

112 If $f: A \rightarrow B$ and $p: B \rightarrow \text{bin}$, prove

(a) $\exists b: f A \cdot p b = \exists a: A \cdot p f a$

(b) $\forall b: f A \cdot p b = \forall a: A \cdot p f a$

After trying the question, scroll down to the solution.

$$\begin{array}{l}
\text{\S(a)} \\
= \\
=
\end{array}
\quad
\begin{array}{l}
\exists b: f A \cdot p b \\
\exists a: A \cdot \langle b: f A \rightarrow p b \rangle (f a) \\
\exists a: A \cdot p f a
\end{array}
\quad
\begin{array}{l}
\text{change of variable law} \\
\text{apply}
\end{array}$$

$$\begin{array}{l}
\text{\S(b)} \\
= \\
=
\end{array}
\quad
\begin{array}{l}
\forall b: f A \cdot p b \\
\forall a: A \cdot \langle b: f A \rightarrow p b \rangle (f a) \\
\forall a: A \cdot p f a
\end{array}
\quad
\begin{array}{l}
\text{change of variable law} \\
\text{apply}
\end{array}$$