490   Exercise 161 asks for a program to compute cumulative sums (running total). Write a program that can be transformed from sequential to concurrent execution with $log\ n$ time where n is the length of the list.

After trying the question, scroll down to the solution.

§ Let L be a list variable. The result we want is $R\ 0\ (\#L)$ where

$$R\ i\ k\ =\ \#L'=\#L\ \wedge\ \forall j\colon i,..k\cdot L'j = \Sigma\, L\,[i;..j{+}1]$$

We refine it as follows.

$$R\ i\ k\ \Longleftarrow\ \textbf{if}\ k{-}i \le 1\ \textbf{then}\ ok$$

       $\textbf{else}\quad\textbf{var}\ m\colon= div\ (k{+}i)\ 2.$

         $(R\ i\ m\ \|\ R\ m\ k).$

         $\textbf{for}\ j\colon= m;..k\ \textbf{do}\ L\ j\colon= L\ j + L\ (m{-}1)\ \textbf{od}\ \textbf{fi}$

The final **for**-loop can be concurrent, so that it takes no time. The computation then takes $log\ (\#L)$ time.