132 What is wrong with the following proof:

$$
\begin{array}{lll}
 & (R \Longleftarrow R.\,S) & \text{use context rule} \\
= & (R \Longleftarrow \bot.\,S) & \bot \text{ is base for } . \\
= & (R \Longleftarrow \bot) & \text{base law for } \Leftarrow \\
= & \top &
\end{array}
$$

After trying the question, scroll down to the solution.

§ The sequential composition operator is defined as

$$R.S \;=\; \exists\sigma''\cdot\ (\text{substitute } \sigma'' \text{ for } \sigma' \text{ in } R) \wedge (\text{substitute } \sigma'' \text{ for } \sigma \text{ in } S)$$

so the context rule cannot be used. The R which is the consequent of the implication is a binary expression in variables σ and σ'. The R which is the left operand of the sequential composition, after substitution, is a binary expression in variables σ and σ''. So they are not the same expression. However, if σ' does not appear in R, then they are the same expression, and the context rule can be used. For example,

$$\begin{array}{lll}
 & (x{=}2 \;\Longleftarrow\; x{=}2.\ x' = x{+}1) & \\
= & (x{=}2 \;\Longleftarrow\; (\exists x''\cdot\ x{=}2 \wedge x' = x''{+}1)) & \text{use one-point} \\
= & (x{=}2 \;\Longleftarrow\; x{=}2) & \text{reflexive law for } \Leftarrow \\
= & \top &
\end{array}$$