

151 Let x and y be natural variables. Here is a refinement.

$A \Leftarrow \mathbf{if } x=0 \vee y=0 \mathbf{ then } ok \mathbf{ else } x:=x-1. y:=y-1. A \mathbf{ fi}$

- (a) Add recursive time.
- (b) Find specification A that gives the exact execution time.
- (c) Prove the execution time.

After trying the question, scroll down to the solution.

(a) Add recursive time.

§ $A \Leftarrow \mathbf{if} \ x=0 \vee y=0 \ \mathbf{then} \ ok \ \mathbf{else} \ x:=x-1. \ y:=y-1. \ t:=t+1. \ A \ \mathbf{fi}$

(b) Find specification A that gives the exact execution time.

§ $A = t' = t + x \downarrow y$

(c) Prove the execution time.

§ Proof by cases. First,

$$(x=0 \vee y=0) \wedge ok$$

$$\Rightarrow x \downarrow y = 0 \wedge t' = t$$

$$\Rightarrow A$$

Second case:

$$\neg(x=0 \vee y=0) \wedge (x:=x-1. \ y:=y-1. \ t:=t+1. \ A) \quad \text{replace } A \text{ and substitute}$$

$$= \neg(x=0 \vee y=0) \wedge t' = t+1 + (x-1) \downarrow (y-1)$$

$$= \neg(x=0 \vee y=0) \wedge t' = t+1 + x \downarrow y - 1$$

$$\Rightarrow A$$