469 (row major) The usual way to represent a 2-dimensional array in a computer's memory is in row major order, stringing the rows together. For example, the $3\times4$ array

[ [5; 2; 7; 3] ;
[8; 4; 2; 0] ;
[9; 2; 7; 7] ]

is represented as 5; 2; 7; 3; 8; 4; 2; 0; 9; 2; 7; 7 .

(a) Given naturals $n$ and $m$ , find a data transformer that transforms an $n\times m$ array $A$ to its row major representation $B$ .

(b) Using your transformer, transform $x:= A\ y\ z$ where $x$ , $y$ , and $z$ are user's variables.

After trying the question, scroll down to the solution.

(a) Given naturals $n$ and $m$, find a data transformer that transforms an $n{\times}m$ array $A$ to its row major representation $B$.

§ $\forall i: 0,..n\cdot \forall j: 0,..m\cdot A\ i\ j = B_{i\times m+j}$

(b) Using your transformer, transform $x{:=}\ A\ y\ z$ where $x$, $y$, and $z$ are user's variables.

§ $\forall A\cdot\ (\forall i: 0,..n\cdot \forall j: 0,..m\cdot A\ i\ j = B_{i\times m+j})$
$\Rightarrow \exists A'\cdot (\forall i: 0,..n\cdot \forall j: 0,..m\cdot A'\,i\ j = B'_{i\times m+j}) \wedge (x{:=}\ A\ y\ z)$ expand assignment

$=$ $\forall A\cdot\ (\forall i: 0,..n\cdot \forall j: 0,..m\cdot A\ i\ j = B_{i\times m+j})$
$\Rightarrow \exists A'\cdot (\forall i: 0,..n\cdot \forall j: 0,..m\cdot A'\,i\ j = B'_{i\times m+j}) \wedge x'{=}A\ y\ z \wedge y'{=}y \wedge z'{=}z \wedge A'{=}A$
one-point

$=$ $\forall A\cdot\ (\forall i: 0,..n\cdot \forall j: 0,..m\cdot A\ i\ j = B_{i\times m+j})$
$\Rightarrow (\forall i: 0,..n\cdot \forall j: 0,..m\cdot A\ i\ j = B'_{i\times m+j}) \wedge x'{=}A\ y\ z \wedge y'{=}y \wedge z'{=}z$
use antecedent as context

$=$ $\forall A\cdot\ (\forall i: 0,..n\cdot \forall j: 0,..m\cdot A\ i\ j = B_{i\times m+j})$
$\Rightarrow (\forall i: 0,..n\cdot \forall j: 0,..m\cdot B_{i\times m+j} = B'_{i\times m+j}) \wedge x'{=}B_{y\times m+z} \wedge y'{=}y \wedge z'{=}z$
the consequent no longer uses $A$ so $\forall A\cdot$ and the antecedent can be dropped

$=$ $(\forall i: 0,..n\cdot \forall j: 0,..m\cdot B_{i\times m+j} = B'_{i\times m+j}) \wedge x'{=}B_{y\times m+z} \wedge y'{=}y \wedge z'{=}z$
contract the first conjunct

$=$ $B'{=}B \wedge x'{=}B_{y\times m+z} \wedge y'{=}y \wedge z'{=}z$

$=$ $x{:=}\ B_{y\times m+z}$