71   Write an expression equivalent to each of the following without using §.
(a) $¢(§x{:}\ D{\cdot}\ P\ x) = 0$
(b) $¢(§x{:}\ D{\cdot}\ P\ x) = 1$
(c) $¢(§x{:}\ D{\cdot}\ P\ x) = 2$

After trying the question, scroll down to the solution.

(a) $\mathcal{C}(\S x{:}\ D{\cdot}\ P\ x) = 0$

§ $\neg\exists x{:}\ D{\cdot}\ P\ x$

(b) $\mathcal{C}(\S x{:}\ D{\cdot}\ P\ x) = 1$

§ $(\exists x{:}\ D{\cdot}\ P\ x)\ \wedge\ \neg(\exists x, y{:}\ D{\cdot}\ x{\neq}y \wedge P\ x \wedge P\ y)$

or $(\exists x{:}\ D{\cdot}\ P\ x)\ \wedge\ (\forall x, y{:}\ D{\cdot}\ P\ x \wedge P\ y \Rightarrow x{=}y)$

or $\exists x{:}\ D{\cdot}\ \forall y{:}\ D{\cdot}\ P\ y = (x{=}y)$

(c) $\mathcal{C}(\S x{:}\ D{\cdot}\ P\ x) = 2$

§ $(\exists x, y{:}\ D{\cdot}\ x{\neq}y \wedge P\ x \wedge P\ y)\ \wedge\ \neg(\exists x, y, z{:}\ D{\cdot}\ x{\neq}y{\neq}z{\neq}x \wedge P\ x \wedge P\ y \wedge P\ z)$