82 There are some people, some keys, and some doors. Let p **holds** k mean that person p holds key k. Let k **unlocks** d mean that key k unlocks door d. Let p **opens** d mean that person p can open door d. Formalize

(a) Anyone can open any door if they have the appropriate key.

(b) At least one door can be opened by anyone without a key.

(c) The locksmith can open any door even without a key.

After trying the question, scroll down to the solution.

§ The solutions offered formalize one understanding of the sentences, but not the only reasonable understanding.

(a) Anyone can open any door if they have the appropriate key.

§ $\forall p\text{: } \mathit{people}\cdot\ \forall d\text{: } \mathit{doors}\cdot\ \forall k\text{: } \mathit{keys}\cdot\ (p \ \mathbf{opens}\ d) \Leftarrow (p \ \mathbf{holds}\ k) \wedge (k \ \mathbf{unlocks}\ d)$

(b) At least one door can be opened by anyone without a key.

§ This is slightly ambiguous. It might mean that anyone who doesn't have a key can open at least one door.

$\forall p\text{: } \mathit{people}\cdot\ (\forall k\text{: } \mathit{keys}\cdot\ \neg(p \ \mathbf{holds}\ k)) \Rightarrow \exists d\text{: } \mathit{doors}\cdot\ (p \ \mathbf{opens}\ d)$

But more likely it means that anyone can open at least one door even if they don't have a key.

$\forall p\text{: } \mathit{people}\cdot\ \exists d\text{: } \mathit{doors}\cdot\ (p \ \mathbf{opens}\ d)$

(c) The locksmith can open any door even without a key.

§ $\forall d\text{: } \mathit{doors}\cdot\ \mathit{locksmith}\ \mathbf{opens}\ d$