137 Let a, b, and c be integer variables. Express as simply as possible without using quantifiers, assignments, or sequential compositions

(a) $b:= a-b.\ b:= a-b$

(b) $a:= a+b.\ b:= a-b.\ a:= a-b$

(c) $c:= a-b-c.\ b:= a-b-c.\ a:= a-b-c.\ c:= a+b+c$

(d) $a:= a+b.\ b:= a+b.\ c:= a+b$

(e) $a:= a+b.\ b' = a+b.\ c:= a+b$

(f) $a:= a+b+1.\ b:= a-b-1.\ a:= a-b-1$

(g) $a' = a+b+1.\ b' = a-b-1$

(h) $a:= a-b.\ b:= a-b.\ a:= a+b$

After trying the question, scroll down to the solution.

§(a)

$$\begin{array}{lll}
 & b:= a-b.\ b:= a-b & \text{expand last assignment}\\
= & b:= a-b.\ a'=a \ \wedge\ b' = a-b \ \wedge\ c'=c & \text{Substitution Law}\\
= & a'=a \ \wedge\ b' = a-(a-b) \ \wedge\ c'=c & \\
= & a'=a \ \wedge\ b'=b \ \wedge\ c'=c & \\
= & ok &
\end{array}$$

§(b)

$$\begin{array}{lll}
 & a:= a+b.\ b:= a-b.\ a:= a-b & \text{expand last assignment}\\
= & a:= a+b.\ b:= a-b.\ a' = a-b \ \wedge\ b'=b \ \wedge\ c'=c & \text{Substitution Law}\\
= & a:= a+b.\ a' = a-(a-b) \ \wedge\ b' = a-b \ \wedge\ c'=c & \text{Substitution Law}\\
= & a'=b \ \wedge\ b' = (a+b)-b \ \wedge\ c'=c & \text{subtract}\\
= & a'=b \ \wedge\ b'=a \ \wedge\ c'=c &
\end{array}$$

§(c)

$$\begin{array}{lll}
 & c:= a-b-c.\ b:= a-b-c.\ a:= a-b-c.\ c:= a+b+c & \text{expand last assignment}\\
= & c:= a-b-c.\ b:= a-b-c.\ a:= a-b-c.\ a'=a \ \wedge\ b'=b \ \wedge\ c'=a+b+c & \text{Substitution Law}\\
= & c:= a-b-c.\ b:= a-b-c.\ a'=a-b-c \ \wedge\ b'=b \ \wedge\ c'=(a-b-c)+b+c & \text{arithmetic}\\
= & c:= a-b-c.\ b:= a-b-c.\ a'=a-b-c \ \wedge\ b'=b \ \wedge\ c'=a & \text{Substitution Law}\\
= & c:= a-b-c.\ a'=a-(a-b-c)-c \ \wedge\ b'=a-b-c \ \wedge\ c'=a & \text{arithmetic}\\
= & c:= a-b-c.\ a'=b \ \wedge\ b'=a-b-c \ \wedge\ c'=a & \text{Substitution Law}\\
= & a'=b \ \wedge\ b'=a-b-(a-b-c) \ \wedge\ c'=a & \text{arithmetic}\\
= & a'=b \ \wedge\ b'=c \ \wedge\ c'=a &
\end{array}$$

§(d)

$$\begin{array}{lll}
 & a:= a+b.\ b:= a+b.\ c:= a+b & \text{expand last assignment}\\
= & a:= a+b.\ b:= a+b.\ a'=a \wedge b'=b \wedge c' = a+b & \text{substitution law}\\
= & a:= a+b.\ \ a'=a \wedge b' = a+b \ \wedge\ c' = a+a+b & \text{substitution law}\\
= & a' = a+b \ \wedge\ b' = a+b+b \ \wedge\ c' = a+b+a+b+b & \text{arithmetic}\\
= & a'=a+b \ \wedge\ b' = a + 2\times b \ \wedge\ c' = 2\times a + 3\times b &
\end{array}$$

§(e)

$$\begin{array}{lll}
 & a:= a+b.\ b' = a+b.\ c:= a+b & \text{expand last assignment}\\
= & a:= a+b.\ b' = a+b.\ a'=a \wedge b'=b \wedge c' = a+b & \text{sequential composition}\\
= & a:= a+b.\ \exists a'', b'', c''\cdot\ b'' = a+b \wedge a'=a'' \wedge b'=b'' \wedge c' = a''+b'' & \\
 & & \text{one-point for } a'' \text{ and } b''\text{, idempotence for } c''\\
= & a:= a+b.\ b' = a+b \wedge c' = a'+b' & \text{substitution law}\\
= & b' = a+b+b \wedge c' = a'+b' & \text{arithmetic}\\
= & b' = a + 2\times b \ \wedge\ c' = a'+b' &
\end{array}$$

§(f)

$$\begin{array}{lll}
 & a:= a+b+1.\ b:= a-b-1.\ a:= a-b-1 & \text{expand last assignment}\\
= & a:= a+b+1.\ b:= a-b-1.\ a' = a-b-1 \wedge b'=b \wedge c'=c & \text{substitution law once}\\
= & a:= a+b+1.\ a' = a-(a-b-1)-1 \wedge b'=a-b-1 \wedge c'=c & \text{simplify}\\
= & a:= a+b+1.\ a'=b \wedge b'=a-b-1 \wedge c'=c & \text{substitution law}\\
= & a'=b \wedge b'=a+b+1-b-1 \wedge c'=c & \text{simplify}\\
= & a'=b \wedge b'=a \wedge c'=c &
\end{array}$$

§(g)

$$\begin{array}{lll}
 & a' = a+b+1.\ b' = a-b-1 & \text{expand sequential composition}\\
= & \exists a'', b'', c''\cdot\ a'' = a+b+1 \ \wedge\ b' = a''-b''-1 & \text{one point for } a''\text{, identity for } c''\\
= & \exists b''\cdot\ b' = a+b+1-b''-1 & \text{simplify, rearrange, and identity}\\
= & \exists b''\cdot\ b'' = a+b-b' \ \wedge\ \top & \text{one point for } b''\\
= & \top &
\end{array}$$

§(h) $\quad a{:=}\ a{-}b.\ \ b{:=}\ a{-}b.\ \ a{:=}\ a{+}b$ $\quad$ expand last $:=$

$=\quad a{:=}\ a{-}b.\ \ b{:=}\ a{-}b.\ \ a' = a{+}b\ \ \land\ \ b'{=}b\ \ \land\ \ c'{=}c$ $\quad$ substitution law

$=\quad a{:=}\ a{-}b.\ \ a' = a{+}a{-}b\ \ \land\ \ b'{=}a{-}b\ \ \land\ \ c'{=}c$ $\quad$ substitution law

$=\quad a' = a{-}b{+}a{-}b{-}b\ \ \land\ \ b'{=}a{-}b{-}b\ \ \land\ \ c'{=}c$ $\quad$ simplify

$=\quad a'\ =\ 2{\times}a - 3{\times}b\ \ \land\ \ b'\ =\ a - 2{\times}b\ \ \land\ \ c'{=}c$