454 A theory provides three names: *zero* , *increase* , and *inquire* . It is presented by an implementation. Let u: *bin* be the user's variable, and let v: *nat* be the implementer's variable. The axioms are

$$zero \;=\; v:= 0$$
$$increase \;=\; v:= v+1$$
$$inquire \;=\; u:= even\ v$$

Use data transformation to replace v with w: *bin* according to the transformer

(a)√ $w = even\ v$

(b) $\top$

(c) $\bot$ (this isn't a data transformer, since $\forall w\cdot \exists v\cdot \bot$ isn't a theorem, but apply it anyway to see what happens)

After trying the question, scroll down to the solution.

(a)√ $w = even\ v$

§ see book Section 7.2

(b) $\top$

§ Operation *zero* becomes

$\forall v\cdot\ \top \Rightarrow \exists v'\cdot\ \top \wedge (v{:=}\ 0)$

$=\ \forall v\cdot\ \exists v'\cdot\ u'{=}u \wedge v'{=}0$

$=\ u'{=}u$

Operation *increase* becomes

$\forall v\cdot\ \top \Rightarrow \exists v'\cdot\ \top \wedge (v{:=}\ v{+}1)$

$=\ \forall v\cdot\ \exists v'\cdot\ u'{=}u \wedge v'{=}v{+}1$

$=\ u'{=}u$

Operation *inquire* becomes

$\forall v\cdot\ \top \Rightarrow \exists v'\cdot\ \top \wedge (u{:=}\ even\ v)$ — replace assignment and use identity law

$=\ \forall v\cdot\ \exists v'\cdot\ u' = even\ v \wedge v'{=}v$ — one-point for v'

$=\ \forall v\cdot\ u' = even\ v$ — idempotent

$=\ (\forall v\cdot\ u' = even\ v) \wedge (\forall v\cdot\ u' = even\ v)$ — specialize twice

$\Longrightarrow\ u' = even\ 0\ \wedge\ u' = even\ 1$

$=\ u'{=}\top \wedge u'{=}\bot$

$=\ \bot$

This transformer is so weak that *inquire* becomes unimplementable.

(c) $\bot$ (this isn't a data transformer, since $\forall w\cdot\ \exists v\cdot\ \bot$ isn't a theorem, but apply it anyway to see what happens)

§ Operation *zero* becomes

$\forall v\cdot\ \bot \Rightarrow \exists v'\cdot\ \bot \wedge (v{:=}\ 0)$

$=\ \top$

and the same for any other operation. This “transformer” is so strong that all operations become arbitrary (completely nondeterministic).