434 (general trees) In general, each node of a tree may have any number of subtrees.
(a) Design a data theory for general trees.
(b) Implement your theory.
(c) Prove your implementation.

After trying the question, scroll down to the solution.

(a) Design a data theory for general trees.

§ $tree \neq null$

$L\colon [*tree] \ \wedge \ x\colon X \ \Rightarrow \ construct\ L\ x\colon tree$

$L\colon [*tree] \ \wedge \ x\colon X \ \Rightarrow \ root\ (construct\ L\ x) = x$

$L\colon [*tree] \ \wedge \ x\colon X \ \wedge \ n<\#L \ \Rightarrow \ child\ n\ (construct\ L\ x) = L\ n$

If we also want an empty tree, we can add the axioms

$emptree\colon tree$

$emptree \neq construct\ L\ x$

If we want an arity function, we add

$L\colon [*tree] \ \wedge \ x\colon X \ \Rightarrow \ arity\ (construct\ L\ x) = \#L$

(b) Implement your theory.

(c) Prove your implementation.