112 If $f: A \to B$ and $p: B \to bin$, prove

(a) $\exists b: f\,A \cdot p\,b = \exists a: A \cdot p\,f\,a$

(b) $\forall b: f\,A \cdot p\,b = \forall a: A \cdot p\,f\,a$

After trying the question, scroll down to the solution.

§(a)		$\exists b\colon f\, A\cdot\ p\ b$	change of variable law
	$=$	$\exists a\colon A\cdot\ \langle b\colon f\, A \to p\ b\rangle\ (f\ a)$	apply
	$=$	$\exists a\colon A\cdot\ p\ f\ a$	

§(b)		$\forall b\colon f\, A\cdot\ p\ b$	change of variable law
	$=$	$\forall a\colon A\cdot\ \langle b\colon f\, A \to p\ b\rangle\ (f\ a)$	apply
	$=$	$\forall a\colon A\cdot\ p\ f\ a$	