive Law
if b **then** P **else** Q **fi**$\wedge R \;=\;$ **if** b **then** $P \wedge R$ **else** $Q \wedge R$ **fi**	Distributive Law
if b **then** P **else** Q **fi**$.\,R \;=\;$ **if** b **then** $P.\,R$ **else** $Q.\,R$ **fi**	Distributive Law
$x{:=}$ **if** b **then** e **else** f **fi** $\;=\;$ **if** b **then** $x{:=}\,e$ **else** $x{:=}\,f$ **fi**	Functional-Imperative Law
→ $x{:=}\,e.\,P \;=\;$ (for x substitute e in P)	Substitution Law

substitution law

$x{:=}\, e.\, P \quad = \quad$ (for x substitute e in P)

substitution law

$x := e .\ P \;\; = \;\; (\text{for } x \text{ substitute } e \text{ in } P)$

$x := y+1 .\ \ y' > x' \;\; =$

substitution law

$x := e.\ P \quad = \quad$ (for x substitute e in P)

$$x := y+1.\ y' > x' \quad = \quad y' > x'$$

substitution law

$x := e.\ P \ = \ $ (for x substitute e in P)

$x := y+1.\ \ y' > x' \ = \ y' > x'$

$x := x+1.\ \ y' > x \land x' > x \ =$

substitution law

$$x{:=}\, e.\ P \;\; = \;\; (\text{for } x \text{ substitute } e \text{ in } P\,)$$

$$x{:=}\, y{+}1.\ \ y' > x' \;\; = \;\; y' > x'$$

$$x{:=}\, x{+}1.\ \ y'{>}x \wedge x'{>}x \;\; = \;\; y' > x{+}1 \;\wedge\; x' > x{+}1$$

substitution law

$x{:=}\ e.\ P \quad = \quad$ (for x substitute e in P)

$x{:=}\ y{+}1.\ \ y' > x' \quad = \quad y' > x'$

$x{:=}\ x{+}1.\ \ y'{>}x \wedge x'{>}x \quad = \quad y' > x{+}1 \ \wedge\ x' > x{+}1$

$x{:=}\ y{+}1.\ \ y' = 2{\times}x \quad =$

substitution law

$$x := e.\ P \quad = \quad (\text{for } x \text{ substitute } e \text{ in } P)$$

$$x := y+1.\ y' > x' \quad = \quad y' > x'$$

$$x := x+1.\ y'>x \land x'>x \quad = \quad y' > x+1 \ \land \ x' > x+1$$

$$x := y+1.\ y' = 2\times x \quad = \quad y' = 2\times(y+1)$$

substitution law

$x{:=}\,e.\,P \;=\; (\text{for } x \text{ substitute } e \text{ in } P)$

$x{:=}\,y{+}1.\;\; y' > x' \;=\; y' > x'$

$x{:=}\,x{+}1.\;\; y'{>}x \wedge x'{>}x \;=\; y' > x{+}1 \;\wedge\; x' > x{+}1$

$x{:=}\,y{+}1.\;\; y' = 2{\times}x \;=\; y' = 2{\times}(y{+}1)$

$x{:=}\,1.\;\; x{\ge}1 \Rightarrow \exists x\cdot\, y' = 2{\times}x \;=$

substitution law

$$x{:=}\, e.\ P \ = \ (\text{for } x \text{ substitute } e \text{ in } P)$$

$$x{:=}\, y{+}1.\ \ y' > x' \ = \ y' > x'$$

$$x{:=}\, x{+}1.\ \ y'{>}x \land x'{>}x \ = \ y' > x{+}1 \ \land \ x' > x{+}1$$

$$x{:=}\, y{+}1.\ \ y' = 2{\times}x \ = \ y' = 2{\times}(y{+}1)$$

$$x{:=}\, 1.\ \ x{\ge}1 \Rightarrow \exists x{\cdot}\ y' = 2{\times}x \ = \ 1{\ge}1 \Rightarrow \exists x{\cdot}\ y' = 2{\times}x$$

substitution law

$$x{:=}\, e.\ P \quad = \quad (\text{for } x \text{ substitute } e \text{ in } P\,)$$

$$x{:=}\, y{+}1.\ \ y' > x' \quad = \quad y' > x'$$

$$x{:=}\, x{+}1.\ \ y'{>}x \wedge x'{>}x \quad = \quad y' > x{+}1 \ \wedge\ x' > x{+}1$$

$$x{:=}\, y{+}1.\ \ y' = 2{\times}x \quad = \quad y' = 2{\times}(y{+}1)$$

$$x{:=}\, 1.\ \ x{\ge}1 \Rightarrow \exists x\cdot\ y' = 2{\times}x \quad = \quad 1{\ge}1 \Rightarrow \exists x\cdot\ y' = 2{\times}x \quad = \quad \mathit{even}\ y'$$

substitution law

$x := e .\ P \quad = \quad$ (for x substitute e in P)

$x := y+1 .\ y' > x' \quad = \quad y' > x'$

$x := x+1 .\ y'>x \wedge x'>x \quad = \quad y' > x+1 \ \wedge \ x' > x+1$

$x := y+1 .\ y' = 2 \times x \quad = \quad y' = 2 \times (y+1)$

$x := 1 .\ x \geq 1 \Rightarrow \exists x \cdot y' = 2 \times x \quad = \quad 1 \geq 1 \Rightarrow \exists x \cdot y' = 2 \times x \quad = \quad even\ y'$

$x := y .\ x \geq 1 \Rightarrow \exists y \cdot y' = x \times y \quad =$

substitution law

$x{:=}\, e.\, P \;\;=\;\; (\text{for } x \text{ substitute } e \text{ in } P\,)$

$x{:=}\, y{+}1.\;\; y' > x' \;\;=\;\; y' > x'$

$x{:=}\, x{+}1.\;\; y'{>}x \land x'{>}x \;\;=\;\; y' > x{+}1 \;\land\; x' > x{+}1$

$x{:=}\, y{+}1.\;\; y' = 2{\times}x \;\;=\;\; y' = 2{\times}(y{+}1)$

$x{:=}\, 1.\;\; x{\ge}1 \Rightarrow \exists x\cdot\, y' = 2{\times}x \;\;=\;\; 1{\ge}1 \Rightarrow \exists x\cdot\, y' = 2{\times}x \;\;=\;\; \mathit{even}\; y'$

$x{:=}\, y.\;\; x{\ge}1 \Rightarrow \exists y\cdot\, y' = x{\times}y \;\;=\;\; x{:=}\, y.\;\; x{\ge}1 \Rightarrow \exists k\cdot\, y' = x{\times}k$

substitution law

$x{:=}\, e.\, P \;\; = \;\; (\text{for } x \text{ substitute } e \text{ in } P)$

$x{:=}\, y{+}1.\;\; y' > x' \;\; = \;\; y' > x'$

$x{:=}\, x{+}1.\;\; y'{>}x \land x'{>}x \;\; = \;\; y' > x{+}1 \;\land\; x' > x{+}1$

$x{:=}\, y{+}1.\;\; y' = 2{\times}x \;\; = \;\; y' = 2{\times}(y{+}1)$

$x{:=}\, 1.\;\; x{\ge}1 \Rightarrow \exists x{\cdot}\; y' = 2{\times}x \;\; = \;\; 1{\ge}1 \Rightarrow \exists x{\cdot}\; y' = 2{\times}x \;\; = \;\; even\ y'$

$x{:=}\, y.\;\; x{\ge}1 \Rightarrow \exists y{\cdot}\; y' = x{\times}y \;\; = \;\; x{:=}\, y.\;\; x{\ge}1 \Rightarrow \exists k{\cdot}\; y' = x{\times}k$

$= \;\; y{\ge}1 \Rightarrow \exists k{\cdot}\; y' = y{\times}k$

substitution law

$x{:=}\,e.\,P \;\;=\;\; (\text{for } x \text{ substitute } e \text{ in } P\,)$

$x{:=}\,y{+}1.\;\; y' > x' \;\;=\;\; y' > x'$

$x{:=}\,x{+}1.\;\; y'{>}x \wedge x'{>}x \;\;=\;\; y' > x{+}1 \;\wedge\; x' > x{+}1$

$x{:=}\,y{+}1.\;\; y' = 2{\times}x \;\;=\;\; y' = 2{\times}(y{+}1)$

$x{:=}\,1.\;\; x{\ge}1 \Rightarrow \exists x\cdot\, y' = 2{\times}x \;\;=\;\; 1{\ge}1 \Rightarrow \exists x\cdot\, y' = 2{\times}x \;\;=\;\; \mathit{even}\; y'$

$x{:=}\,y.\;\; x{\ge}1 \Rightarrow \exists y\cdot\, y' = x{\times}y \;\;=\;\; x{:=}\,y.\;\; x{\ge}1 \Rightarrow \exists k\cdot\, y' = x{\times}k$

$\;\;=\;\; y{\ge}1 \Rightarrow \exists k\cdot\, y' = y{\times}k$

$x{:=}\,1.\;\; \mathit{ok} \;\;=$

substitution law

$x{:=}\,e.\,P \;\;=\;\; (\text{for } x \text{ substitute } e \text{ in } P)$

$x{:=}\,y{+}1.\;\; y' > x' \;\;=\;\; y' > x'$

$x{:=}\,x{+}1.\;\; y'{>}x \wedge x'{>}x \;\;=\;\; y' > x{+}1 \;\wedge\; x' > x{+}1$

$x{:=}\,y{+}1.\;\; y' = 2{\times}x \;\;=\;\; y' = 2{\times}(y{+}1)$

$x{:=}\,1.\;\; x{\ge}1 \Rightarrow \exists x{\cdot}\; y' = 2{\times}x \;\;=\;\; 1{\ge}1 \Rightarrow \exists x{\cdot}\; y' = 2{\times}x \;\;=\;\; \mathit{even}\; y'$

$x{:=}\,y.\;\; x{\ge}1 \Rightarrow \exists y{\cdot}\; y' = x{\times}y \;\;=\;\; x{:=}\,y.\;\; x{\ge}1 \Rightarrow \exists k{\cdot}\; y' = x{\times}k$

$\;\;=\;\; y{\ge}1 \Rightarrow \exists k{\cdot}\; y' = y{\times}k$

$x{:=}\,1.\;\; ok \;\;=\;\; x{:=}\,1.\;\; x'{=}x \wedge y'{=}y$

substitution law

$$x{:=}\, e.\ P \;=\; (\text{for } x \text{ substitute } e \text{ in } P)$$

$$x{:=}\, y{+}1.\ \ y' > x' \;=\; y' > x'$$

$$x{:=}\, x{+}1.\ \ y'{>}x \wedge x'{>}x \;=\; y' > x{+}1 \;\wedge\; x' > x{+}1$$

$$x{:=}\, y{+}1.\ \ y' = 2{\times}x \;=\; y' = 2{\times}(y{+}1)$$

$$x{:=}\, 1.\ \ x{\ge}1 \Rightarrow \exists x\cdot\ y' = 2{\times}x \;=\; 1{\ge}1 \Rightarrow \exists x\cdot\ y' = 2{\times}x \;=\; \mathit{even}\ y'$$

$$x{:=}\, y.\ \ x{\ge}1 \Rightarrow \exists y\cdot\ y' = x{\times}y \;=\; x{:=}\, y.\ \ x{\ge}1 \Rightarrow \exists k\cdot\ y' = x{\times}k$$

$$=\; y{\ge}1 \Rightarrow \exists k\cdot\ y' = y{\times}k$$

$$x{:=}\, 1.\ \ \mathit{ok} \;=\; x{:=}\, 1.\ \ x'{=}x \wedge y'{=}y \;=\; x'{=}1 \wedge y'{=}y$$

substitution law

$x{:=}\, e.\ P \quad = \quad$ (for x substitute e in P)

$x{:=}\, y{+}1.\ \ y' > x' \quad = \quad y' > x'$

$x{:=}\, x{+}1.\ \ y'{>}x \land x'{>}x \quad = \quad y' > x{+}1 \ \land\ x' > x{+}1$

$x{:=}\, y{+}1.\ \ y' = 2{\times}x \quad = \quad y' = 2{\times}(y{+}1)$

$x{:=}\, 1.\ \ x{\ge}1 \Rightarrow \exists x{\cdot}\ y' = 2{\times}x \quad = \quad 1{\ge}1 \Rightarrow \exists x{\cdot}\ y' = 2{\times}x \quad = \quad \mathit{even}\ y'$

$x{:=}\, y.\ \ x{\ge}1 \Rightarrow \exists y{\cdot}\ y' = x{\times}y \quad = \quad x{:=}\, y.\ \ x{\ge}1 \Rightarrow \exists k{\cdot}\ y' = x{\times}k$

$\quad = \quad y{\ge}1 \Rightarrow \exists k{\cdot}\ y' = y{\times}k$

$x{:=}\, 1.\ \ ok \quad = \quad x{:=}\, 1.\ \ x'{=}x \land y'{=}y \quad = \quad x'{=}1 \land y'{=}y$

$x{:=}\, 1.\ \ y{:=}\, 2 \quad =$

substitution law

$x{:=}\, e.\ P \ \ = \ \ (\text{for } x \text{ substitute } e \text{ in } P\,)$

$x{:=}\, y{+}1.\ \ y' > x' \ \ = \ \ y' > x'$

$x{:=}\, x{+}1.\ \ y'{>}x \land x'{>}x \ \ = \ \ y' > x{+}1 \ \land \ x' > x{+}1$

$x{:=}\, y{+}1.\ \ y' = 2{\times}x \ \ = \ \ y' = 2{\times}(y{+}1)$

$x{:=}\, 1.\ \ x{\ge}1 \Rightarrow \exists x\cdot\ y' = 2{\times}x \ \ = \ \ 1{\ge}1 \Rightarrow \exists x\cdot\ y' = 2{\times}x \ \ = \ \ \mathit{even}\ y'$

$x{:=}\, y.\ \ x{\ge}1 \Rightarrow \exists y\cdot\ y' = x{\times}y \ \ = \ \ x{:=}\, y.\ \ x{\ge}1 \Rightarrow \exists k\cdot\ y' = x{\times}k$

$= \ \ y{\ge}1 \Rightarrow \exists k\cdot\ y' = y{\times}k$

$x{:=}\, 1.\ \ \mathit{ok} \ \ = \ \ x{:=}\, 1.\ \ x'{=}x \land y'{=}y \ \ = \ \ x'{=}1 \land y'{=}y$

$x{:=}\, 1.\ \ y{:=}\, 2 \ \ = \ \ x{:=}\, 1.\ \ x'{=}x \land y'{=}2$

substitution law

$x := e .\ P \quad = \quad$ (for x substitute e in P)

$x := y+1 .\ y' > x' \quad = \quad y' > x'$

$x := x+1 .\ y' > x \land x' > x \quad = \quad y' > x+1 \ \land \ x' > x+1$

$x := y+1 .\ y' = 2 \times x \quad = \quad y' = 2 \times (y+1)$

$x := 1 .\ x \ge 1 \Rightarrow \exists x \cdot y' = 2 \times x \quad = \quad 1 \ge 1 \Rightarrow \exists x \cdot y' = 2 \times x \quad = \quad even\ y'$

$x := y .\ x \ge 1 \Rightarrow \exists y \cdot y' = x \times y \quad = \quad x := y .\ x \ge 1 \Rightarrow \exists k \cdot y' = x \times k$

$\quad = \quad y \ge 1 \Rightarrow \exists k \cdot y' = y \times k$

$x := 1 .\ ok \quad = \quad x := 1 .\ x' = x \land y' = y \quad = \quad x' = 1 \land y' = y$

$x := 1 .\ y := 2 \quad = \quad x := 1 .\ x' = x \land y' = 2 \quad = \quad x' = 1 \land y' = 2$

substitution law

$x{:=}\,e.\,P \;\; = \;\; (\text{for } x \text{ substitute } e \text{ in } P)$

substitution law

$x{:=}\, e.\, P \;\; = \;\; (\text{for } x \text{ substitute } e \text{ in } P)$

$x{:=}\, 1.\;\; y{:=}\, 2.\;\; x{:=}\, x{+}y$

substitution law

$$x{:=}\, e.\, P \quad = \quad (\text{for } x \text{ substitute } e \text{ in } P\,)$$

$$x{:=}\,1.\;\; y{:=}\,2.\;\; x{:=}\,x{+}y$$

$$=\qquad x{:=}\,1.\;\; y{:=}\,2.\;\; x' = x{+}y \;\wedge\; y'{=}y$$

substitution law

$$x{:=}\, e.\, P \;\;=\;\; (\text{for } x \text{ substitute } e \text{ in } P\,)$$

$$x{:=}\,1.\;\; y{:=}\,2.\;\; x{:=}\,x{+}y$$

$$=\qquad x{:=}\,1.\;\; y{:=}\,2.\;\; x' = x{+}y \;\wedge\; y'{=}y$$

$$=\qquad x{:=}\,1.\;\; x' = x{+}2 \;\wedge\; y'{=}2$$

substitution law

$$x{:=}\, e.\, P \;\; = \;\; (\text{for } x \text{ substitute } e \text{ in } P)$$

$$x{:=}\,1.\;\; y{:=}\,2.\;\; x{:=}\,x{+}y$$

$$=\quad x{:=}\,1.\;\; y{:=}\,2.\;\; x' = x{+}y \;\wedge\; y'{=}y$$

$$=\quad x{:=}\,1.\;\; x' = x{+}2 \;\wedge\; y'{=}2$$

$$=\quad x'{=}3 \wedge y'{=}2$$

substitution law

$x{:=}\,e.\;P \;\;=\;\; (\text{for } x \text{ substitute } e \text{ in } P)$

$$x{:=}\,1.\;\; y{:=}\,2.\;\; x{:=}\,x{+}y$$

$$=\quad x{:=}\,1.\;\; y{:=}\,2.\;\; x' = x{+}y \;\wedge\; y'{=}y$$

$$=\quad x{:=}\,1.\;\; x' = x{+}2 \;\wedge\; y'{=}2$$

$$=\quad x'{=}3 \wedge y'{=}2$$

$$x{:=}\,1.\;\; x' > x.\;\; x' = x{+}1$$

substitution law

$x{:=}\, e.\ P \;\;=\;\; (\text{for } x \text{ substitute } e \text{ in } P)$

$$x{:=}\,1.\;\; y{:=}\,2.\;\; x{:=}\,x{+}y$$

$$=\quad x{:=}\,1.\;\; y{:=}\,2.\;\; x' = x{+}y \;\wedge\; y'{=}y$$

$$=\quad x{:=}\,1.\;\; x' = x{+}2 \;\wedge\; y'{=}2$$

$$=\quad x'{=}3 \wedge y'{=}2$$

$$x{:=}\,1.\;\; x' > x.\;\; x' = x{+}1$$

$$=\quad x' > 1.\;\; x' = x{+}1$$

substitution law

$$x{:=}\,e.\,P \;\;=\;\; (\text{for } x \text{ substitute } e \text{ in } P\,)$$

$$x{:=}\,1.\;\; y{:=}\,2.\;\; x{:=}\,x{+}y$$

$$=\qquad x{:=}\,1.\;\; y{:=}\,2.\;\; x' = x{+}y \;\;\land\;\; y'{=}y$$

$$=\qquad x{:=}\,1.\;\; x' = x{+}2 \;\;\land\;\; y'{=}2$$

$$=\qquad x'{=}3 \land y'{=}2$$

$$x{:=}\,1.\;\; x' > x.\;\; x' = x{+}1$$

$$=\qquad x' > 1.\;\; x' = x{+}1$$

$$=\qquad \exists x'', y''\cdot\, x''{>}1 \;\;\land\;\; x' = x''{+}1$$

substitution law

$$x{:=}\,e.\,P \;\;=\;\; (\text{for } x \text{ substitute } e \text{ in } P\,)$$

$$x{:=}\,1.\;\; y{:=}\,2.\;\; x{:=}\,x{+}y$$

$$=\qquad x{:=}\,1.\;\; y{:=}\,2.\;\; x' = x{+}y \;\wedge\; y'{=}y$$

$$=\qquad x{:=}\,1.\;\; x' = x{+}2 \;\wedge\; y'{=}2$$

$$=\qquad x'{=}3 \wedge y'{=}2$$

$$x{:=}\,1.\;\; x' > x.\;\; x' = x{+}1$$

$$=\qquad x' > 1.\;\; x' = x{+}1$$

$$=\qquad \exists x'', y''\cdot\, x''{>}1 \;\wedge\; x' = x''{+}1$$

$$=\qquad \exists x''\cdot\, x''{>}1 \;\wedge\; x' = x''{+}1$$

substitution law

$$x{:=}\,e.\,P \;=\; (\text{for } x \text{ substitute } e \text{ in } P\,)$$

$$
\begin{array}{ll}
 & x{:=}\,1.\;\; y{:=}\,2.\;\; x{:=}\,x{+}y \\
= & x{:=}\,1.\;\; y{:=}\,2.\;\; x' = x{+}y \;\land\; y'{=}y \\
= & x{:=}\,1.\;\; x' = x{+}2 \;\land\; y'{=}2 \\
= & x'{=}3 \land y'{=}2
\end{array}
$$

$$
\begin{array}{ll}
 & x{:=}\,1.\;\; x' > x.\;\; x' = x{+}1 \\
= & x' > 1.\;\; x' = x{+}1 \\
= & \exists x'', y''\cdot\; x''{>}1 \;\land\; x' = x''{+}1 \\
= & \exists x''\cdot\; x''{>}1 \;\land\; x' = x''{+}1 \\
= & \exists x''\cdot\; x''{>}1 \;\land\; x'' = x'{-}1
\end{array}
$$

substitution law

$$x{:=}\, e.\ P \quad = \quad (\text{for } x \text{ substitute } e \text{ in } P\,)$$

$x{:=}\,1.\ \ y{:=}\,2.\ \ x{:=}\,x{+}y$

$=\quad x{:=}\,1.\ \ y{:=}\,2.\ \ x' = x{+}y \ \wedge\ y'{=}y$

$=\quad x{:=}\,1.\ \ x' = x{+}2 \ \wedge\ y'{=}2$

$=\quad x'{=}3 \wedge y'{=}2$

$x{:=}\,1.\ \ x' > x.\ \ x' = x{+}1$

$=\quad x' > 1.\ \ x' = x{+}1$

$=\quad \exists x'', y''\cdot\ x''{>}1 \ \wedge\ x' = x''{+}1$

$=\quad \exists x''\cdot\ x''{>}1 \ \wedge\ x' = x''{+}1$

$=\quad \exists x''\cdot\ x''{>}1 \ \wedge\ x'' = x'{-}1$

$=\quad x'{-}1 > 1$

substitution law

$x{:=}\,e.\,P \;=\; (\text{for } x \text{ substitute } e \text{ in } P)$

$$
\begin{aligned}
& x{:=}\,1.\; y{:=}\,2.\; x{:=}\,x{+}y \\
= \quad & x{:=}\,1.\; y{:=}\,2.\; x'=x{+}y \;\land\; y'{=}y \\
= \quad & x{:=}\,1.\; x'=x{+}2 \;\land\; y'{=}2 \\
= \quad & x'{=}3 \land y'{=}2
\end{aligned}
$$

$$
\begin{aligned}
& x{:=}\,1.\; x'>x.\; x'=x{+}1 \\
= \quad & x'>1.\; x'=x{+}1 \\
= \quad & \exists x'', y''\cdot\; x''{>}1 \;\land\; x'=x''{+}1 \\
= \quad & \exists x''\cdot\; x''{>}1 \;\land\; x'=x''{+}1 \\
= \quad & \exists x''\cdot\; x''{>}1 \;\land\; x''=x'{-}1 \\
= \quad & x'{-}1>1 \\
= \quad & x'{>}2
\end{aligned}
$$