351 (dice) If you repeatedly throw a pair of six-sided dice until they are equal,

(a)√ prove the distribution of final times is $(t'\geq t) \times (5/6)^{t'-t} \times 1/6$.

(b) prove the average final time is $t+5$.

After trying the question, scroll down to the solution.

(a)√ prove the distribution of final times is $(t'{\ge}t) \times (5/6)^{t'-t} \times 1/6$.

§ see book Subsection 5.7.0

(b) prove the average final time is $t{+}5$.

§ $(t'{\ge}t) \times (5/6)^{t'-t} / 6.\ t$ — remove .

$=$ $\Sigma t''\cdot\ (t''{\ge}t) \times (5/6)^{t''-t} / 6 \times t''$ — factor out /6

$=$ $(\Sigma t''\cdot\ (t''{\ge}t) \times (5/6)^{t''-t} \times t'')/6$ — restate sum informally

$=$ $((5/6)^0{\times}(t{+}0) + (5/6)^1{\times}(t{+}1) + (5/6)^2{\times}(t{+}2) + (5/6)^3{\times}(t{+}3) + ...)/6$

split the sum into two sums

$=$ $((5/6)^0{\times}t + (5/6)^1{\times}t + (5/6)^2{\times}t + (5/6)^3{\times}t + ...)/6$

$+$ $((5/6)^0{\times}0 + (5/6)^1{\times}1 + (5/6)^2{\times}2 + (5/6)^3{\times}3 + ...)/6$

from the top sum factor out $\times t$. In the bottom sum, remove the 0 term.

$=$ $((5/6)^0 + (5/6)^1 + (5/6)^2 + (5/6)^3 + ...){\times}t/6$

$+$ $((5/6)^1{\times}1 + (5/6)^2{\times}2 + (5/6)^3{\times}3 + ...)/6$ — The top sum is a geometric series.

$=$ $1/(1-(5/6)){\times}t/6$

$+$ $((5/6)^1{\times}1 + (5/6)^2{\times}2 + (5/6)^3{\times}3 + ...)/6$

Simplify top line. In the bottom line, let the sum be x .

$=$ $t + x/6$

$x = (5/6)^1{\times}1 + (5/6)^2{\times}2 + (5/6)^3{\times}3 + (5/6)^4{\times}4 + ...$

$(5/6){\times}x = (5/6)^2{\times}1 + (5/6)^3{\times}2 + (5/6)^4{\times}3 + ...$

$x - (5/6){\times}x = (5/6)^1 + (5/6)^2 + (5/6)^3 + ...$ — This is a geometric series.

$x/6 = (5/6)/(1 - (5/6))$

$x/6 = 5$

$x = 30$

Returning to the previous calculation, the average final time is

$t + x/6$

$=$ $t + 30/6$

$=$ $t{+}5$