308 Suppose variable declaration with initialization is defined as

$$\mathbf{new}\ x\colon T := e\cdot P \quad = \quad \mathbf{new}\ x\colon T\cdot x := e.\ P$$

In what way does this differ from the definition given in Subsection 5.0.0?

After trying the question, scroll down to the solution.

§ According to Subsection 5.0.0,

$$
\begin{array}{lll}
& \textbf{new } x\text{: } T := e\cdot\ P & \\
= & \exists x\text{: } e\cdot\ \exists x'\text{: } T\cdot\ P & \\
= & (\text{for } x \text{ substitute } e \text{ in } \exists x'\text{: } T\cdot\ P\ ) & \text{assuming } T \text{ cannot mention } x \\
& & \text{and } e \text{ cannot mention } x' \\
= & \exists x'\text{: } T\cdot\ (\text{for } x \text{ substitute } e \text{ in } P\ ) & \text{assuming } e \text{ cannot mention } x \\
= & \exists x\text{: } T\cdot\ \exists x'\text{: } T\cdot\ (\text{for } x \text{ substitute } e \text{ in } P\ ) & \text{substitution law} \\
= & \exists x, x'\text{: } T\cdot\ (x{:=}\ e.\ P\ ) & \\
= & \textbf{new } x\text{: } T\cdot\ x{:=}\ e.\ P &
\end{array}
$$

With the three assumptions, there's no difference. So let's violate those assumptions. First, let $T = x+1$.

$$
\begin{array}{ll}
& \textbf{new } x\text{: } x+1\cdot\ x{:=}\ e.\ P \\
= & \exists x, x'\text{: } x+1\cdot\ (x{:=}\ e.\ P\ ) \\
= & \exists\langle x\text{: } x+1\cdot\ \exists x'\text{: } x+1\cdot\ (x{:=}\ e.\ P\ )\rangle
\end{array}
$$

Section 3.0 defines a function by saying “Let v be a name, and let D be a bunch of items (possibly using previously introduced names but not using v), ...”. We do not have a definition of $\langle x\text{: } x+1\cdot\ ...\ \rangle$.

Next, suppose $e = x+1$.

$$
\begin{array}{ll}
& \textbf{new } x\text{: } T := x+1\cdot\ P \\
= & \exists x\text{: } x+1\cdot\ \exists x'\text{: } T\cdot\ P \\
= & \exists\langle x\text{: } x+1\cdot\ \exists x'\text{: } T\cdot\ P\rangle
\end{array}
$$

So again we do not have a definition of $\langle x\text{: } x+1\cdot\ ...\ \rangle$.

Last, suppose $e = x'+1$.

$$
\begin{array}{ll}
& \textbf{new } x\text{: } T := x'+1\cdot\ P \\
= & \exists x\text{: } x'+1\cdot\ \exists x'\text{: } T\cdot\ P
\end{array}
$$

The x' appearing first is not the same variable as the x' appearing second.