438 (limited-stack) A stack, according to our axioms, has an unlimited capacity to have items pushed onto it. A limited-stack is a similar data structure but with a limited capacity to have items pushed onto it.

(a) Design axioms for a limited-data-stack.

(b) Design axioms for a limited-program-stack.

(c) Can the limit be 0 ?

After trying the question, scroll down to the solution.

(a) Design axioms for a limited-data-stack.

§ I suppose *limit* is a given natural , and X is a given bunch . I introduce the following new syntax: *stack* , *push* , *pop* , *top* , *empty* , *size* . Here are the axioms: Let s, t: *stack* and x, y: X . Then

$$empty\text{: } stack$$
$$size\ empty = 0$$
$$size\ s > 0 \ \Rightarrow\ pop\ s\text{: } stack$$
$$size\ s > 0 \ \Rightarrow\ size\ (pop\ s) = size\ s - 1$$
$$size\ s < limit \ \Rightarrow\ push\ s\ x\text{: } stack$$
$$size\ s < limit \ \Rightarrow\ size\ (push\ s\ x) = size\ s + 1$$
$$size\ s < limit \ \Rightarrow\ push\ s\ x \neq empty$$
$$size\ s < limit \ \Rightarrow\ (push\ s\ x = push\ t\ y \ = \ s{=}t \land x{=}y)$$
$$size\ s < limit \ \Rightarrow\ pop\ (push\ s\ x) = s$$
$$size\ s < limit \ \Rightarrow\ top\ (push\ s\ x) = x$$

(b) Design axioms for a limited-program-stack.

§ I suppose *limit* is a given natural , and X is a given bunch . I introduce the following new syntax: *push* , *pop* , *top* , *mkempty* , *size* . Here are the axioms: Let x: X . Then

$$top'{=}x \land size'{=}size{+}1 \ \Longleftarrow\ size{<}limit \land push\ x$$
$$size'{=}size{-}1 \ \Longleftarrow\ size{>}0 \land pop$$
$$size'{=}0 \ \Longleftarrow\ mkempty$$
$$ok \ \Longleftarrow\ size{<}limit \land (push\ x.\ pop)$$

(c) Can the limit be 0 ?

§ Sure. Why not? Then the empty stack is also full, and no operations are possible. But there's no logical problem.