

380 (fixed-point theorem) Suppose we define nat by fixed-point construction and induction.

$$nat = 0, nat+1$$

$$B = 0, B+1 \Rightarrow nat: B$$

Prove that ordinary construction and induction

$$0, nat+1: nat$$

$$0, B+1: B \Rightarrow nat: B$$

are theorems. Warning: this is hard, and requires the use of limits.

§ Ordinary construction is immediately implied by fixed-point construction. What remains is to prove ordinary induction from both fixed-point construction and fixed-point induction.