164   (*mod* 2) Let n be a natural variable. The problem to reduce n modulo 2 can be solved as follows:

$n' = mod\ n\ 2 \impliedby$ **if** $n<2$ **then** *ok* **else** $n:= n–2.\ n' = mod\ n\ 2$ **fi**

Using the recursive time measure, find and prove an upper time bound. Make it as small as you can.

After trying the question, scroll down to the solution.

§ $t' \leq t + n/2$ $\Longleftarrow$ **if** $n<2$ **then** *ok* **else** $n := n-2.\ \ t := t+1.\ \ t' \leq t + n/2$ **fi**

Proof, by cases.

     $n<2 \wedge ok$

$=$ $n<2 \wedge n'=n \wedge t'=t$

$\Longrightarrow$ $t' \leq t + n/2$

     $n \geq 2 \wedge (n := n-2.\ \ t := t+1.\ \ t' \leq t + n/2)$ substitution law twice

$=$ $n \geq 2 \wedge t' \leq t+1 + (n-2)/2$ arithmetic

$\Longrightarrow$ $t' \leq t + n/2$