401 (bit strings) Let a, b, c: *(0, 1) . Define operator $\oplus$ (precedence 4) as follows.

$$nil \oplus nil = 0$$
$$a \oplus 0 = a$$
$$(a; 0) \oplus 1 = a; 1$$
$$(a; 1) \oplus 1 = a \oplus 1;\ 0$$
$$a \oplus b = b \oplus a$$
$$(a \oplus b) \oplus c = a \oplus (b \oplus c)$$

Prove

(a) $a \oplus a = a; 0$

(b) $a; 0 = a \oplus a \oplus 0$

(c) $a; 1 = a \oplus a \oplus 1$

After trying the question, scroll down to the start of a solution.

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(a) $a \oplus a \;=\; a;0$

Since a: *(0, 1) then a is one of nil or $b;0$ or $b;1$ for some b: *(0, 1) .

Case $a = nil$

	$a \oplus a$	$a = nil$
=	$nil \oplus nil$	axiom
=	0	string identity law
=	$nil; 0$	$a = nil$
=	$a; 0$	

Case $a = b; 0$

	$a \oplus a$	
=	UNFINISHED	
=	$a; 0$	

Case $a = b; 1$

	$a \oplus a$	
=	UNFINISHED	
=	$a; 0$	

That was an induction, but it was not fully formal because we don't have the induction axiom for * .

(b) $a;0 \;=\; a \oplus a \oplus 0$

	$a \oplus \underline{a \oplus 0}$	axiom
=	$a \oplus a$	part (a)
=	$a; 0$	

(c) $a;1 \;=\; a \oplus a \oplus 1$

	$a \oplus a \oplus 1$	
=	UNFINISHED	
=	$a; 1$	