282 Let n and p be natural variables. Write a program to solve

$$n\ge 2 \;\Rightarrow\; p'\colon 2^{2^{nat}} \;\wedge\; n\le p'<n^2$$

Include a finite upper bound on the execution time, but it doesn't matter how big or small.

After trying the question, scroll down to the solution.

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$$n \ge 2 \;\Rightarrow\; p'\!: 2^{2^{nat}} \wedge n \le p' < n^2 \;\Longleftarrow$$
$$\qquad p := 2.\; p\!: 2^{2^{nat}} \wedge p < n^2 \;\Rightarrow\; p'\!: 2^{2^{nat}} \wedge n \le p' < n^2$$

$$p\!: 2^{2^{nat}} \wedge p < n^2 \;\Rightarrow\; p'\!: 2^{2^{nat}} \wedge n \le p' < n^2 \;\Longleftarrow$$
$$\qquad \textbf{if } n \le p \textbf{ then } ok \textbf{ else } p := p \times p.\; p\!: 2^{2^{nat}} \wedge p < n^2 \;\Rightarrow\; p'\!: 2^{2^{nat}} \wedge n \le p' < n^2 \textbf{ fi}$$

$$t' \le t+n \;\Longleftarrow\; p := 2.\; 2 \le p \;\Rightarrow\; t' \le t+n-p+2$$

$$2 \le p \;\Rightarrow\; t' \le t+n-p+2 \;\Longleftarrow$$
$$\qquad \textbf{if } n \le p \textbf{ then } ok \textbf{ else } p := p \times p.\; t := t+1.\; 2 \le p \;\Rightarrow\; t' \le t+n-p+2 \textbf{ fi}$$