357 (chameleon) There are c chameleons, of which r are red and the remainder are blue. At each tick of the clock, a chameleon is chosen at random, and

(a) it changes color. How long will it be before all chameleons have the same color?

(b) one of the chameleons of the other color changes color. How long will it be before all chameleons have the same color?

After trying the question, scroll down to the solution.

(a) it changes color. How long will it be before all chameleons have the same color?

§ The number of chameleons c is a constant. The variables are r and t. We need to find a distribution P of r' and t' such that

$$P \;=\; \textbf{if } 0<r<c \textbf{ then if } r/c \textbf{ then } r:= r-1 \textbf{ else } r:= r+1 \textbf{ fi}.\; t:= t+1.\; P \textbf{ else } ok \textbf{ fi}$$

In the end (whether in finite or infinite time) we have $t'\ge t \wedge (r'=0 \vee r'=c)$, which can be written more arithmetically as $(t'\ge t) \times ((r'=0) - (r'=0)\times(r'=c) + (r'=c))$, but this is not a distribution.

UNFINISHED

This is an example of a stable system.

(b) one of the chameleons of the other color changes color to match the color of the randomly chosen chameleon. How long will it be before all chameleons have the same color?

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