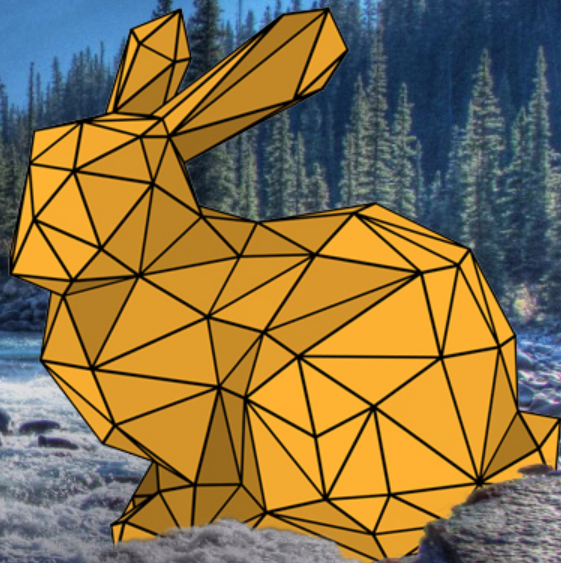


Geometry Processing in the Wild

Alec Jacobson

Canada Research Chair

University of Toronto



Big Data

video
images
audio

today



Big Data

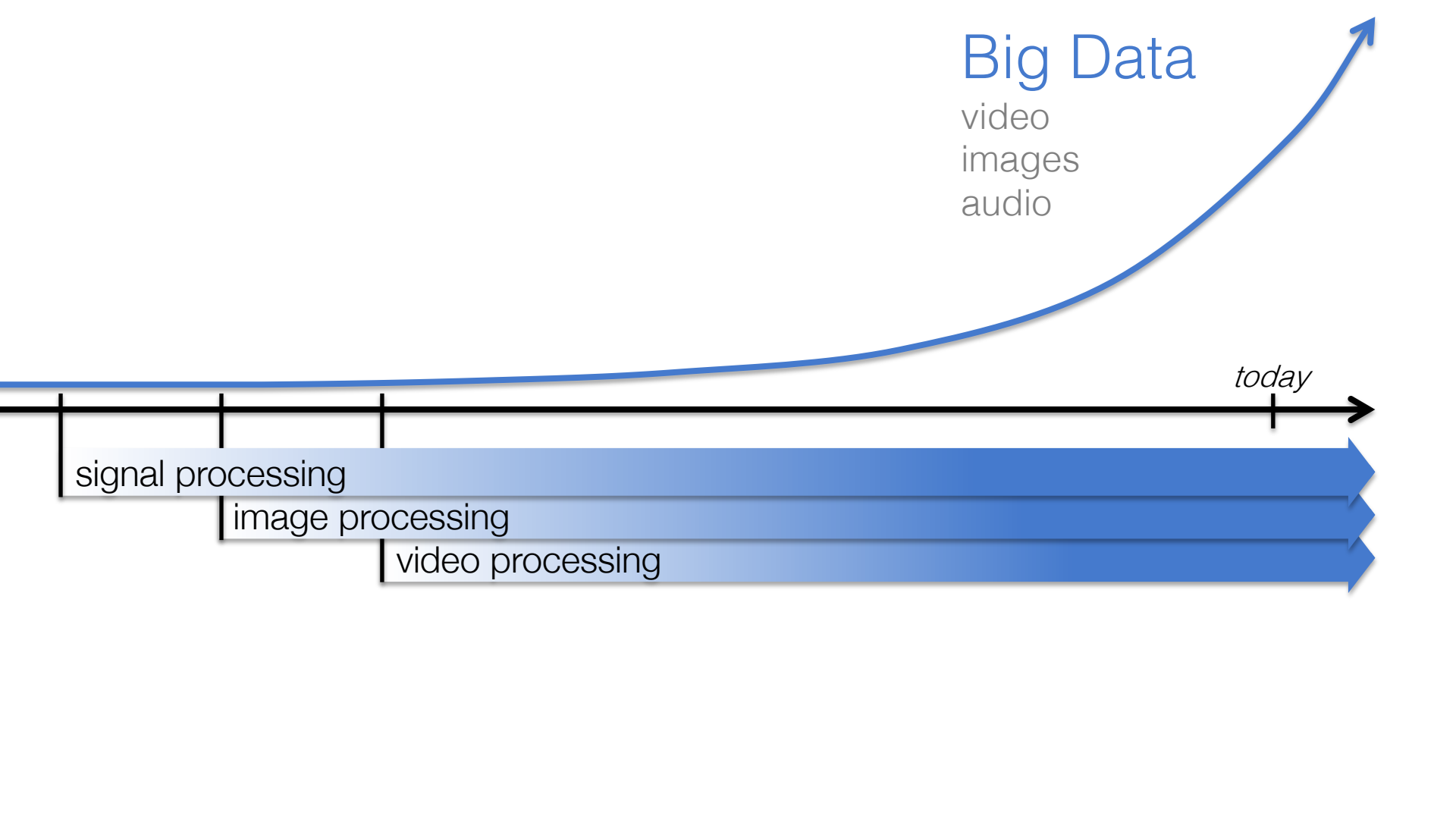
video
images
audio

today

signal processing

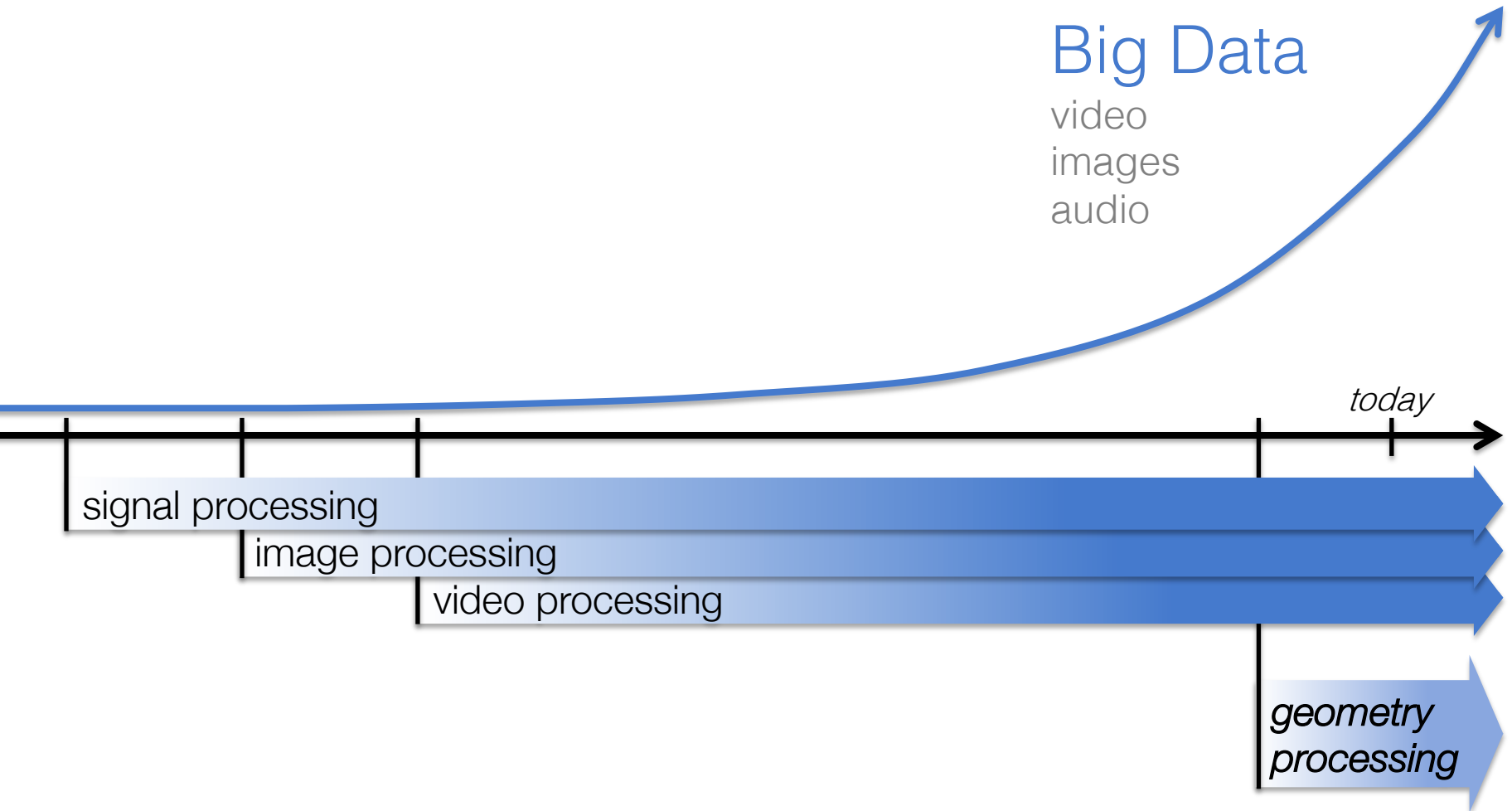
image processing

video processing



Big Data

video
images
audio



signal processing

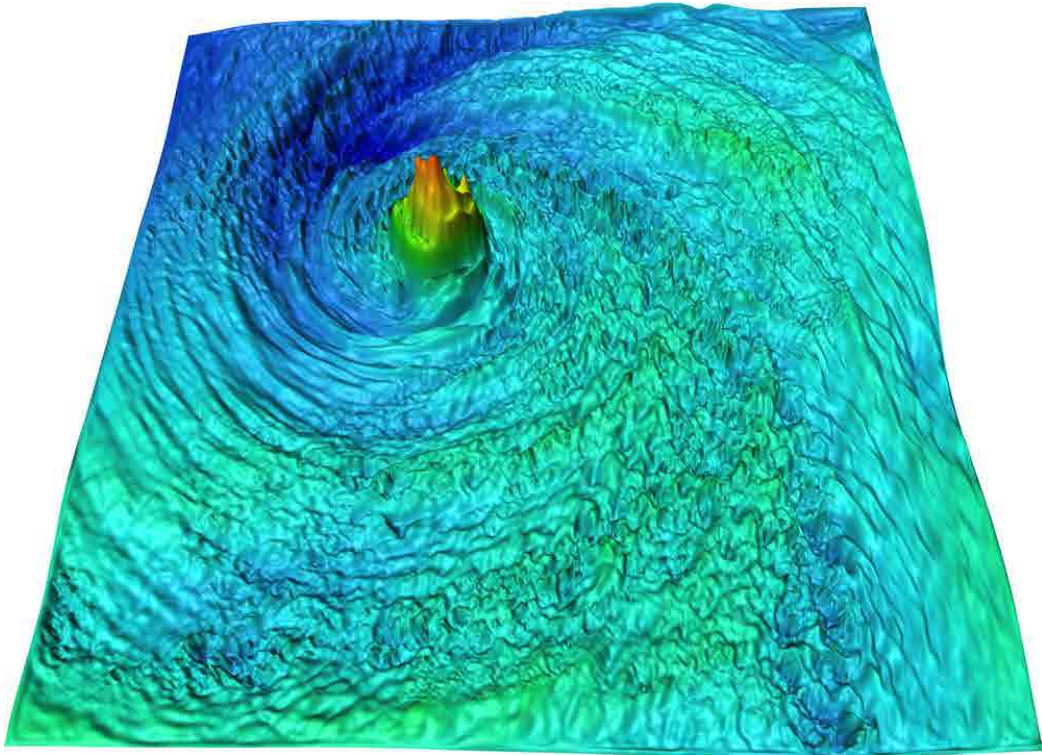
image processing

video processing

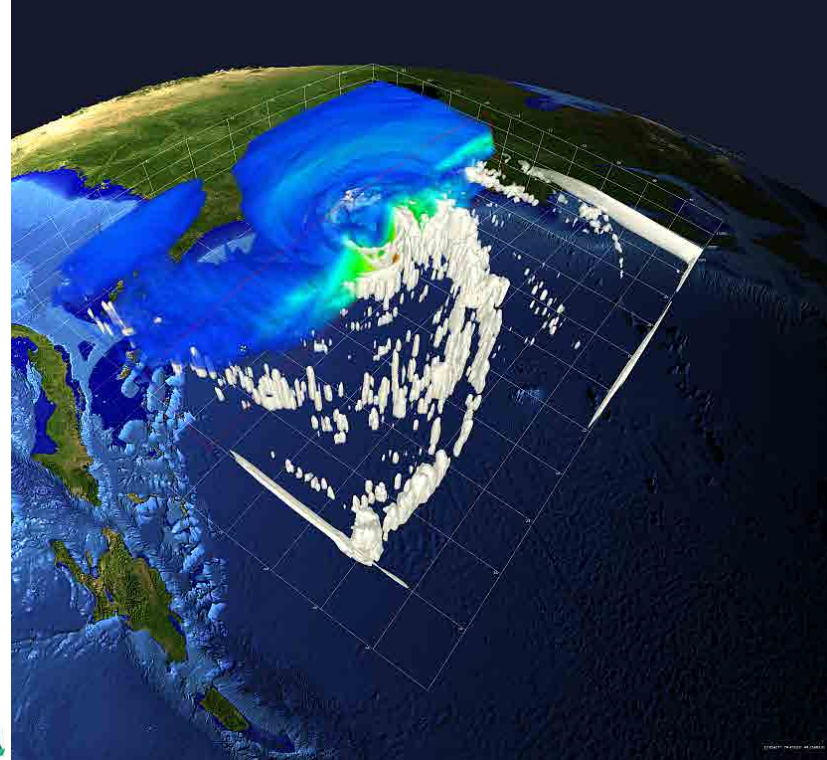
*geometry
processing*

today

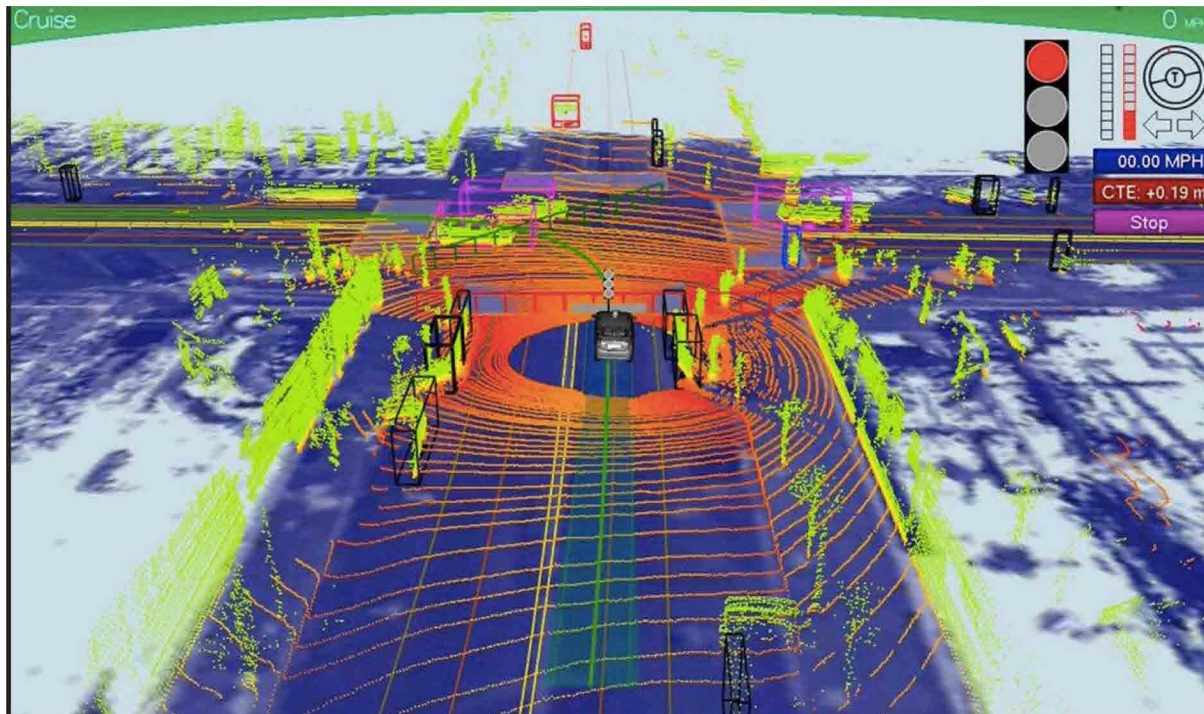
Geometric data are everywhere



Hurricane Isabel



Geometric data are everywhere



Google Self-Driving Car

The New York Times

Self-Driving Uber Car Kills Pedestrian in Arizona, Where Robots Roam



A woman crossing Mill Avenue at its intersection with Curry Road in Tempe, Ariz., on Monday. A pedestrian was struck and killed by a self-driving Uber vehicle at the intersection a night earlier. Caitlin O'Hara for The New York Times

By Daisuke Wakabayashi

March 19, 2018



[Leer en español](#)

SAN FRANCISCO — Arizona officials saw opportunity when Uber and other companies began testing driverless cars a few years ago. Promising to keep oversight light, they invited the companies to test their robotic vehicles on the state's roads.

Then on Sunday night, an autonomous car operated by Uber — and with an emergency backup driver behind the wheel — struck and killed a woman on a street in Tempe, Ariz. It was believed to be the first pedestrian death associated with self-driving technology. The

Geometric data are everywhere



Siemens Medical



R Loganathan

Geometric data are everywhere



Intel RealSense D415

1. Step Back 2. Adjust Garment 3. Colors & Photos

Next ▶

Start Over

Help? x

Haydel Zyher - Black Truffle
\$184.00

Colors

Zugara

Geometric data are everywhere



thealternativelimbproject.com



New Balance

Geometric data are everywhere



Calvino Noir



Red Dead Redemption

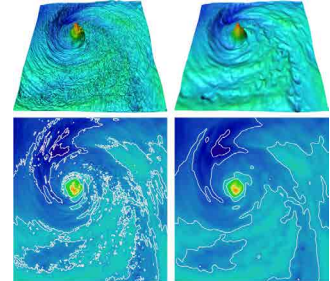
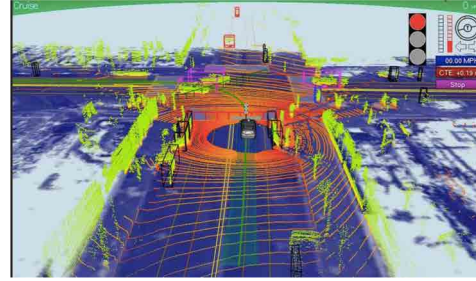
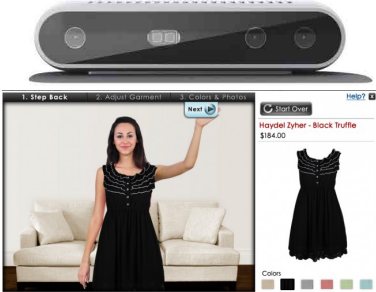
Geometric data are everywhere



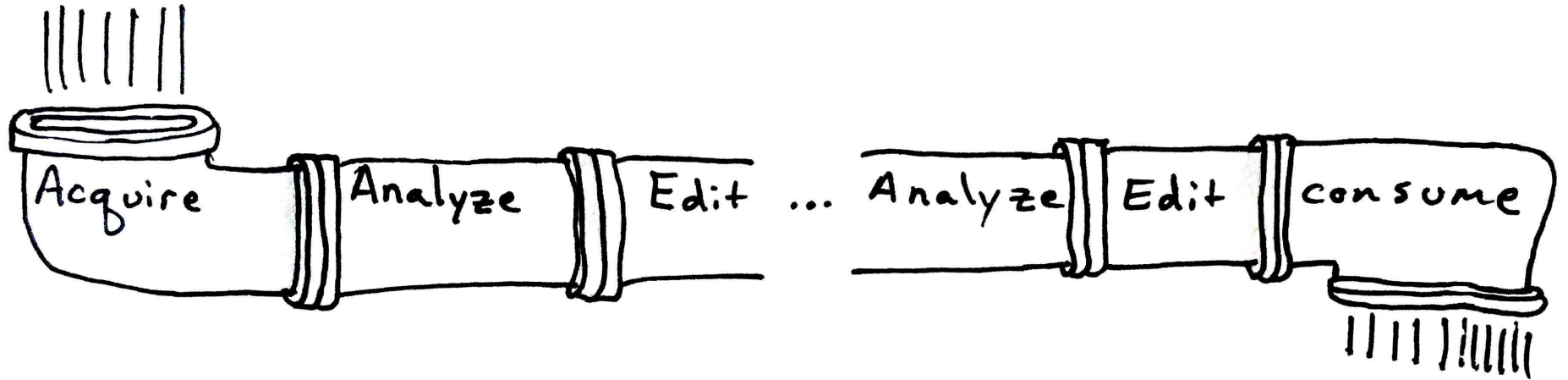
Ugly Betty

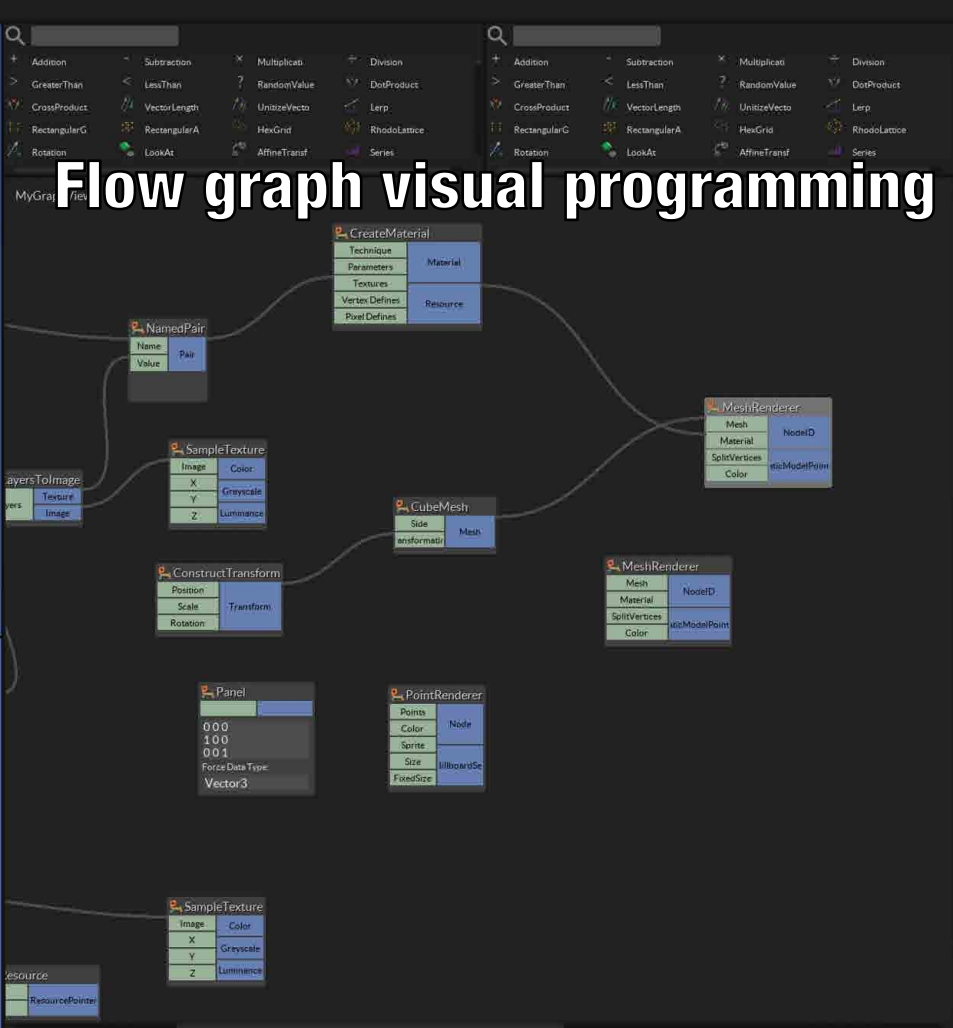
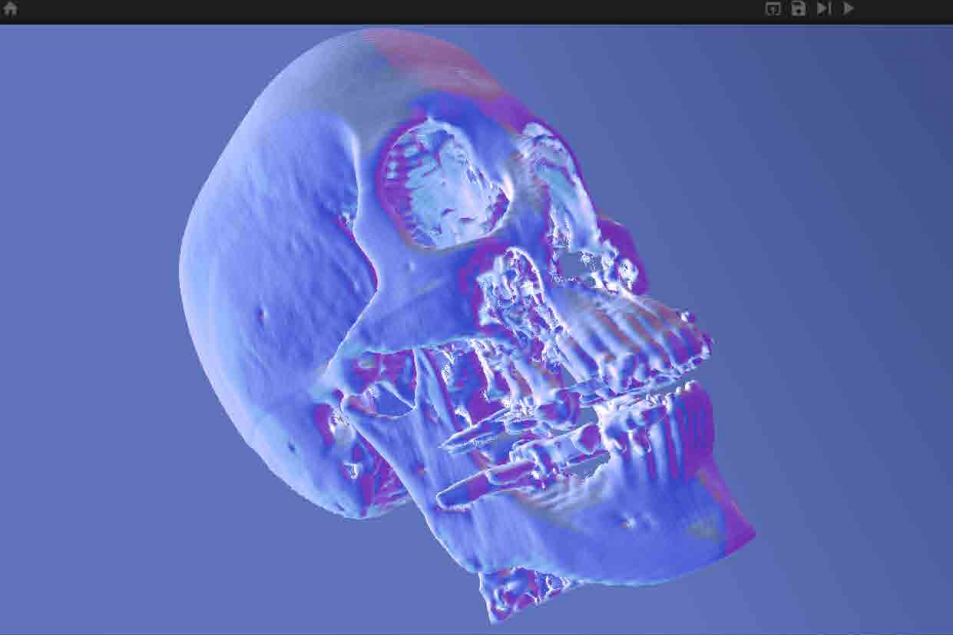
El secreto de sus ojos

All require collecting, processing and using geometric data

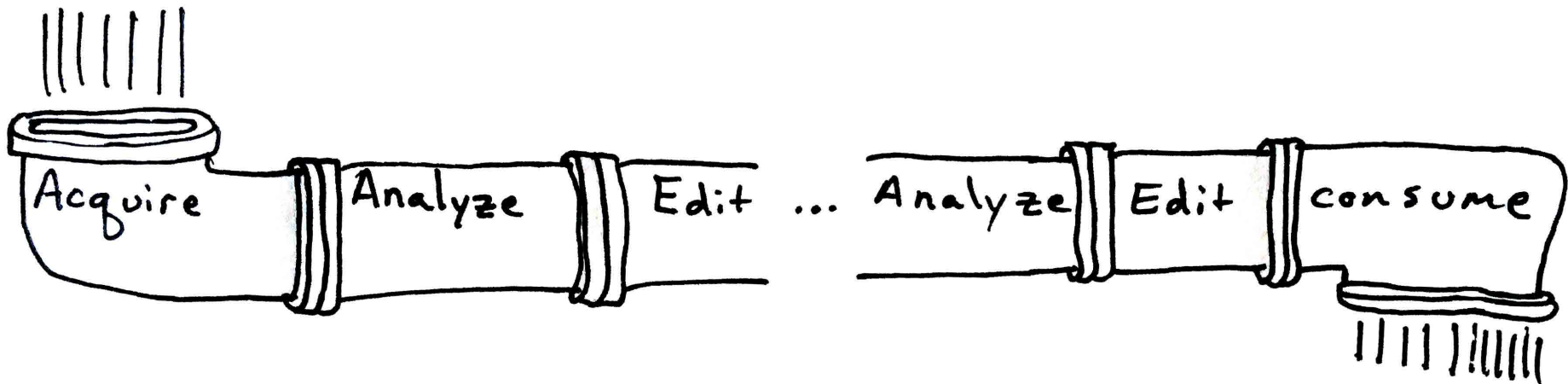


Traditionally we think of the geometry processing *pipeline*...

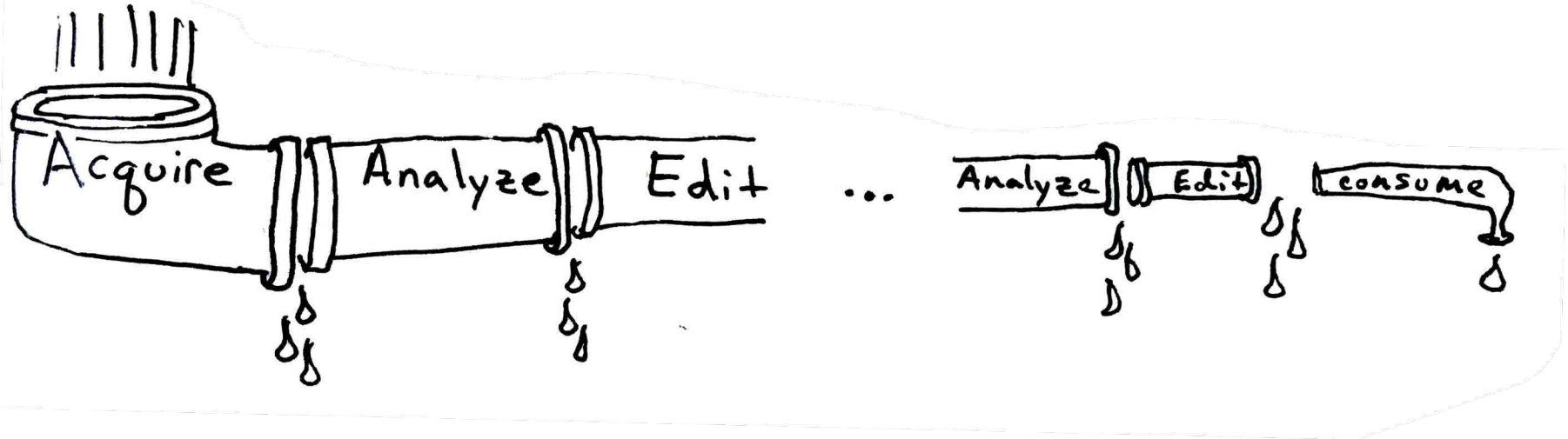




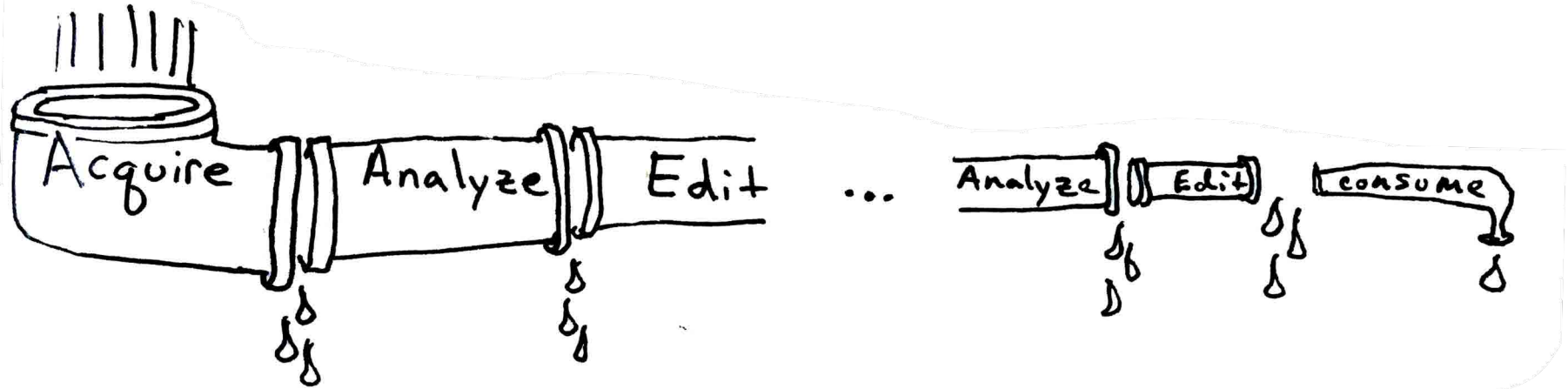
Traditionally we think of the geometry processing *pipeline*...



... in the *wild* this pipeline is leaky

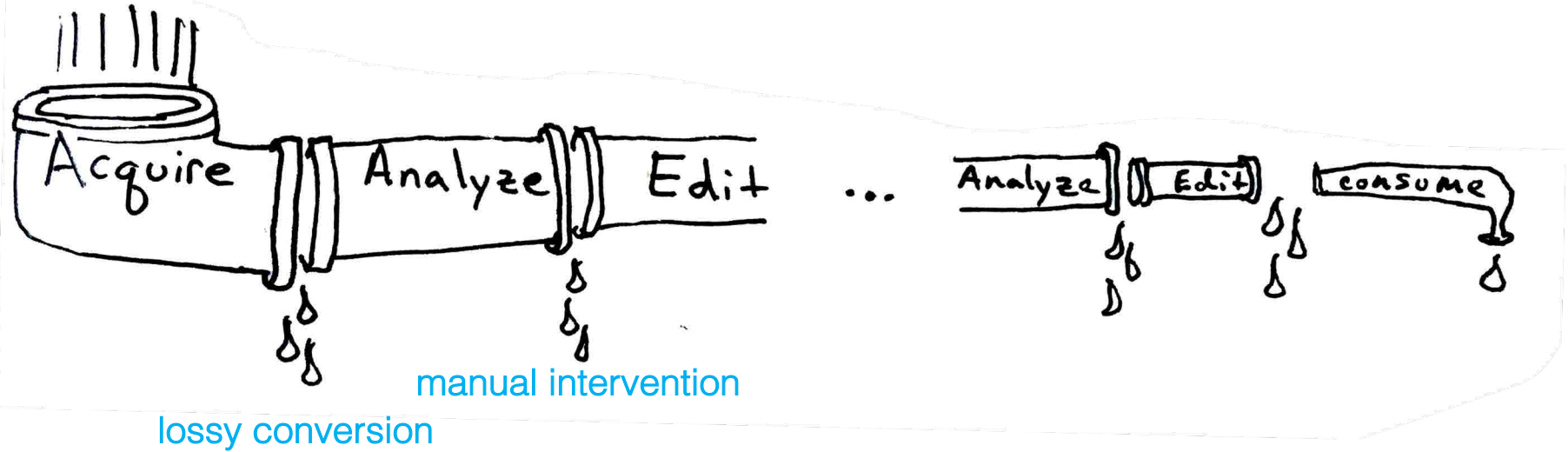


... in the *wild* this pipeline is leaky

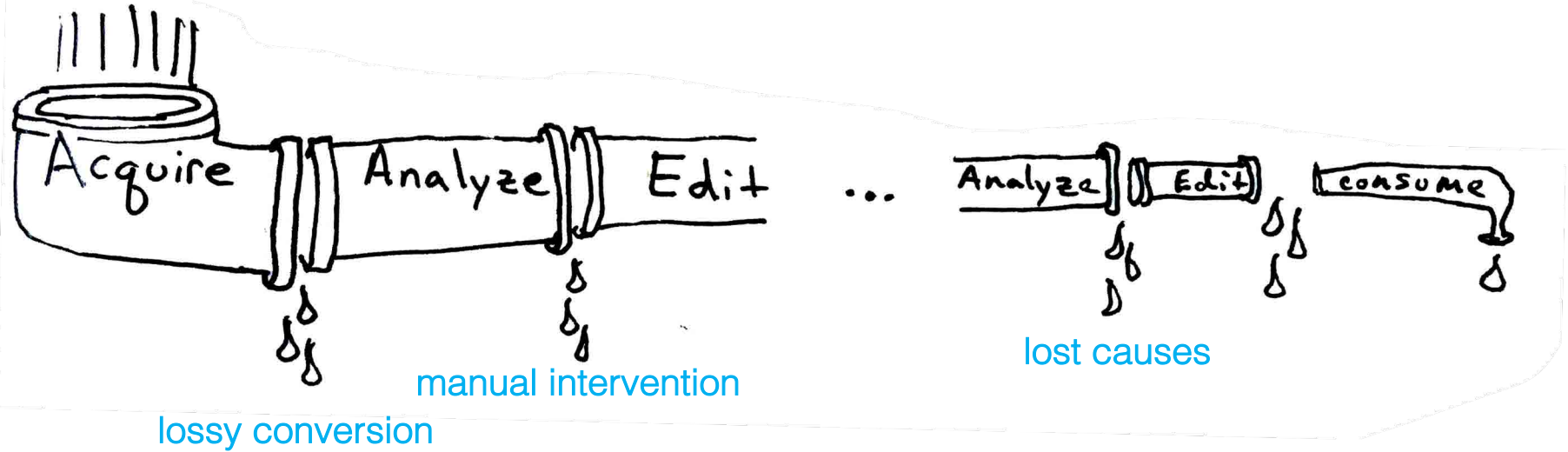


lossy conversion

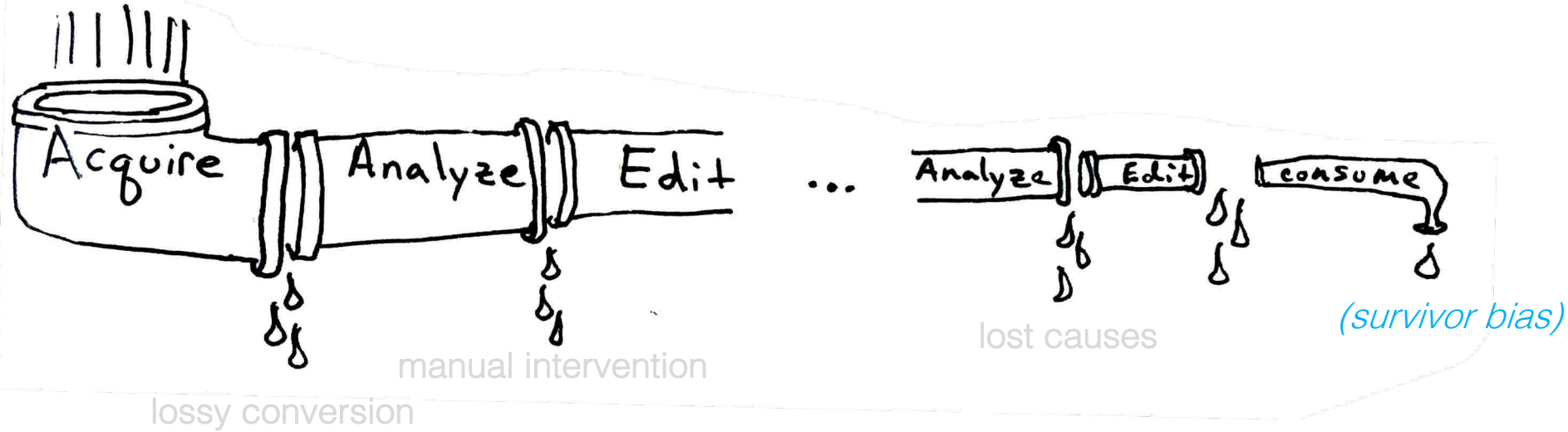
... in the *wild* this pipeline is leaky



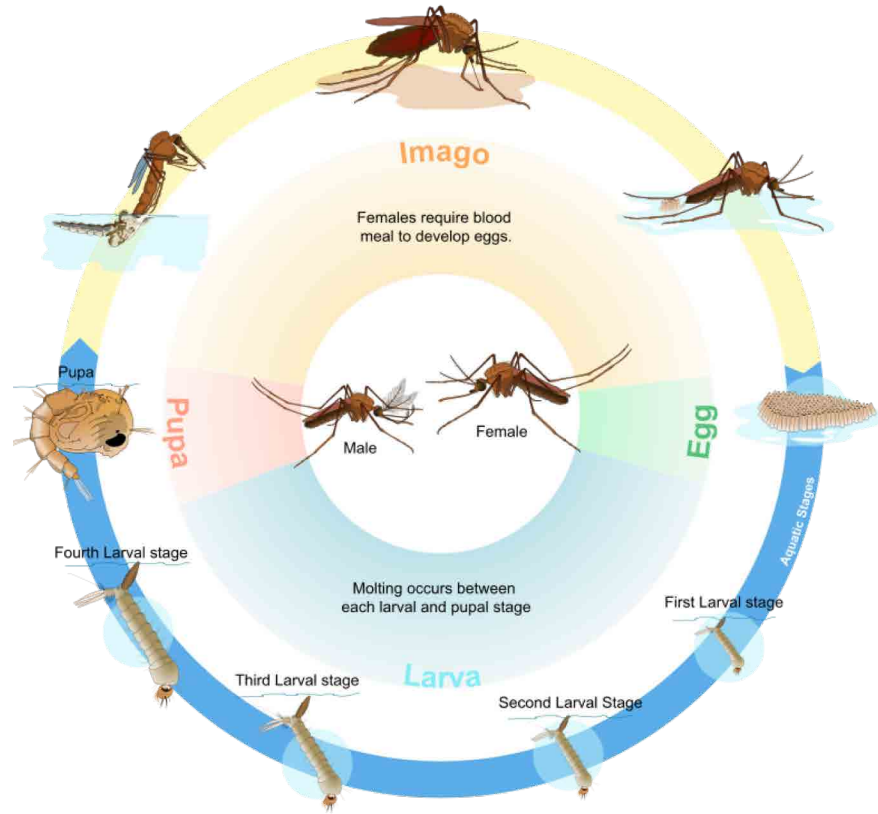
... in the *wild* this pipeline is leaky



... in the *wild* this pipeline is leaky

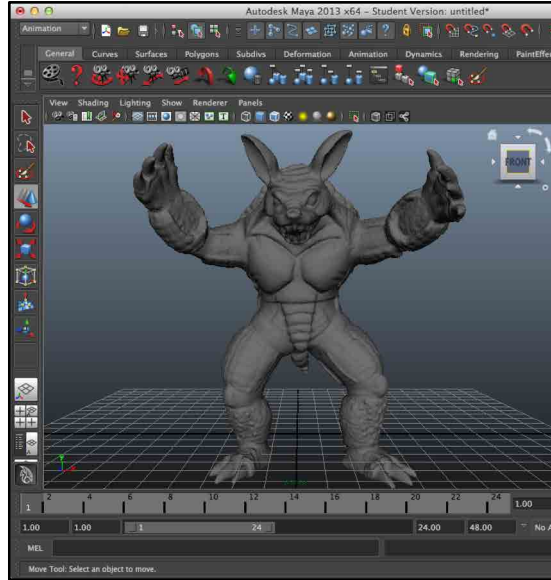


Geometry Processing is biology



Geometry processing studies the *life of a shape*

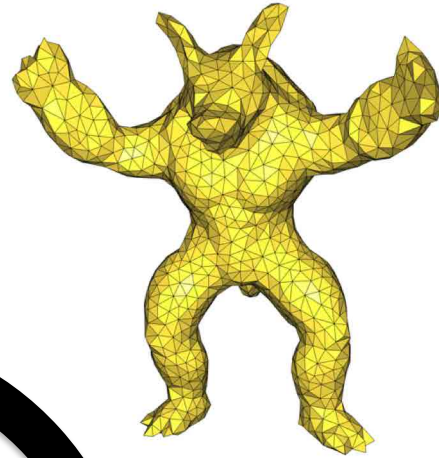
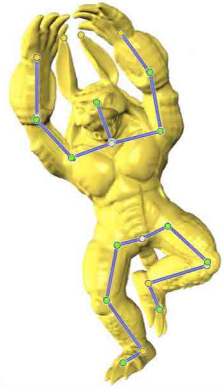
birth



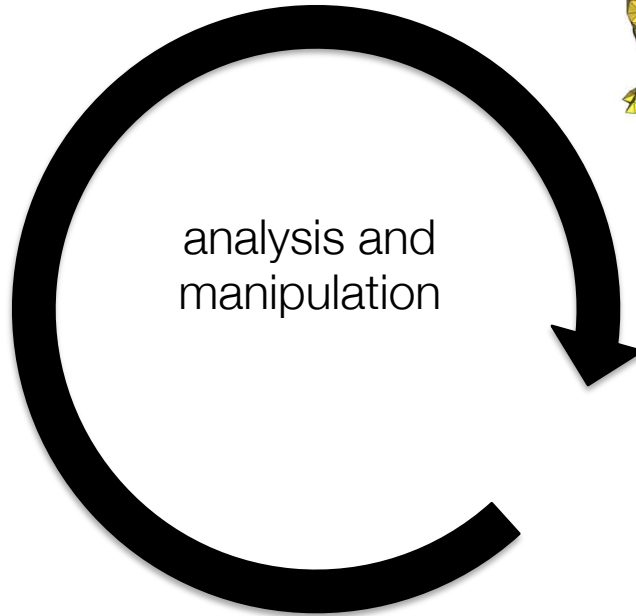
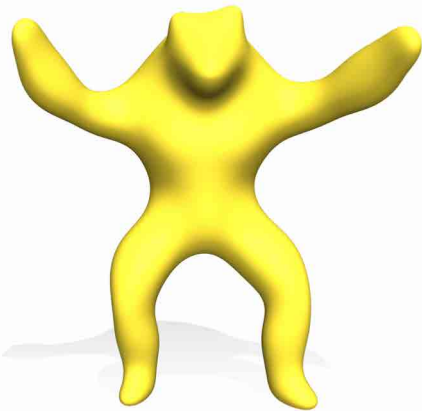
...

e.g., scan of a physical object or modeling in Maya

Geometry processing studies the *life of a shape*



...



...

Geometry processing studies the *life of a shape*

consumption



animation

...



Geometry processing studies the *life of a shape*

...



consumption



3d printing

We should be suspicious of the conventional assumptions

“we can use any type or resolution of discretization any time we want”

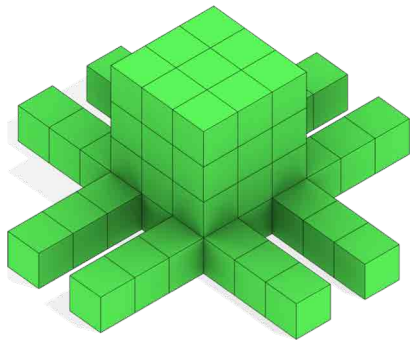
“geometric domain is given with certainty”

Conventional assumptions make
problems well-posed and simpler

Conventional assumptions make problems well-posed and simpler

“Construct a coarse-to-fine multigrid hierarchy for an octopus 🐙 shape”

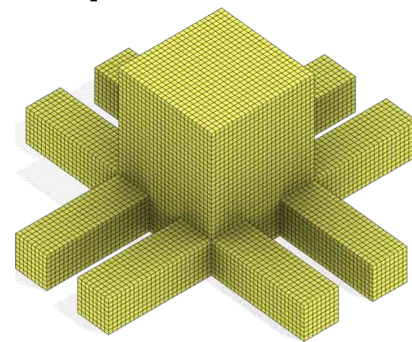
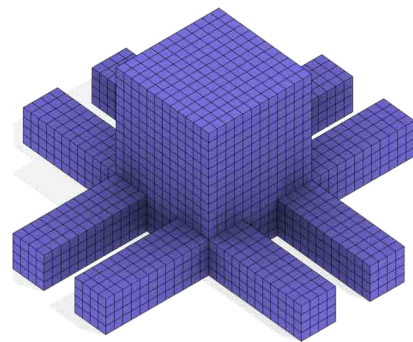
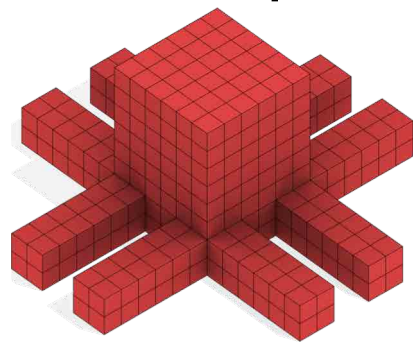
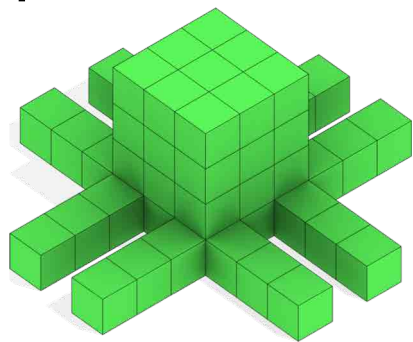
Conventional assumptions make problems well-posed and simpler



assume geometry is
representable on coarsest level

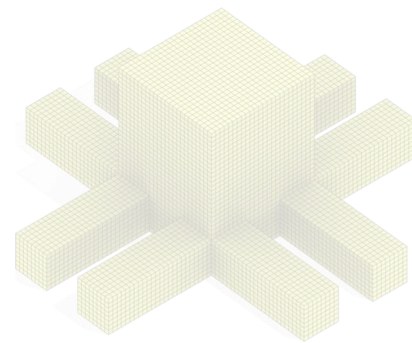
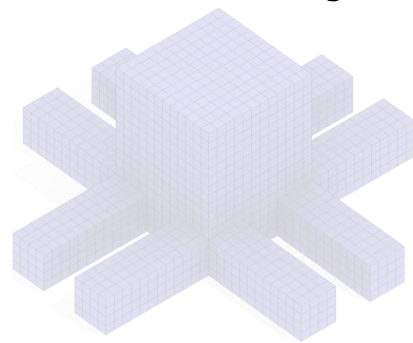
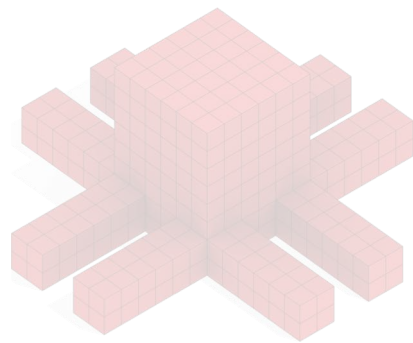
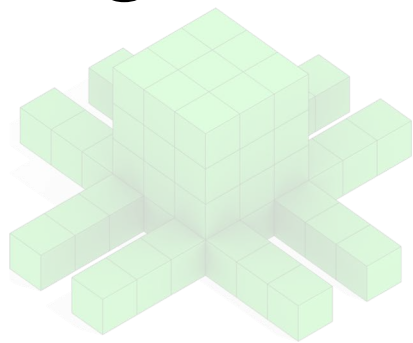
“Construct a coarse-to-fine
multigrid hierarchy for an octopus 🐙 shape”

Conventional assumptions make problems well-posed and simpler



“Construct a coarse-to-fine multigrid hierarchy for an octopus 🐙 shape”

In the *wild*, geometry is high-resolution and messy

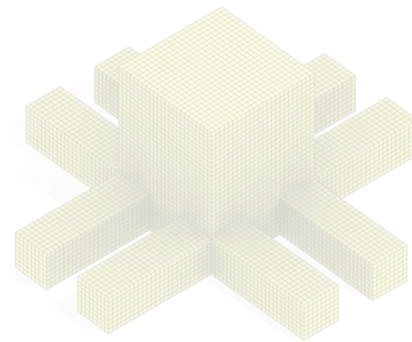
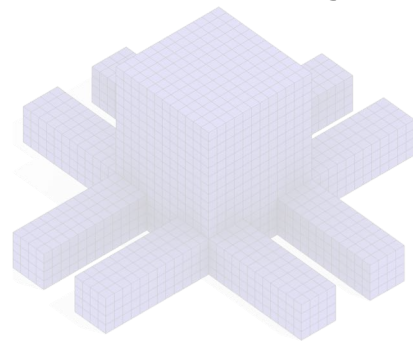
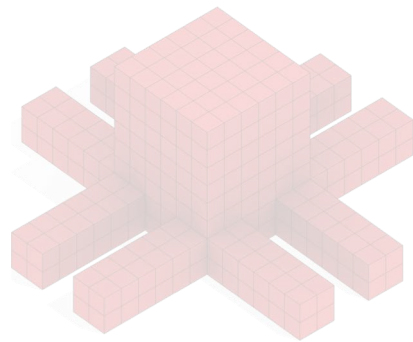
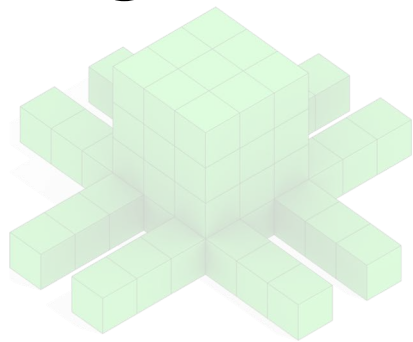


“Construct a coarse-to-fine multigrid hierarchy for an octopus 🐙 shape”

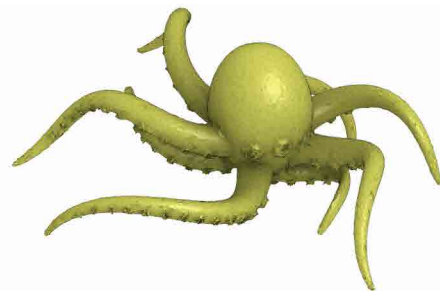
need to construct coarse domains *given* a fine domain



In the *wild*, geometry is high-resolution and messy



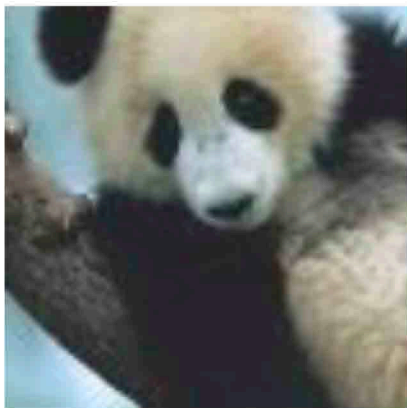
“Construct a coarse-to-fine multigrid hierarchy for an octopus 🐙 shape”



“Nested Cages” [Sacht, Vouga, & J. 2015]

Images are often easier to work with
than geometry...

Images are often easier to work with than geometry...



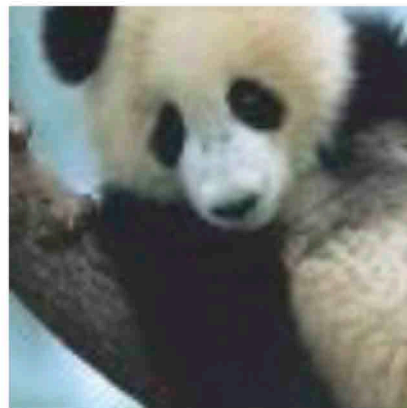
“panda”
57.7% confidence

+ .007 ×



“nematode”
8.2% confidence

=



“gibbon”
99.3 % confidence

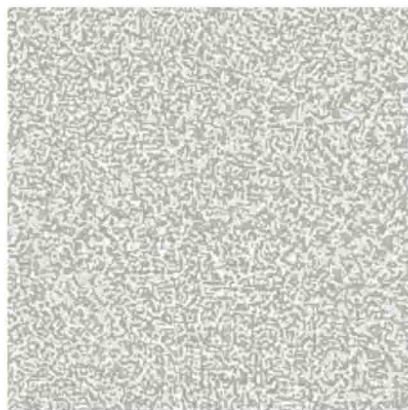
Images are often easier to work with than geometry...



“panda”

57.7% confidence

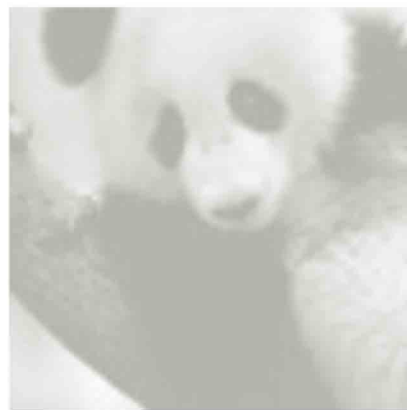
+ .007 ×



“nematode”

8.2% confidence

=



“gibbon”

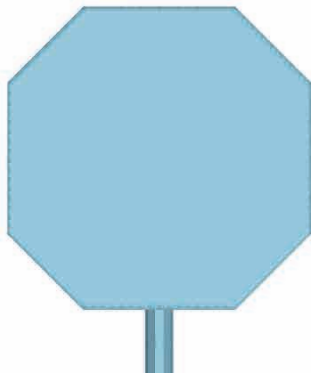
99.3 % confidence

“Most adversarial example research today is based on a specific *toy game* in the context of visual object recognition.”

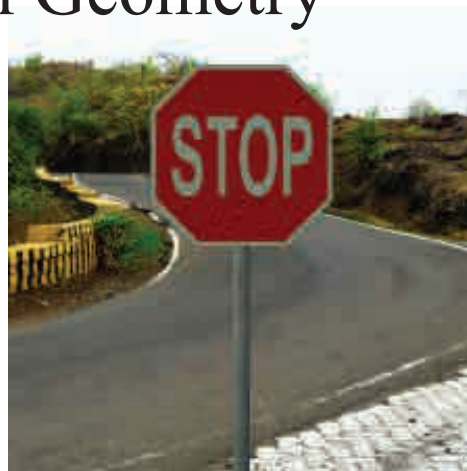
-- Goodfellow, *three years later*

Geometry and light are the primary degrees of freedom behind images

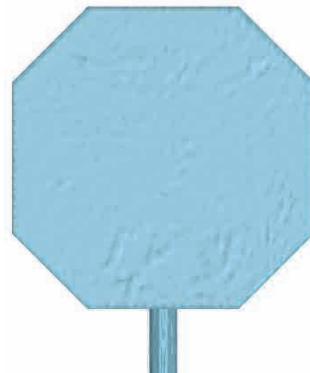
Adversarial Geometry



street sign 98%

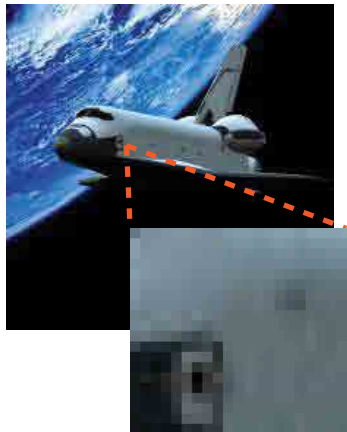


mailbox 83%

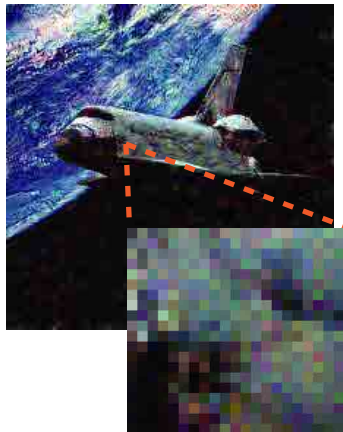


Geometry and light are the primary degrees of freedom behind images

original
image



one-step pixel
[Goodfellow 14]

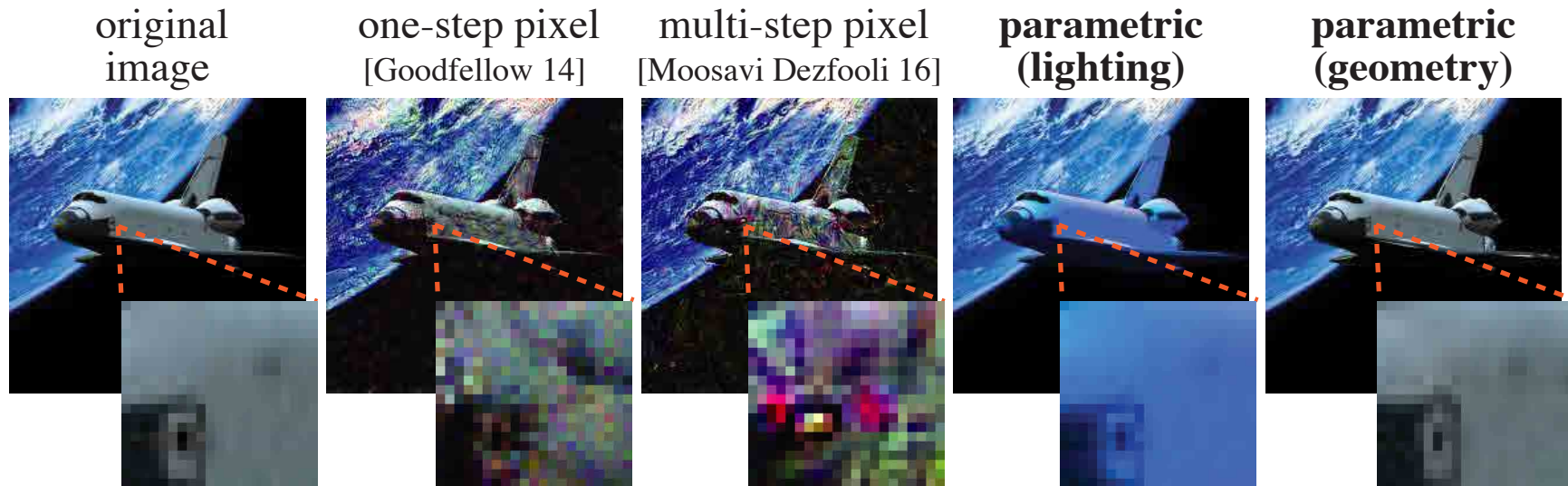


multi-step pixel
[Moosavi Dezfouli 16]



"Beyond Pixel Norm-Balls: Parametric Adversaries using an Analytically Differentiable Renderer"
[Liu, Tao, Chun-Liang Li, Nowrouzezahrai, & J. 2019]

Geometry and light are the primary degrees of freedom behind images



"Beyond Pixel Norm-Balls: Parametric Adversaries using an Analytically Differentiable Renderer"
[Liu, Tao, Chun-Liang Li, Nowrouzezahrai, & J. 2019]

Linear models are straightforward and simple to work with...

PCA - 62 dof



95Hz

"Latent-space Dynamics for Reduced Deformable Simulation"
[Fulton, Modi, Duvenaud, Levin, J. 2019]

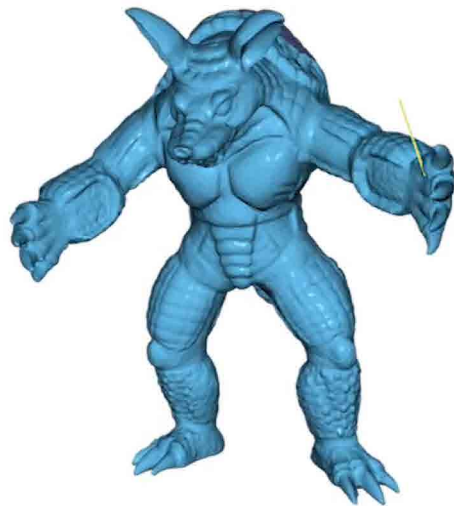
... but ultimately an inefficient use of data

PCA - 62 dof



95Hz

Ours - 20 dof



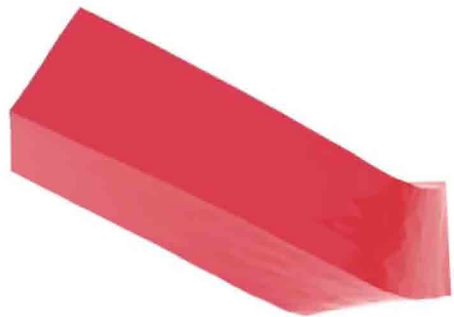
159 Hz

"Latent-space Dynamics for Reduced-Deformation Simulation"
[Fulton, Modi, Duvenaud, Levin, J. 2019]

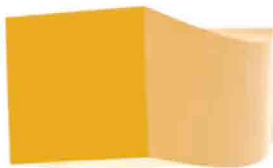
SCREEN CAPTURE

... but ultimately an inefficient use of data

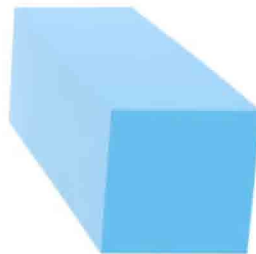
Quasi-Static Convergence



Full



Linear



Ours

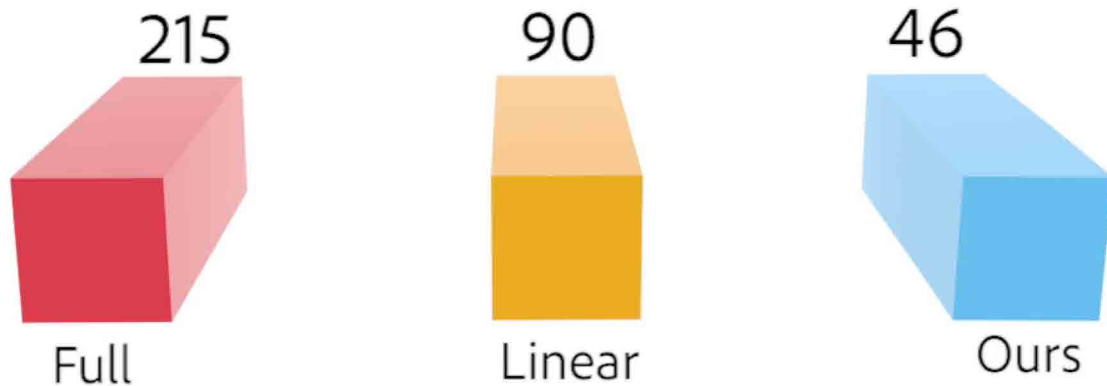
Iterations

40

"Latent-space Dynamics for Reduced Deformable Simulation"
[Fulton, Modi, Duvenaud, Levin, J. 2019]

... but ultimately an inefficient use of data

Quasi-Static Convergence



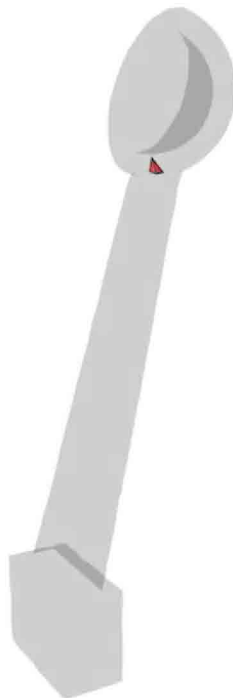
Iterations

"Latent-space Dynamics for Reduced Deformable Simulation"
[Fulton, Modi, Duvenaud, Levin, J. 2019]

... but ultimately an inefficient use of data

Stability Test

Single Cubature Point



"Latent-space Dynamics for Reduced Deformable Simulation"
[Fulton, Modi, Duvenaud, Levin, J. 2019]

... but ultimately an inefficient use of data



6 dof linear subspace

SCREEN CAPTURE

"Latent-space Dynamics for Reduced Deformable Simulation"
[Fulton, Modi, Duvenaud, Levin, J. 2019]

... but ultimately an inefficient use of data



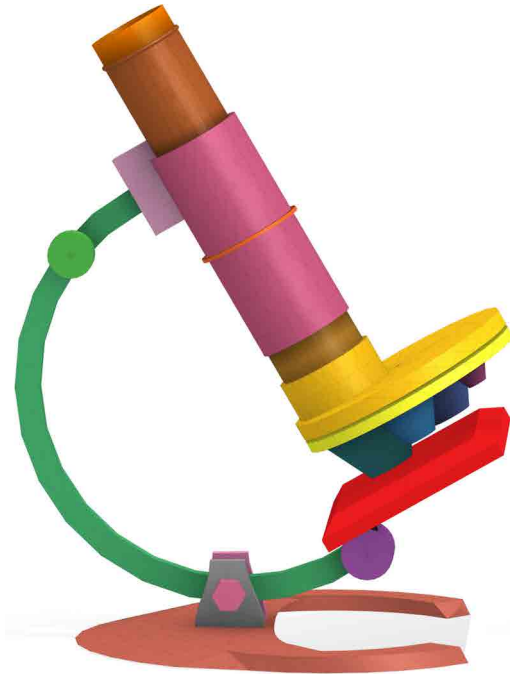
2 dof Autoencoder subspace (ours)

SCREEN CAPTURE

"Latent-space Dynamics for Reduced Deformable Simulation"
[Fulton, Modi, Duvenaud, Levin, J. 2019]

Isolating subproblems from full pipeline
can make robustness harder

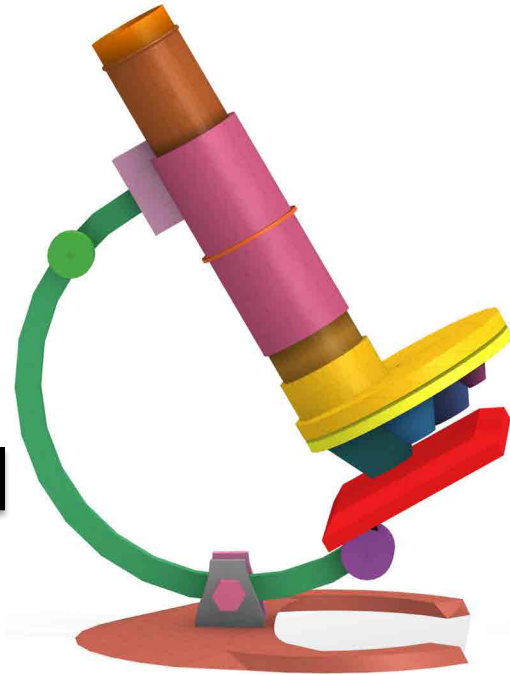
Isolating subproblems from full pipeline can make robustness harder



Solve heat equation on
this microscope
modeled as union of 24
overlapping parts

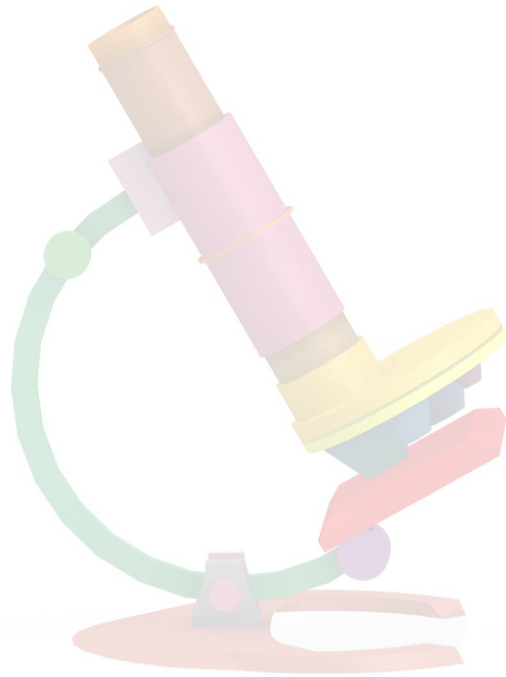
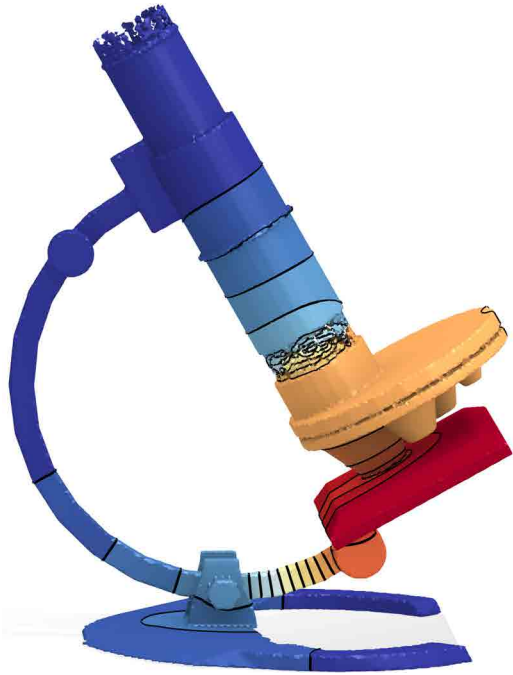
Isolating subproblems from full pipeline can make robustness harder

Discretize volume and
use finite element method



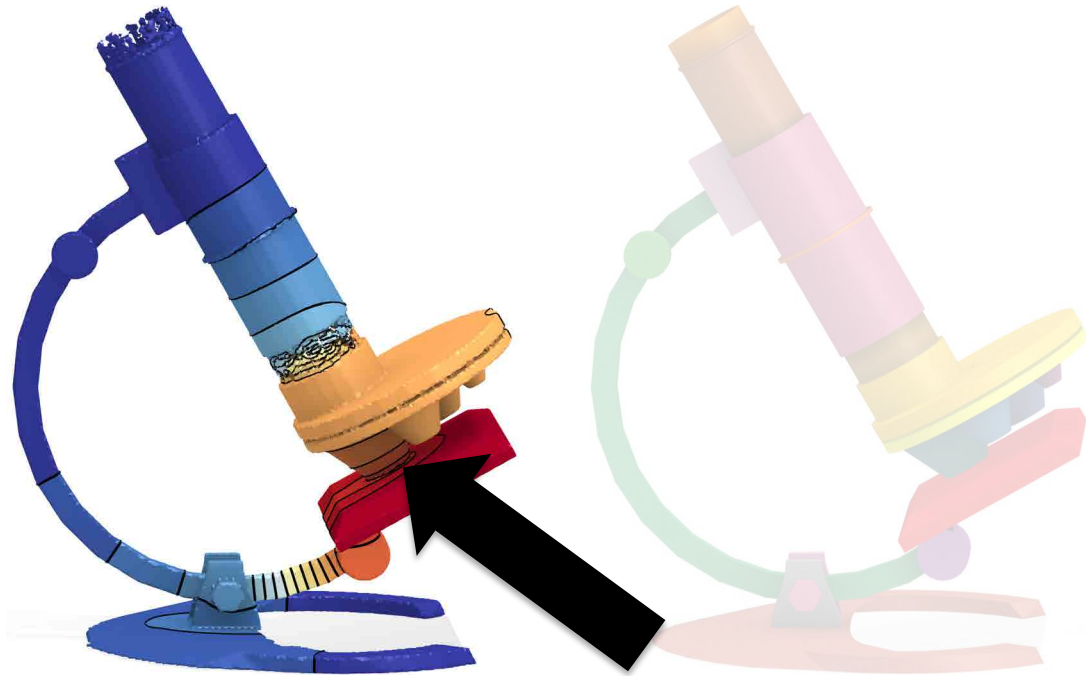
Solve heat equation on
this microscope
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Isolating subproblems from full pipeline can make robustness harder



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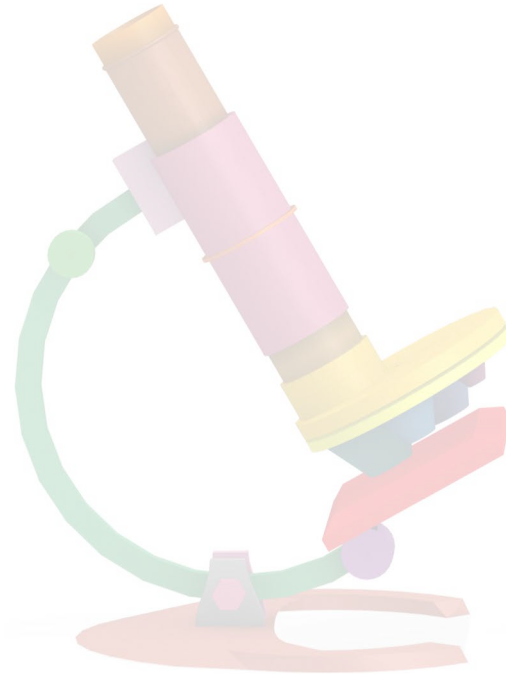
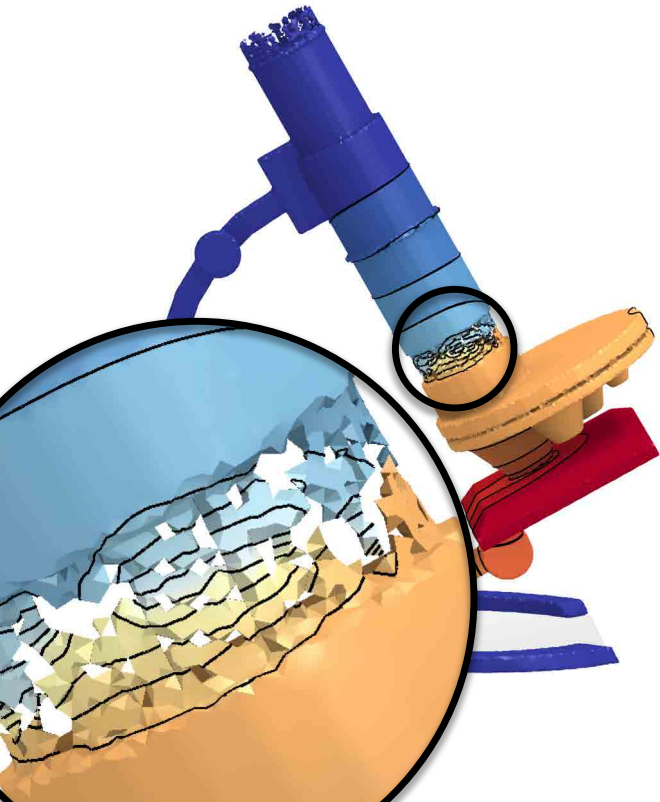
Isolating subproblems from full pipeline can make robustness harder



close-but-separate parts
fused together

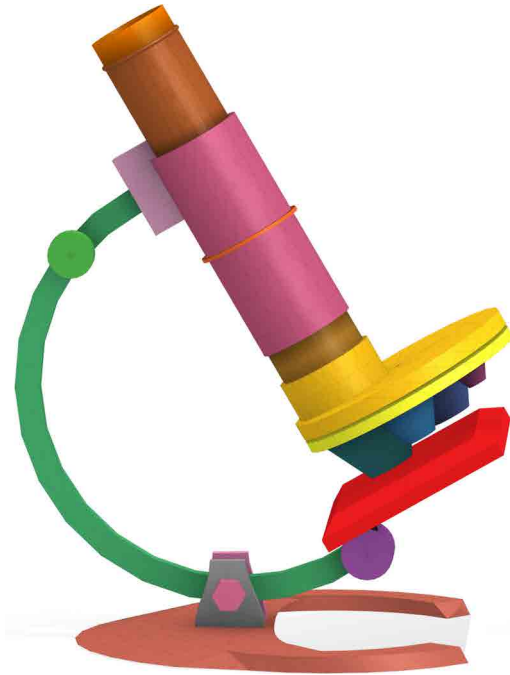
Solve heat equation on
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Isolating subproblems from full pipeline can make robustness harder



Solve heat equation on
this microscope
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overlapping parts

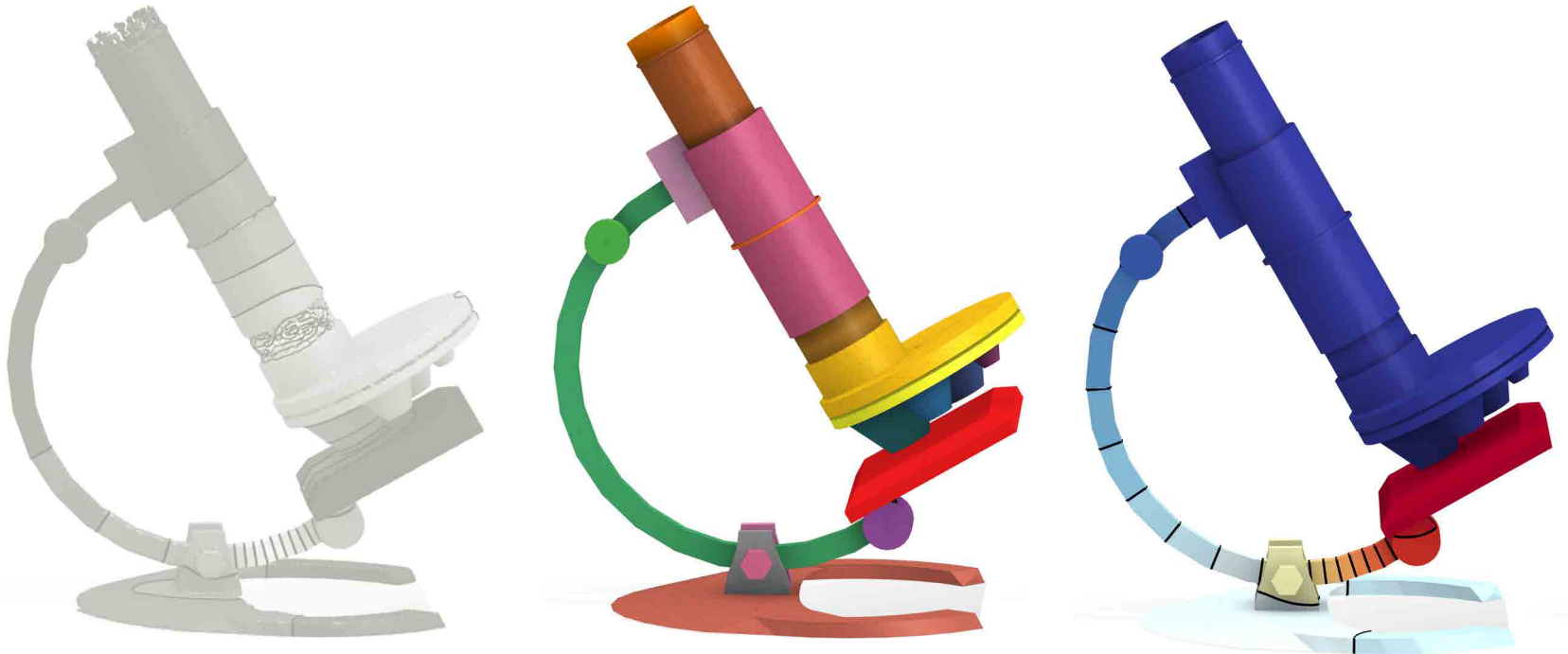
Isolating subproblems from full pipeline can make robustness harder



Discretize *each* volume
then solve coupled PDE



Isolating subproblems from full pipeline can make robustness harder



"Solid Geometry Processing on Deconstructed Domains"
[Sellán, Cheng, Ma, Dembowski, & J. 2019]

Return to fundamental questions about shapes
and consider generalization to *messy* shapes

Return to fundamental questions about shapes
and consider generalization to *messy* shapes

Am I inside or outside of a shape?

Return to fundamental questions about shapes
and consider generalization to *messy* shapes

Am I inside or outside of a shape?

What is the combination of two shapes?

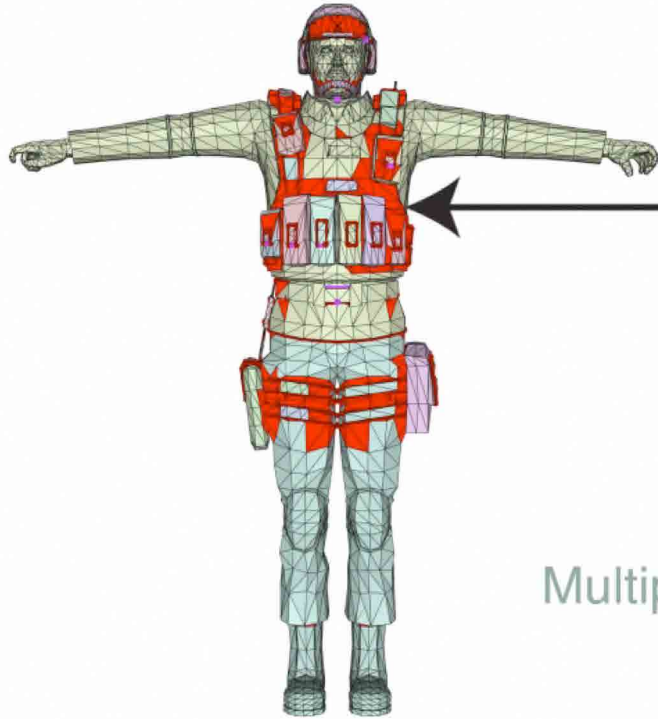
Return to fundamental questions about shapes
and consider generalization to *messy* shapes

Am I inside or outside of a shape?

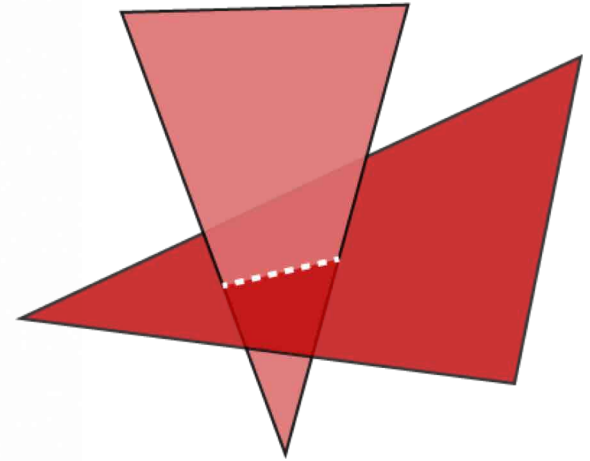
What is the combination of two shapes?

Given a solid object's surface,
can I represent the volume?

Good enough for visualization does not imply
good enough for admit geometric computation

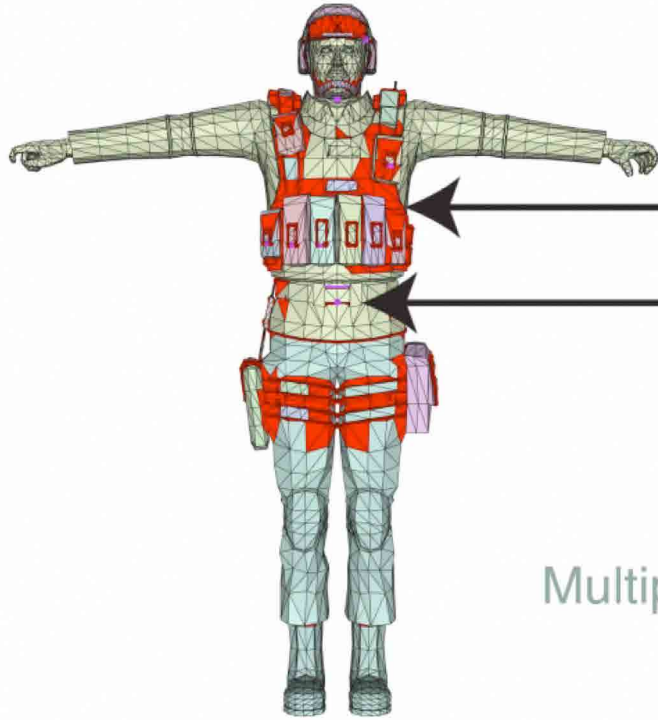


Self-intersections



Multiple connected components

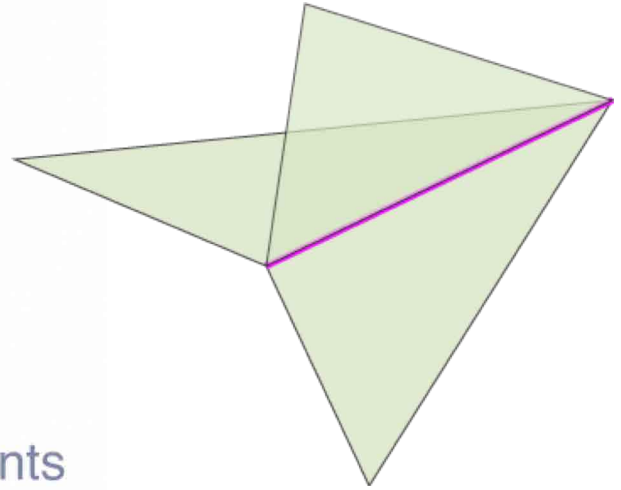
Good enough for visualization does not imply
good enough for admit geometric computation



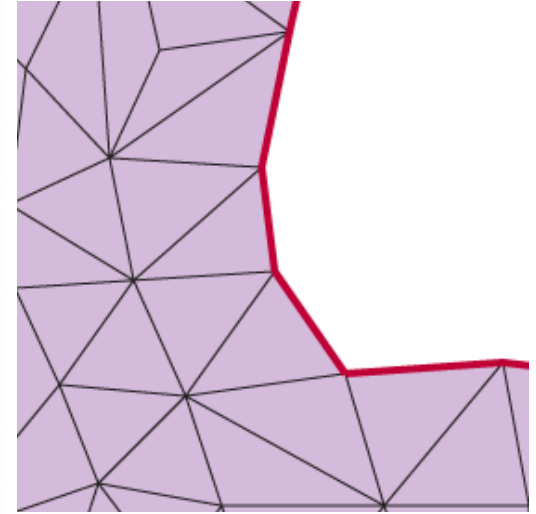
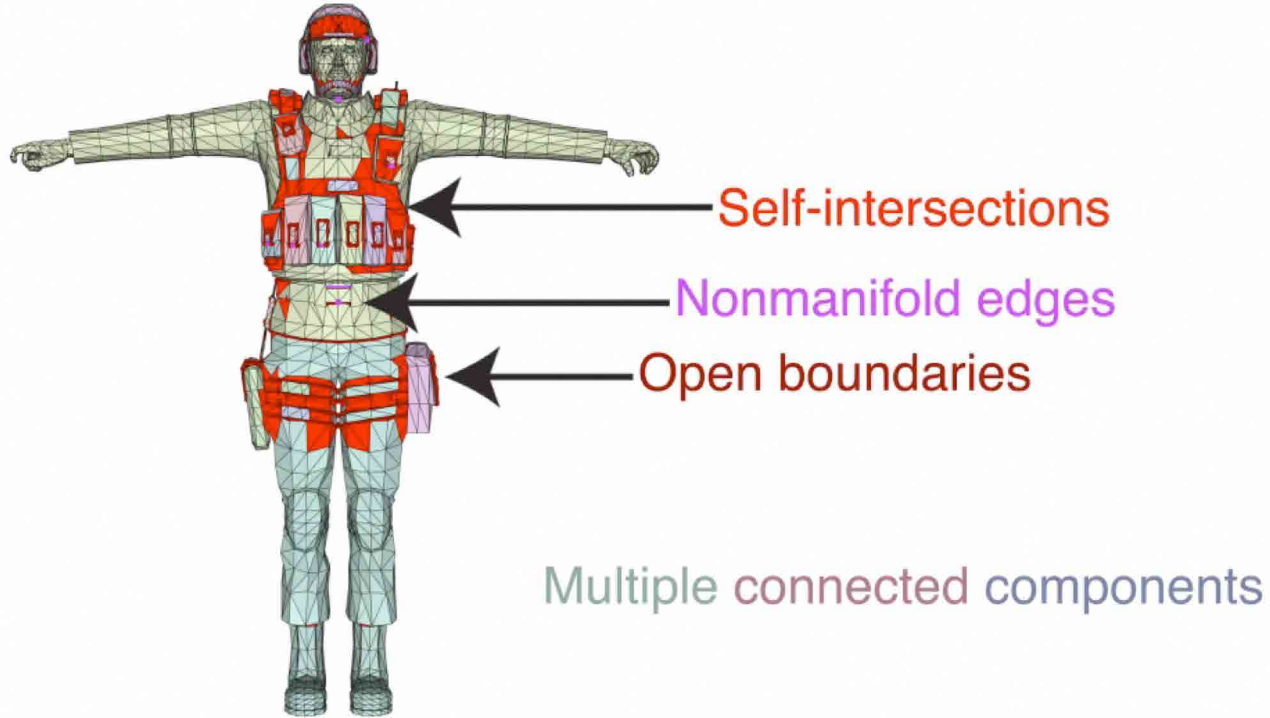
Self-intersections

Nonmanifold edges

Multiple connected components



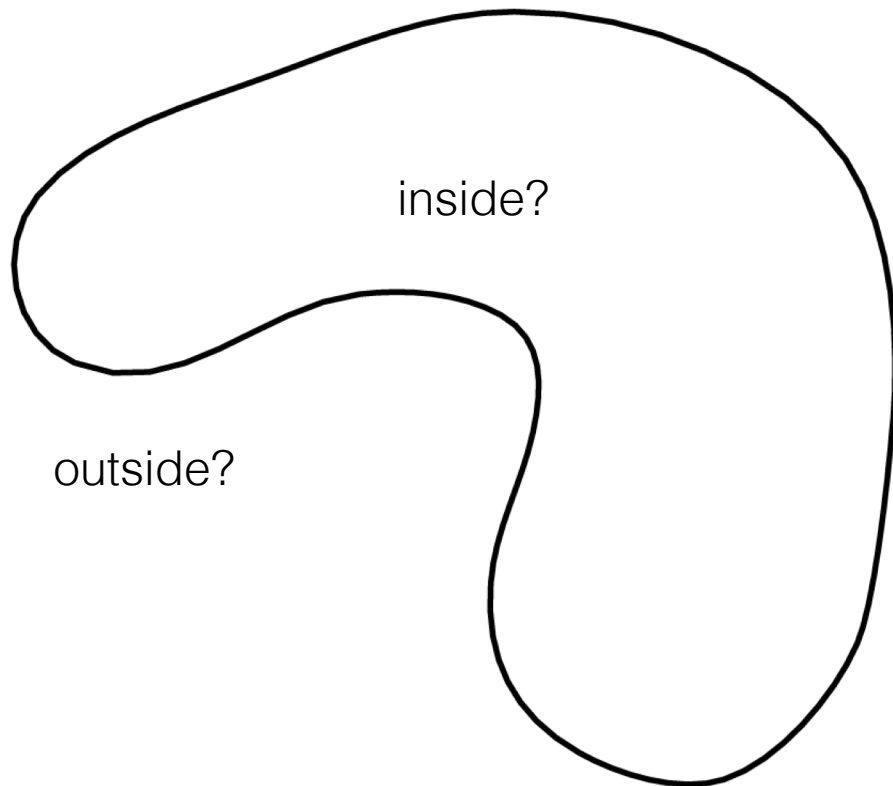
Good enough for visualization does not imply
good enough for admit geometric computation



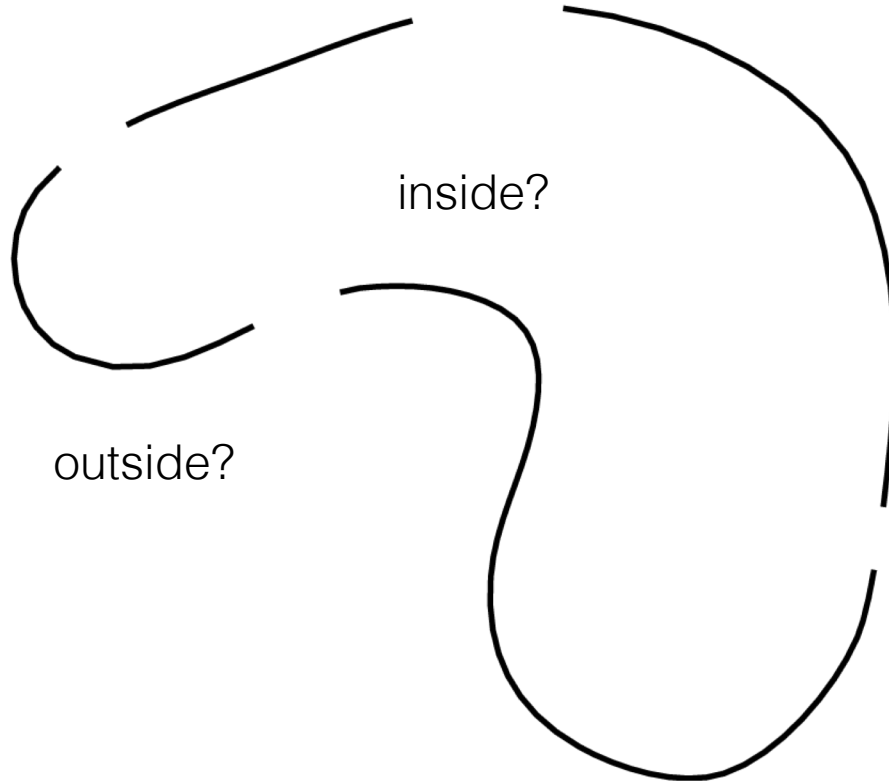
Fatal error

command exited with status 1

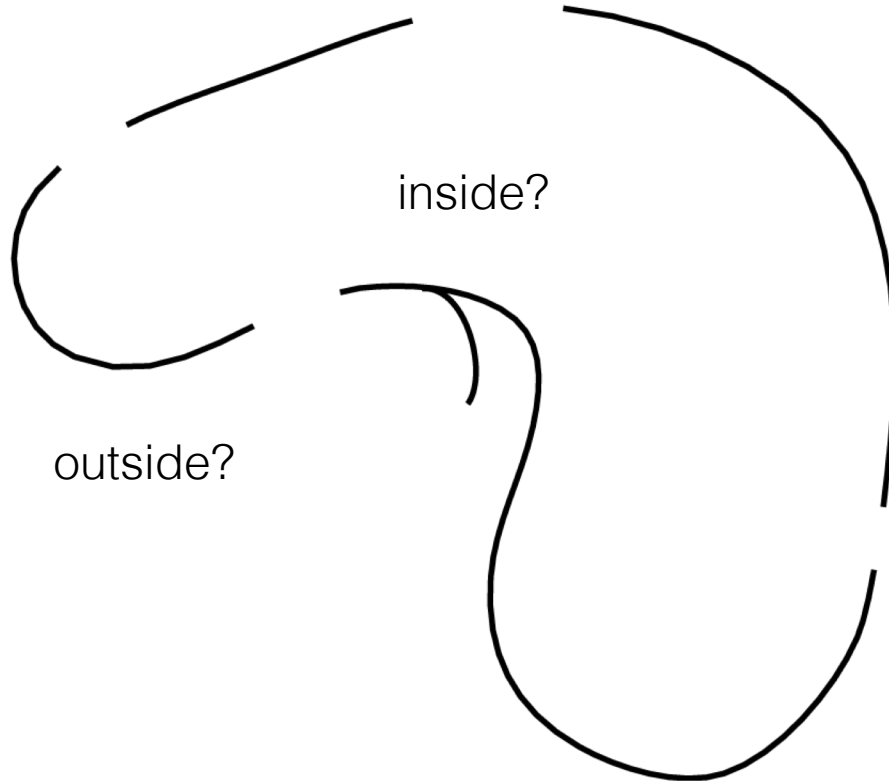
Adapt traditional algorithms and theory
to work even in the presence of messy data



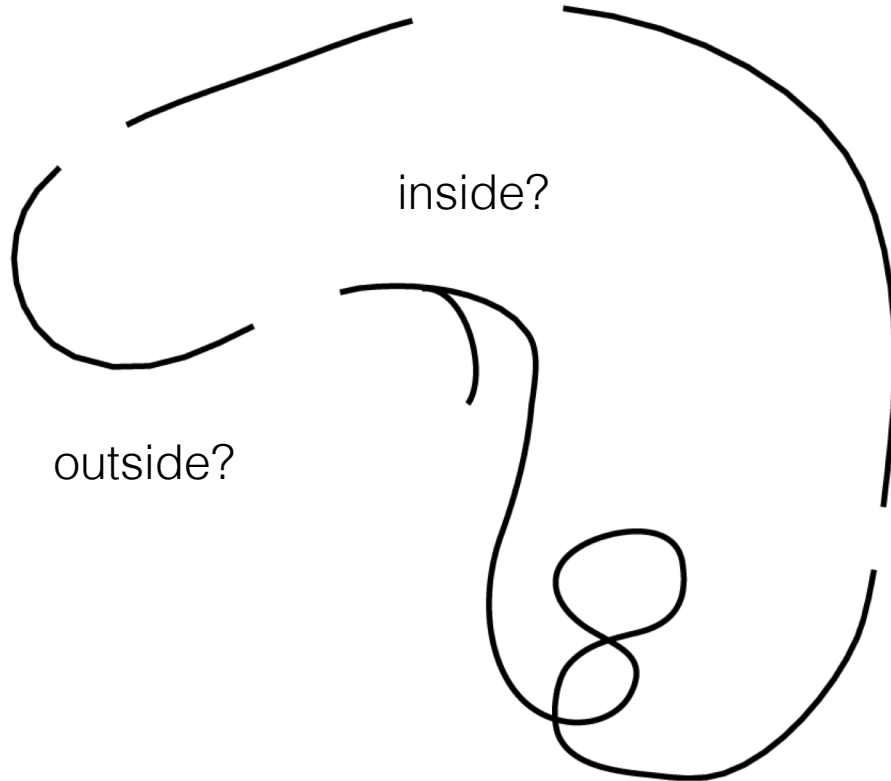
Adapt traditional algorithms and theory
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Adapt traditional algorithms and theory
to work even in the presence of messy data

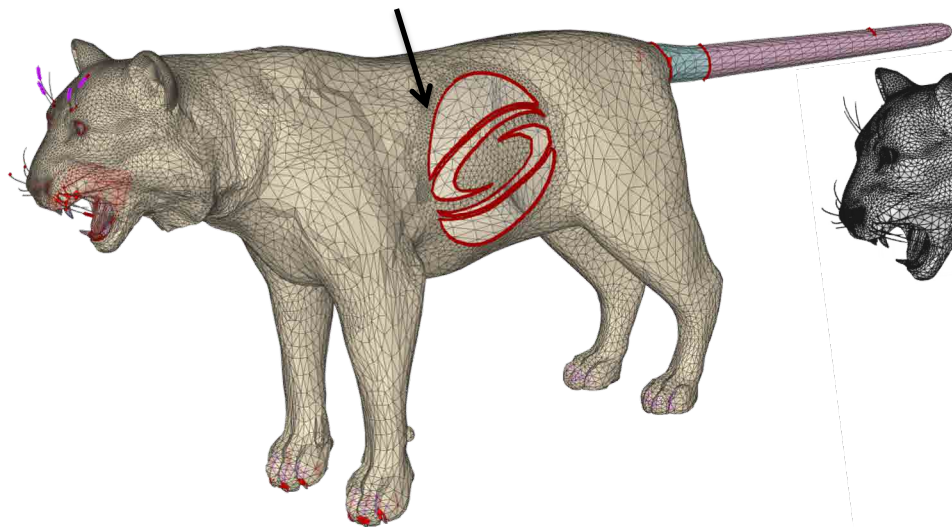


Adapt traditional algorithms and theory
to work even in the presence of messy data

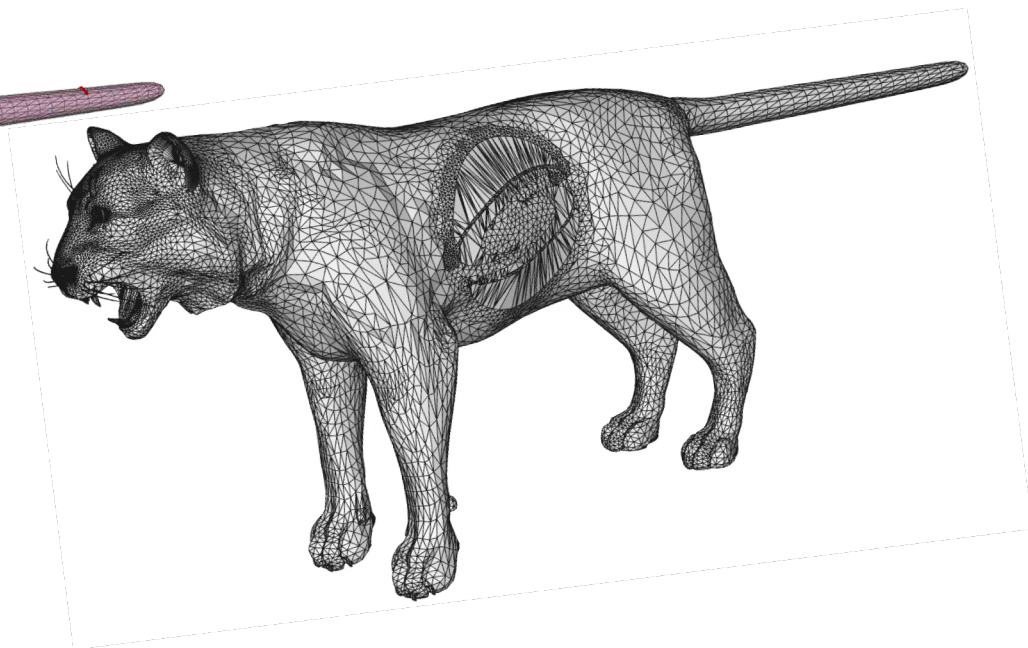


Determining insiderness is *fundamental*

open boundaries



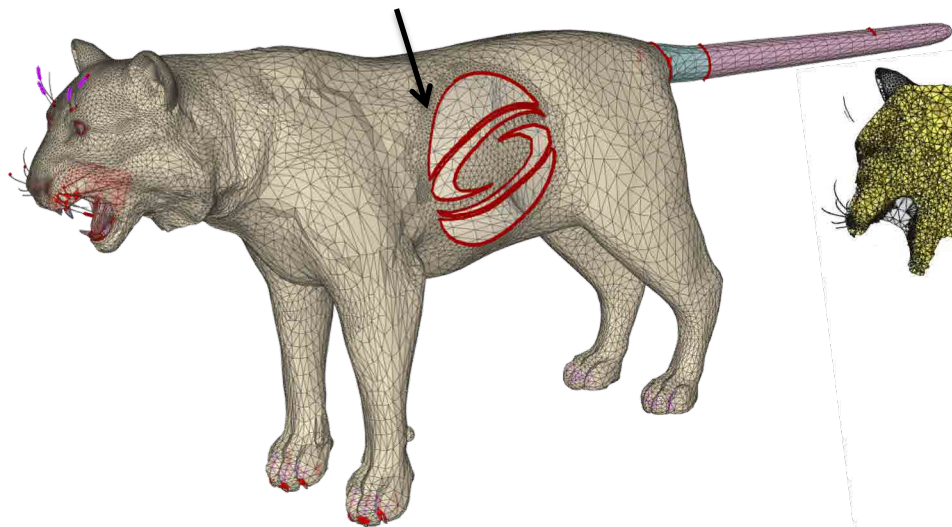
“messy” input mesh



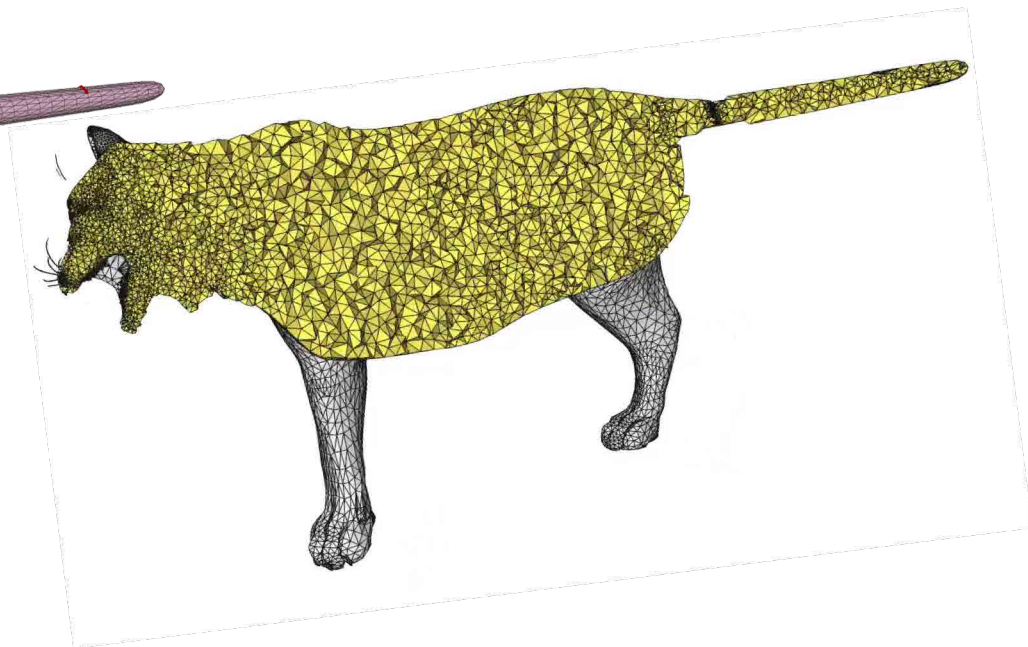
volume discretization

Determining insiderness is *fundamental*

open boundaries

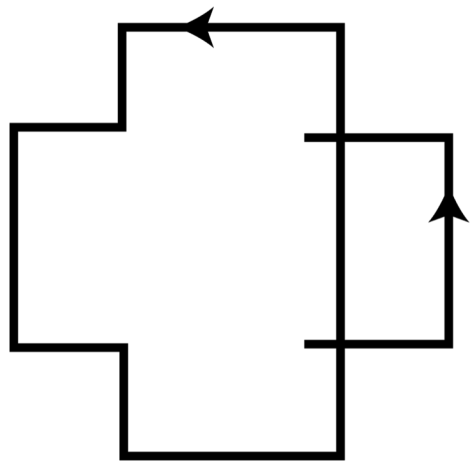


“messy” input mesh

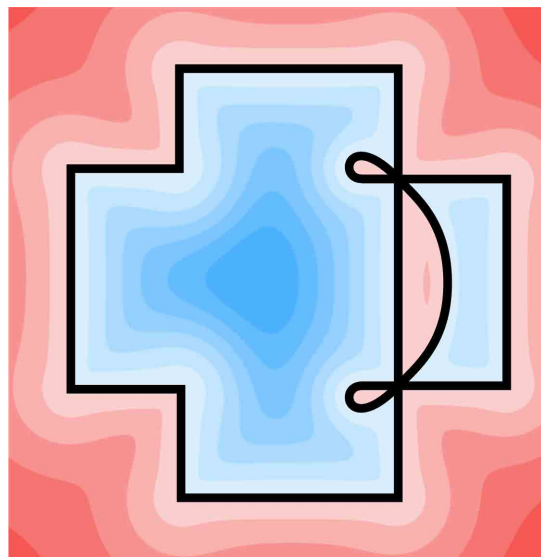


volume discretization

Previous solutions are unsatisfactory

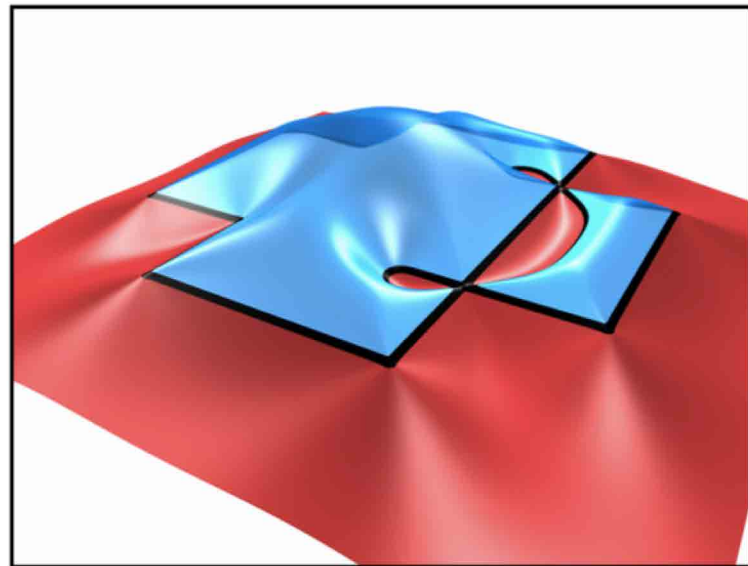


Input



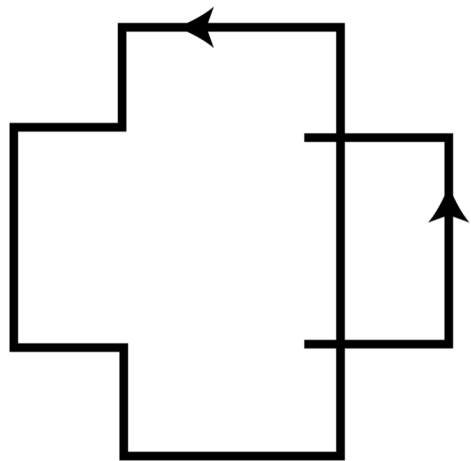
Interpolating function

[Shen et al. 2004]

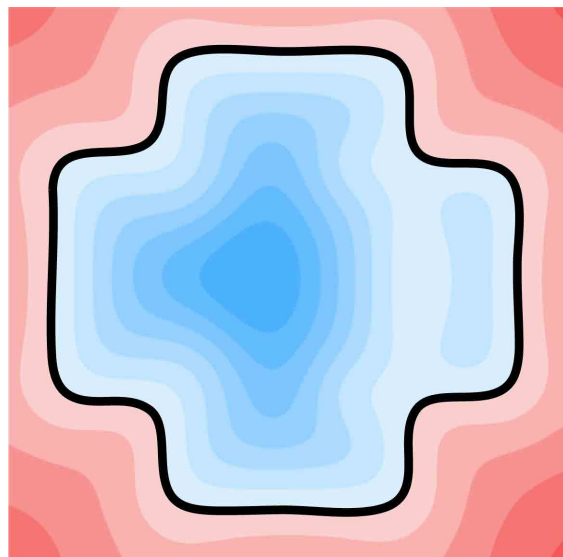


Height field visualization

Previous solutions are unsatisfactory

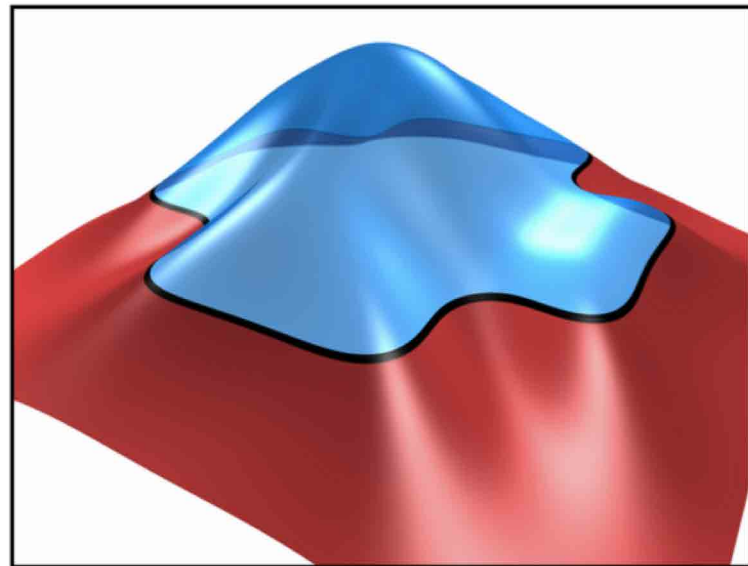


Input



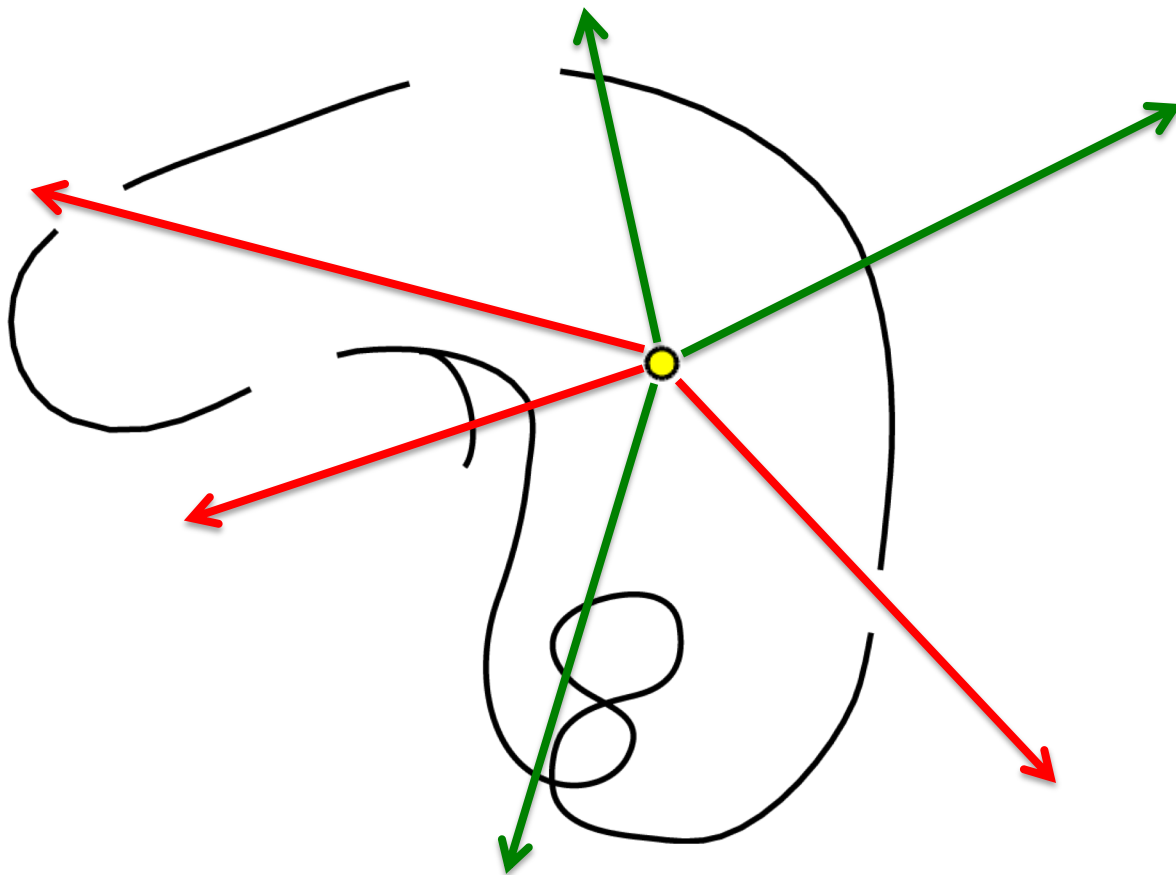
Approximating function

[Shen et al. 2004]



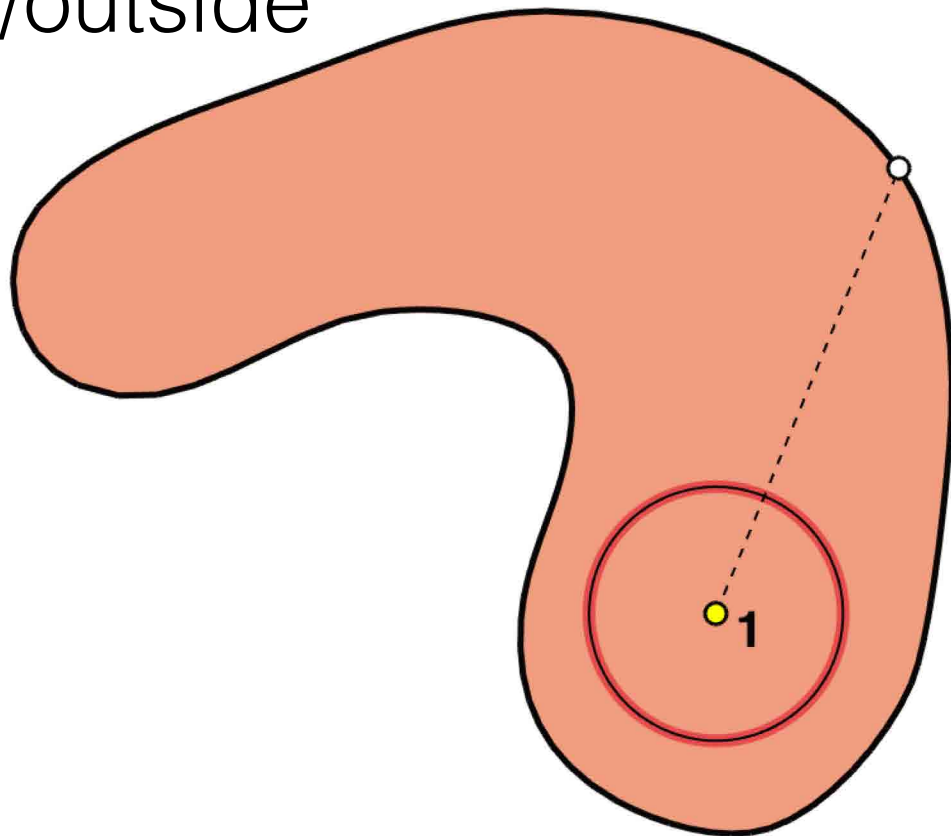
Height field visualization

Previous solutions are unsatisfactory



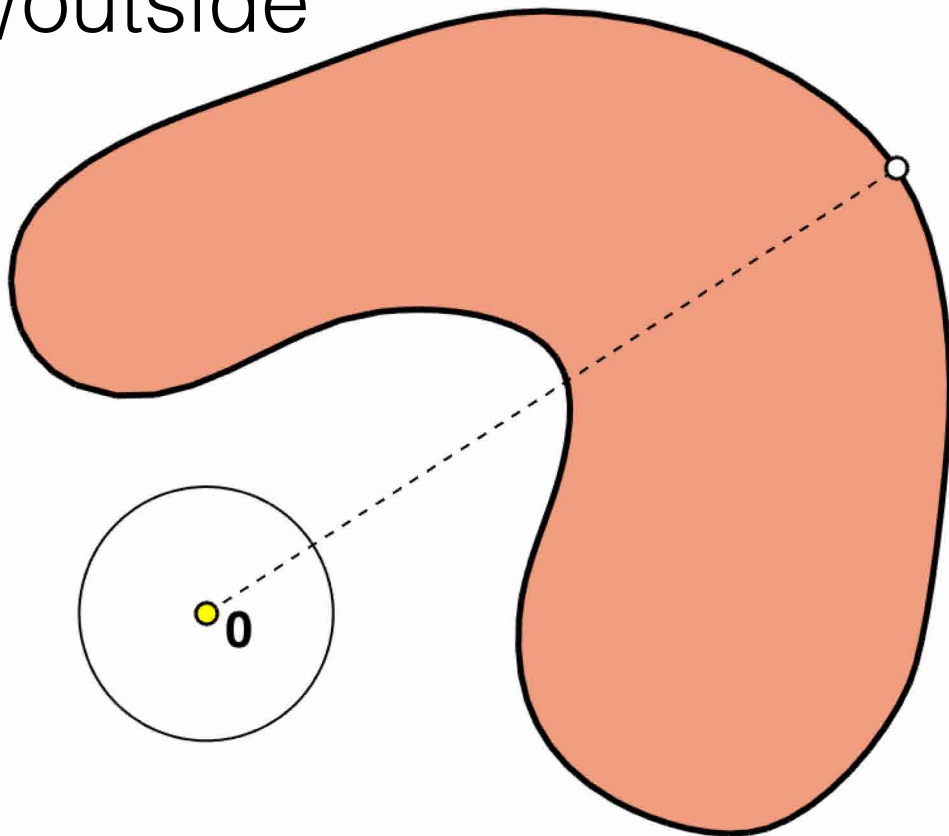
For “clean” shapes, classic winding number indicates inside/outside

$$w(\mathbf{p}) = \frac{1}{2\pi} \oint_C d\theta$$



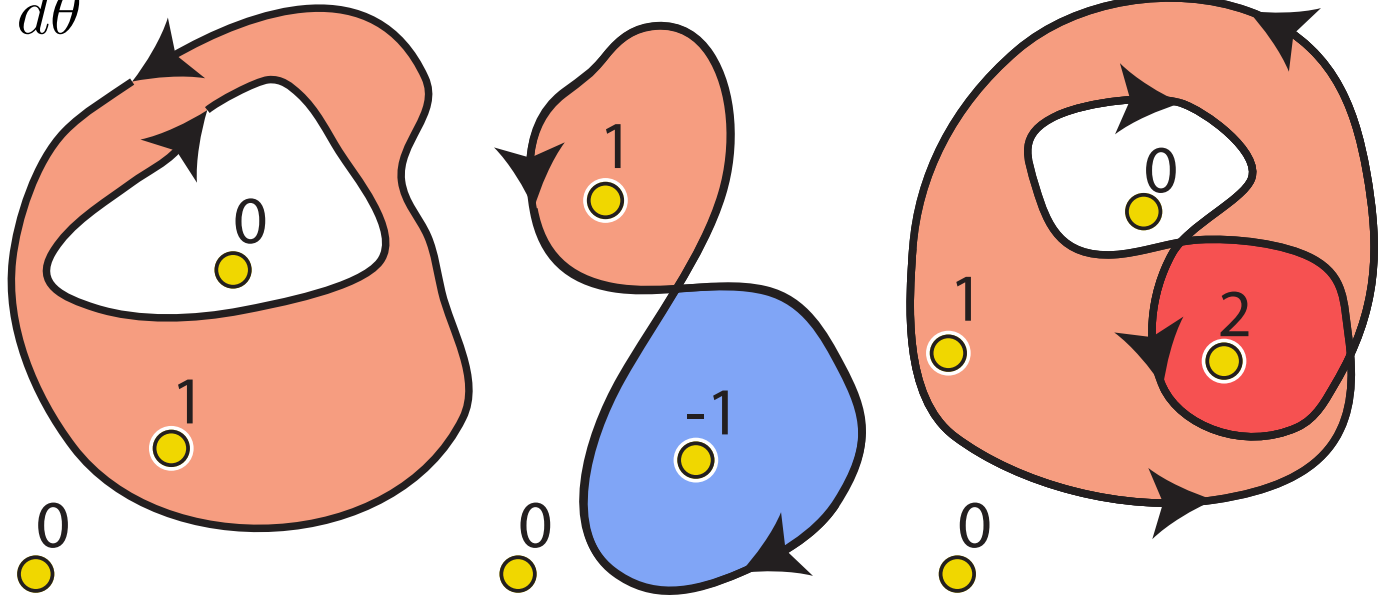
For “clean” shapes, classic winding number indicates inside/outside

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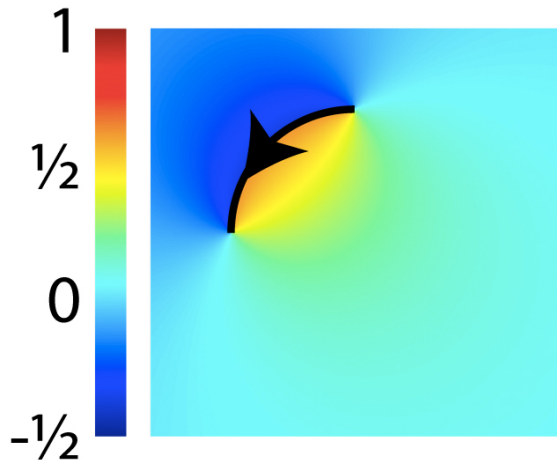
Classic winding number already handles a wide variety of shapes

$$w(\mathbf{p}) = \frac{1}{2\pi} \oint_C d\theta$$



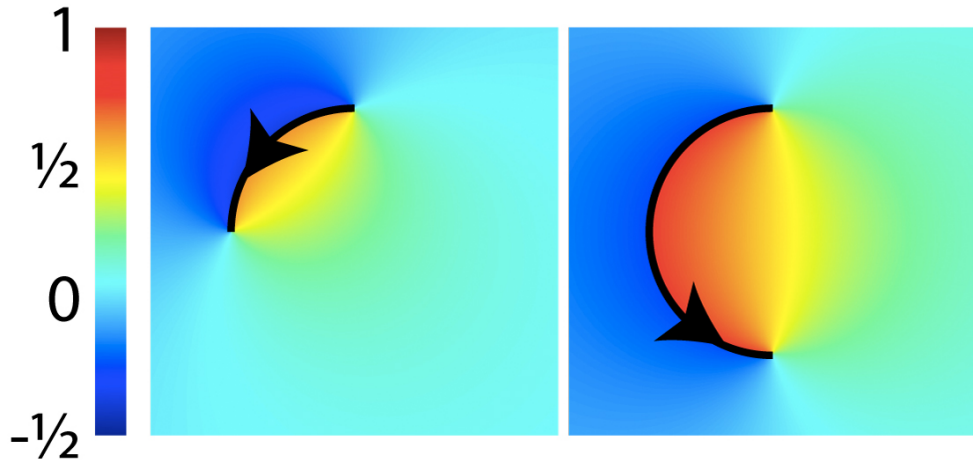
What happens if the shape is open?

$$w(\mathbf{p}) = \frac{1}{2\pi} \oint_C d\theta$$



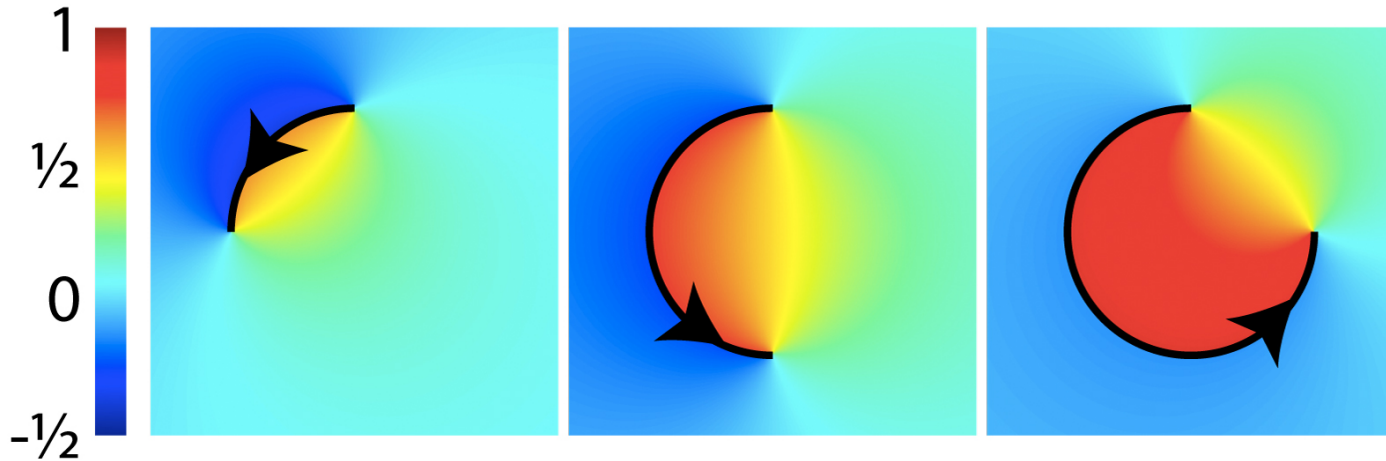
What happens if the shape is open?

$$w(\mathbf{p}) = \frac{1}{2\pi} \oint_C d\theta$$



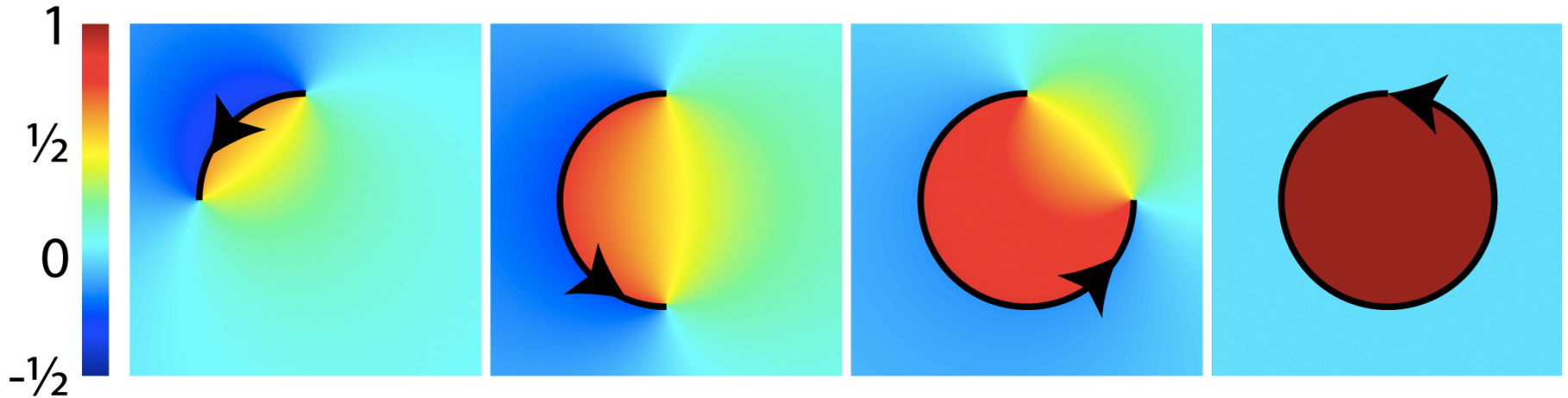
What happens if the shape is open?

$$w(\mathbf{p}) = \frac{1}{2\pi} \oint_C d\theta$$



What happens if the shape is open?

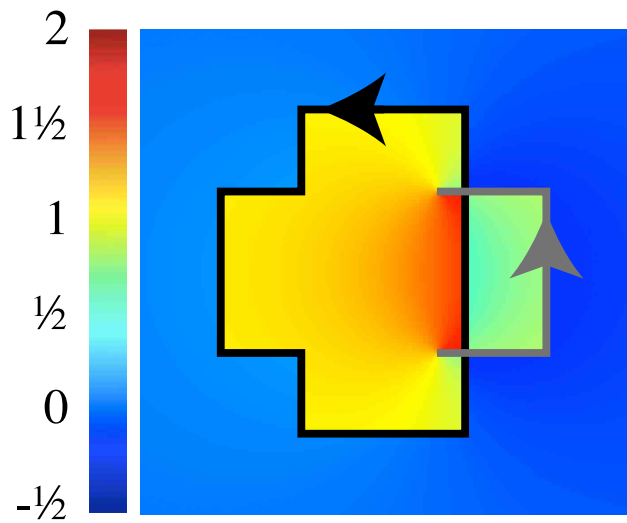
$$w(\mathbf{p}) = \frac{1}{2\pi} \oint_C d\theta$$



Gracefully tends toward perfect indicator

What if the shape is self-intersecting?

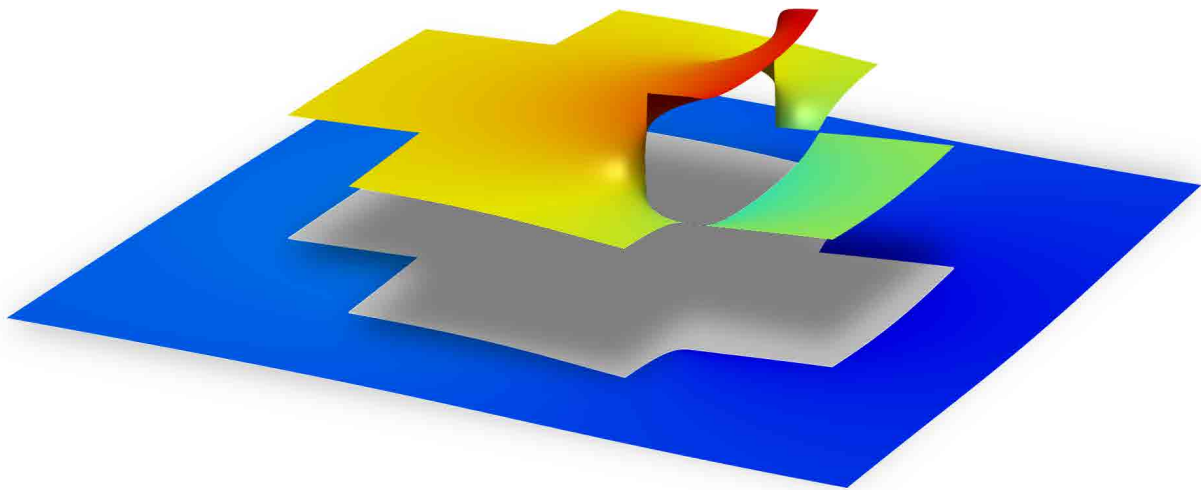
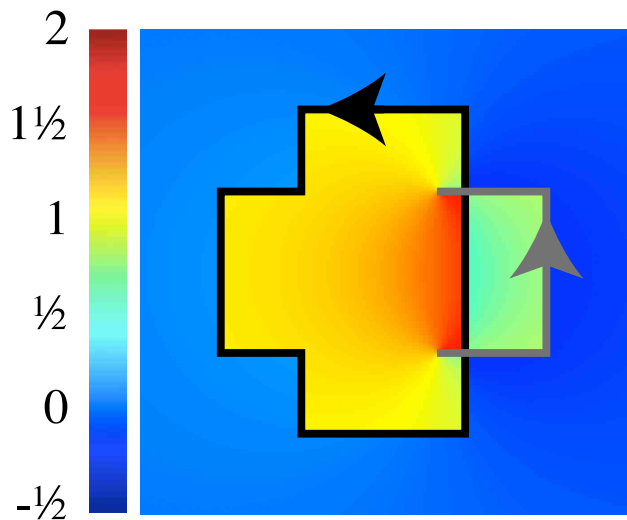
$$w(\mathbf{p}) = \frac{1}{2\pi} \oint_{\mathcal{C}} d\theta$$



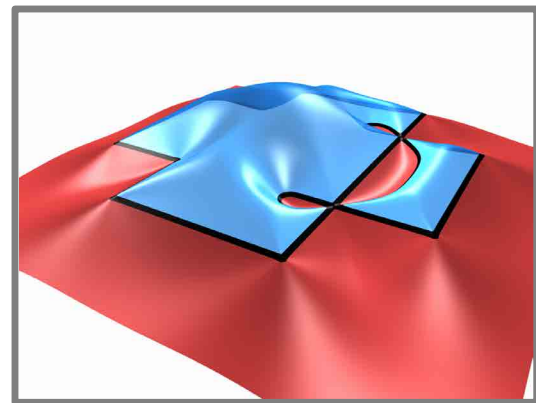
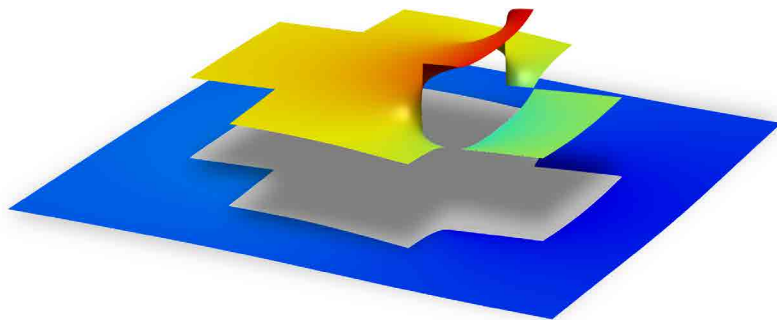
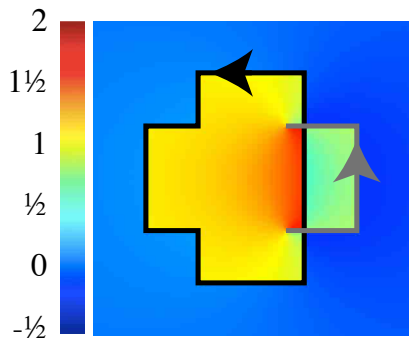
Winding number jumps across boundaries,
otherwise harmonic!

$$w(\mathbf{p}) = \frac{1}{2\pi} \oint_{\mathcal{C}} d\theta$$

$$\Delta w = 0$$



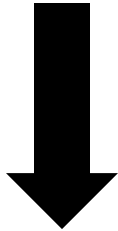
Other interpolating implicit functions may contain “surprise” oscillations



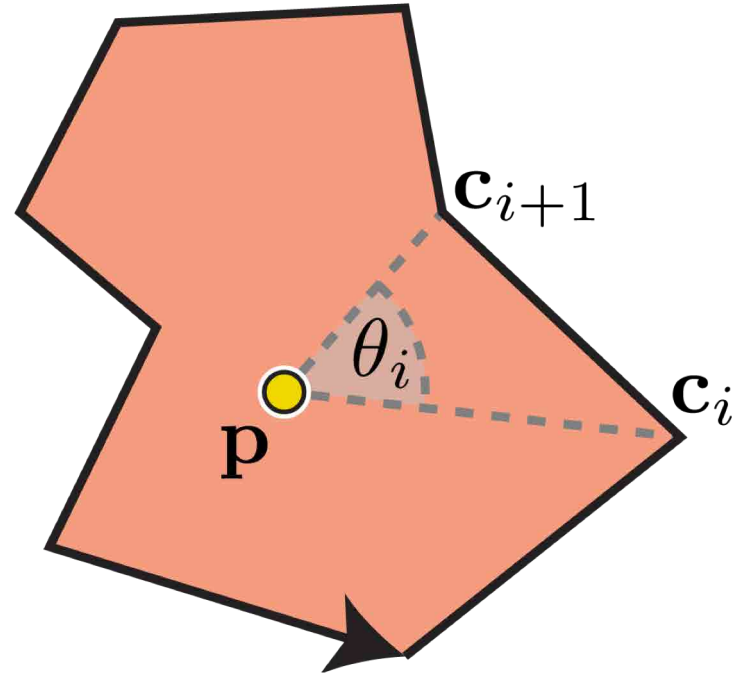
[Shen et al. 2004]

Discretization is simple and exact

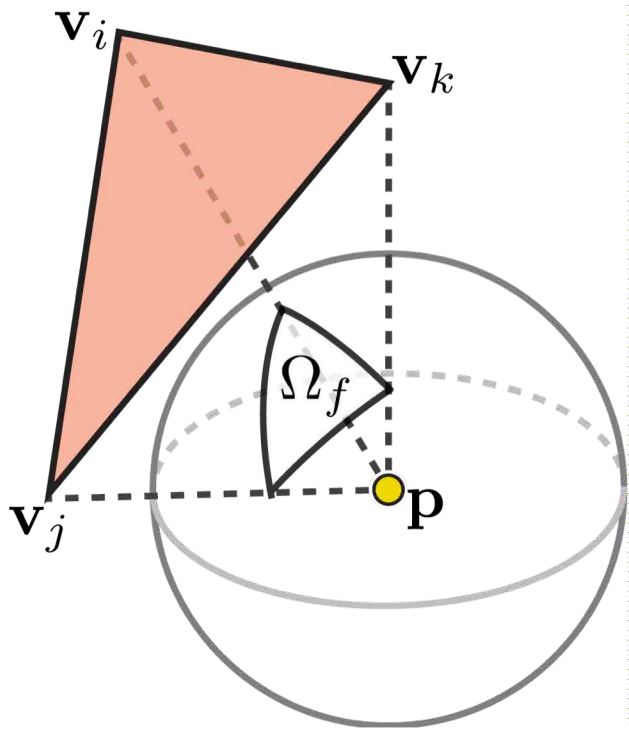
$$w(\mathbf{p}) = \frac{1}{2\pi} \oint_C d\theta$$



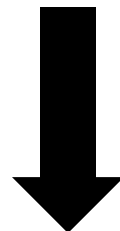
$$w(\mathbf{p}) = \frac{1}{2\pi} \sum_{i=1}^n \theta_i$$



Generalizes elegantly to 3D via solid angle

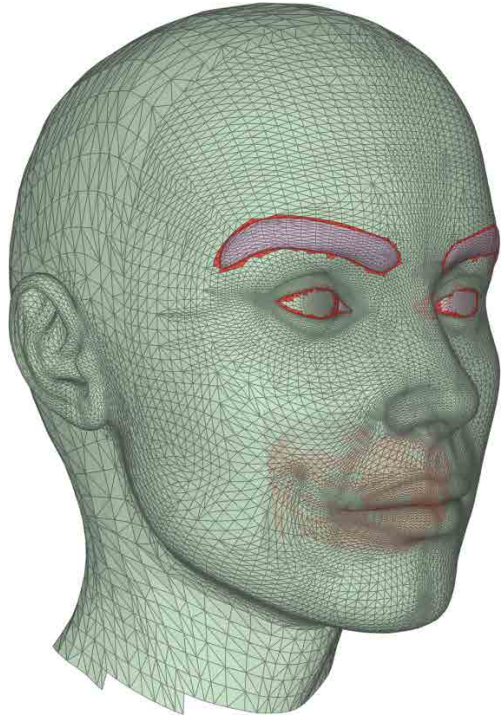


$$w(\mathbf{p}) = \frac{1}{4\pi} \iint_{\mathcal{S}} \sin(\phi) d\theta d\phi$$



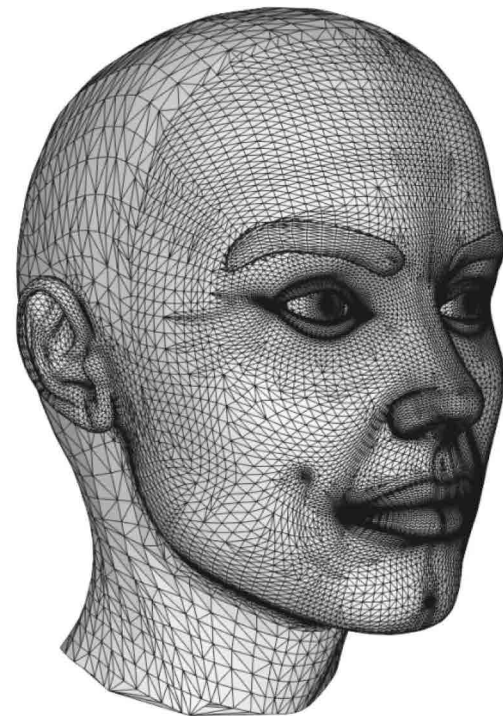
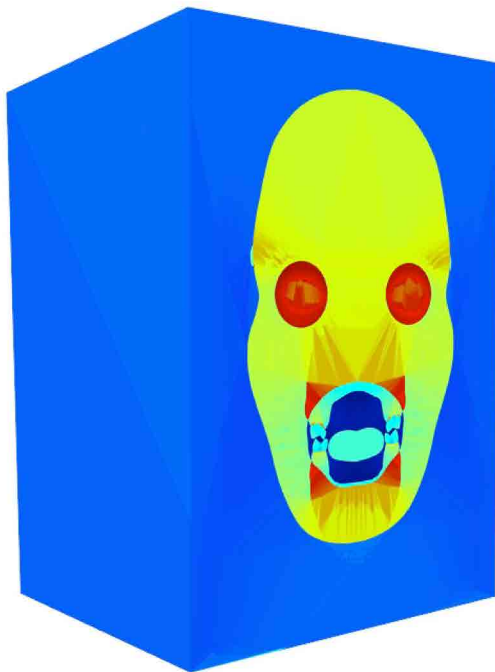
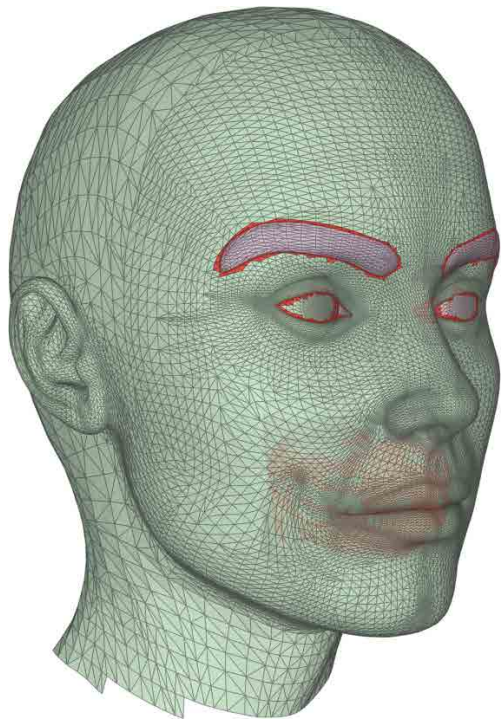
$$w(\mathbf{p}) = \frac{1}{4\pi} \sum_{f=1}^m \Omega_f$$

I adapt traditional algorithms and theory
to work even in the presence of messy data



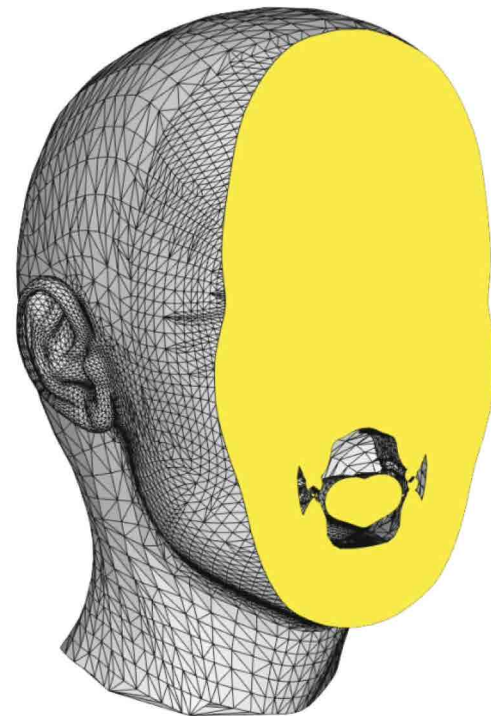
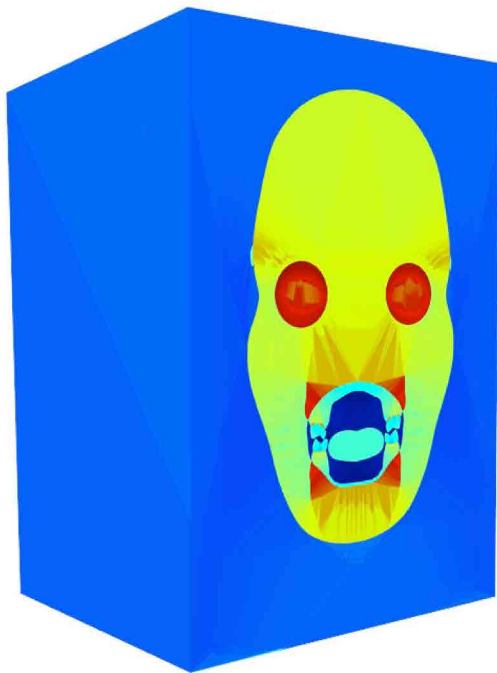
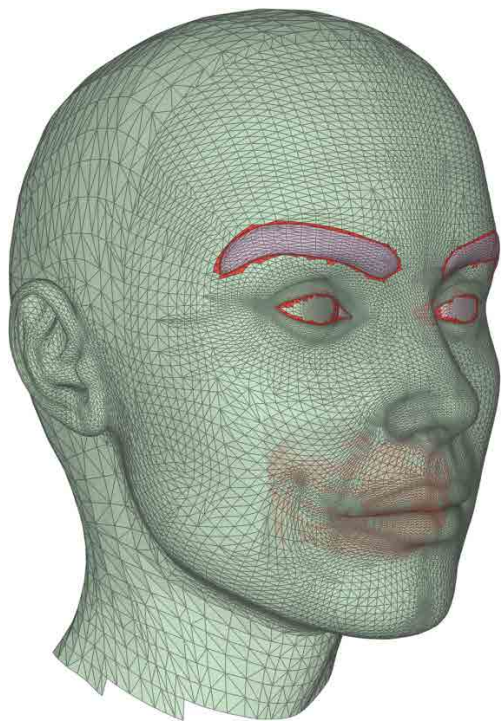
"Robust Inside-Outside Segmentation using Generalized Winding Numbers"
[J., Kavan, & Sorkine-Hornung 2013]

Enables volumetric discretization, in turn
enables better physics, rendering, ...



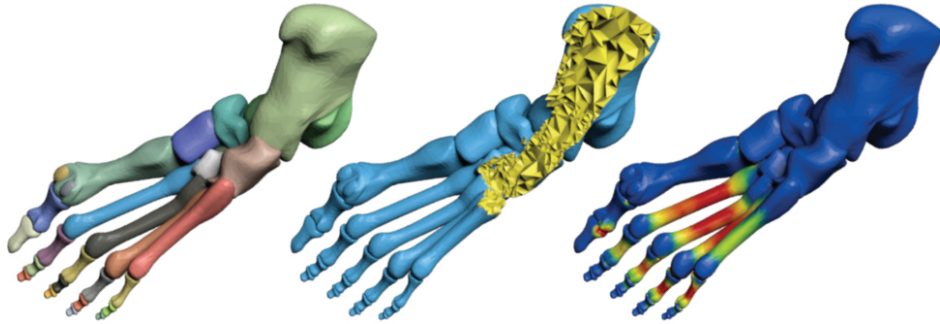
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"Robust Inside-Outside Segmentation using Generalized Winding Numbers"
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Robust inside/outside enables high-level processing



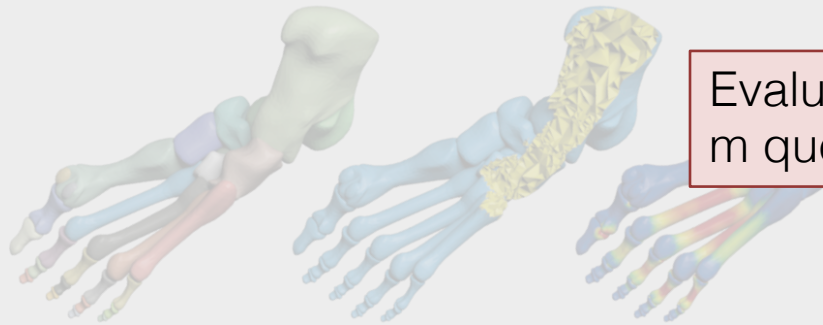
Structural analysis



Functionality-driven deformation

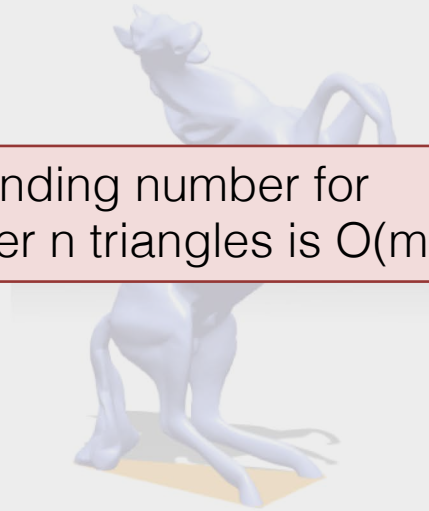


Robust inside/outside enables high-level processing

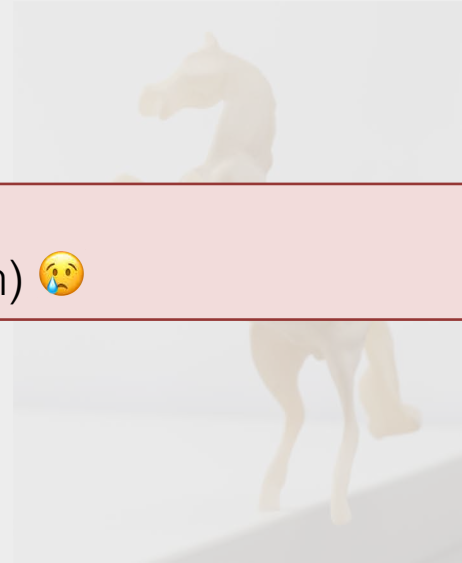


Structural analysis

Evaluating winding number for m queries over n triangles is $O(mn)$ 😞

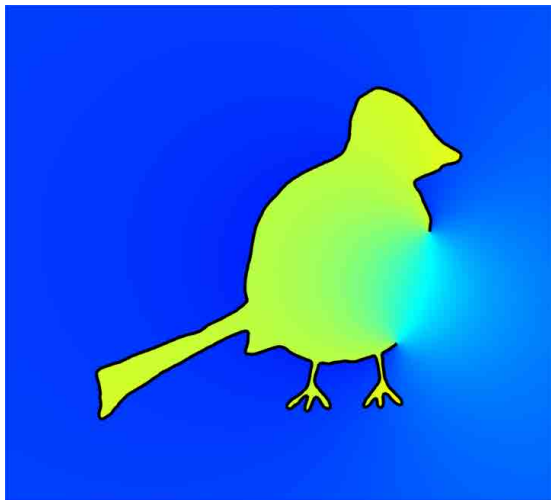


Functionality-driven deformation



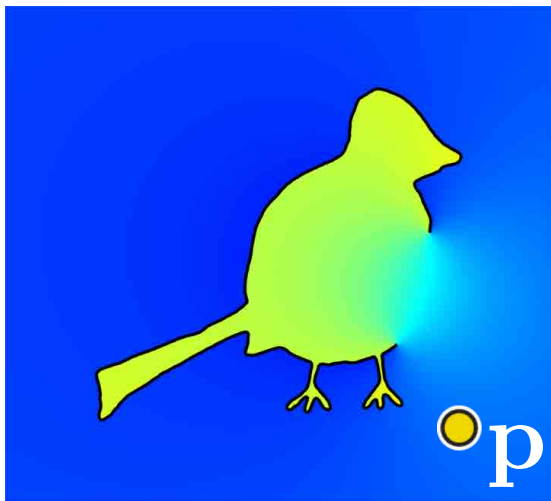
Exploit interesting fact for speedup

c



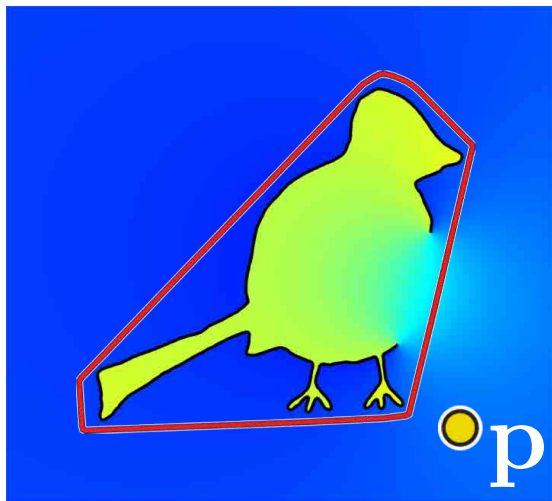
Exploit interesting fact for speedup

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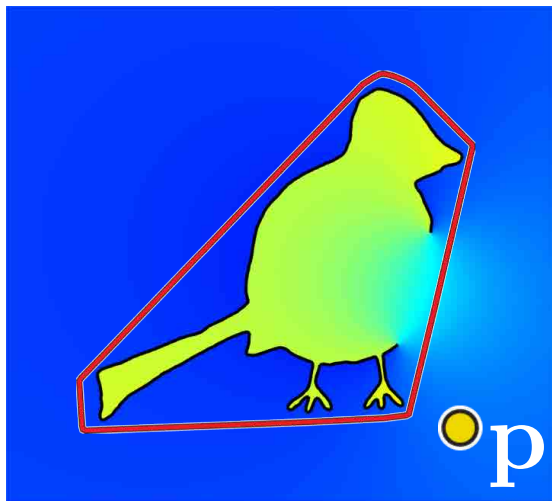


Exploit interesting fact for speedup

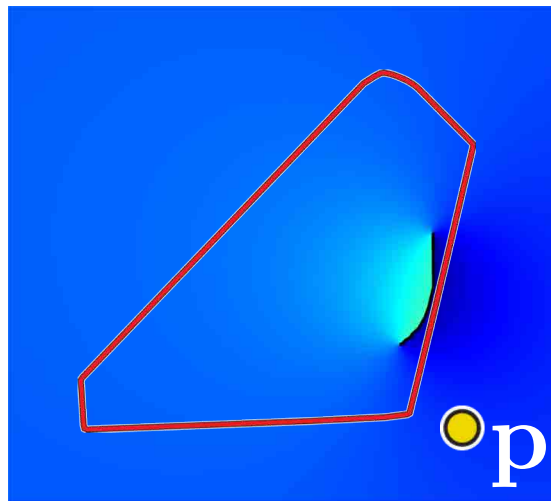
c



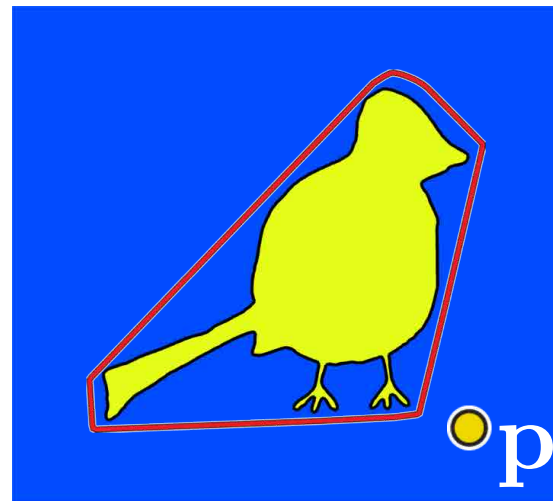
Exploit interesting fact for speedup

 c 

+

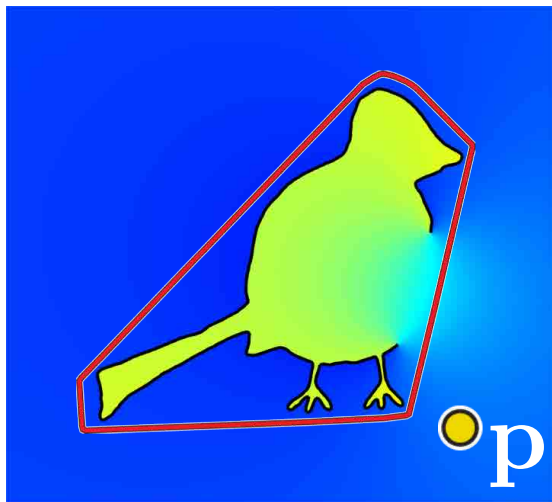
 \bar{c} 

=

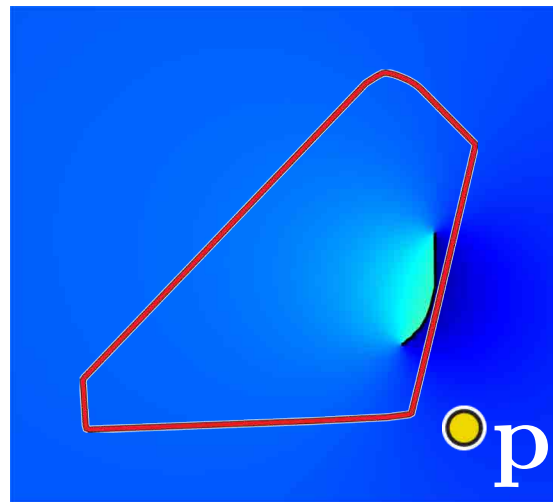
 $c \cup \bar{c}$ 

$$w_{c \cup \bar{c}}(\mathbf{p}) = 0$$

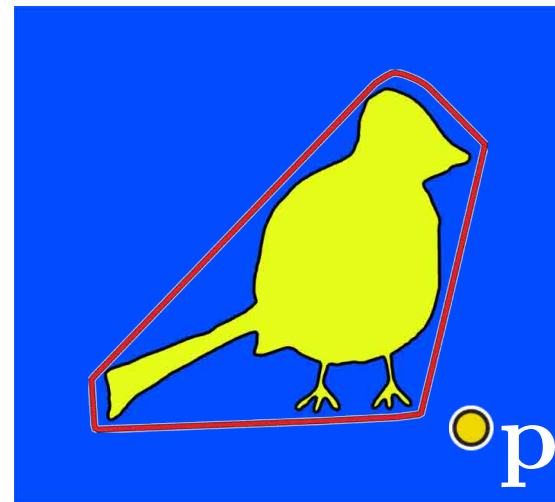
Exploit interesting fact for speedup

 c 

+

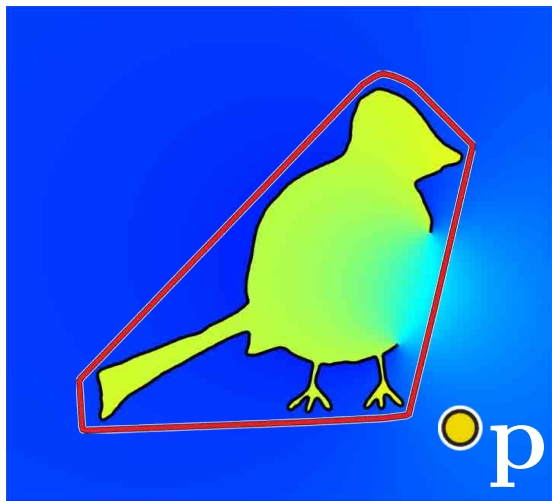
 \bar{c} 

=

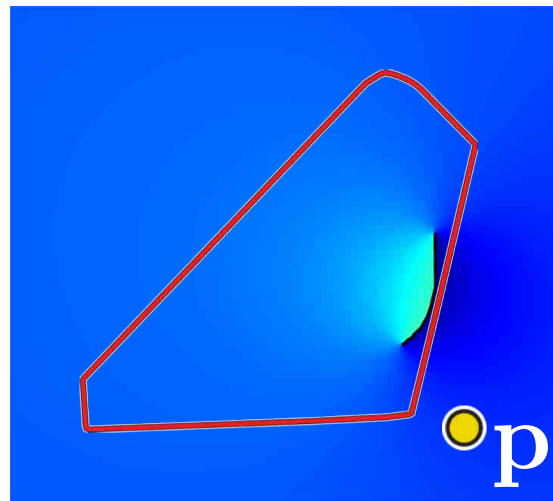
 $c \cup \bar{c}$ 

$$w_c(\mathbf{p}) + w_{\bar{c}}(\mathbf{p}) = w_{c \cup \bar{c}}(\mathbf{p}) = 0$$

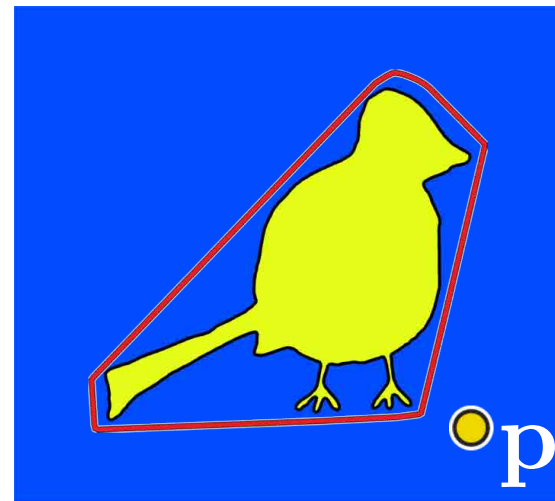
Exploit interesting fact for speedup

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+

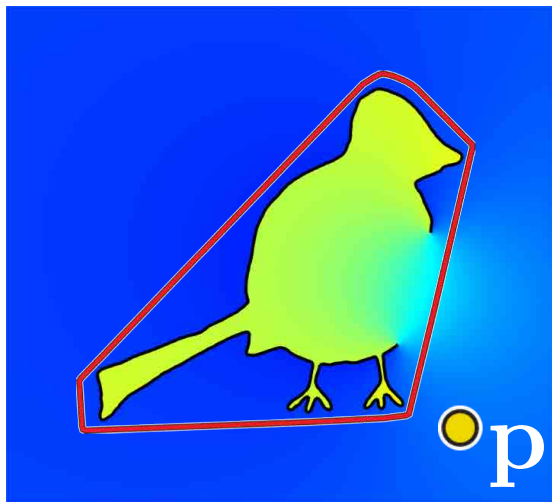
 \bar{c} 

=

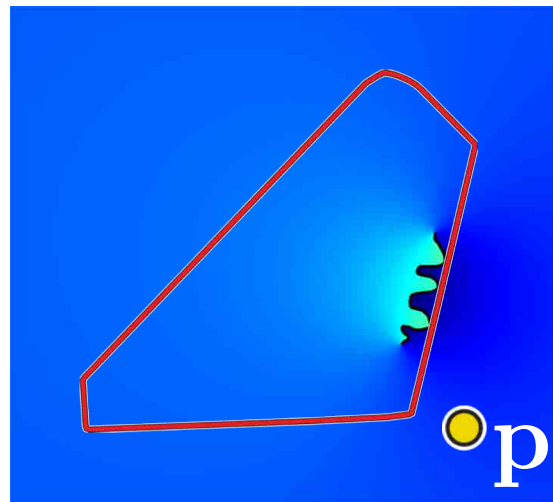
 $c \cup \bar{c}$ 

$$w_c(\mathbf{p}) = -w_{\bar{c}}(\mathbf{p})$$

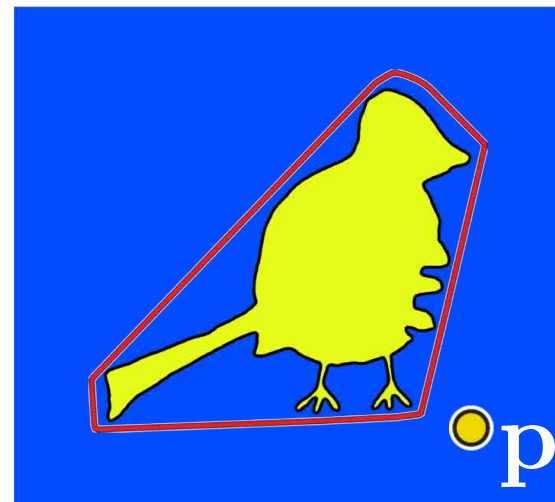
Exploit interesting fact for speedup

 c 

+

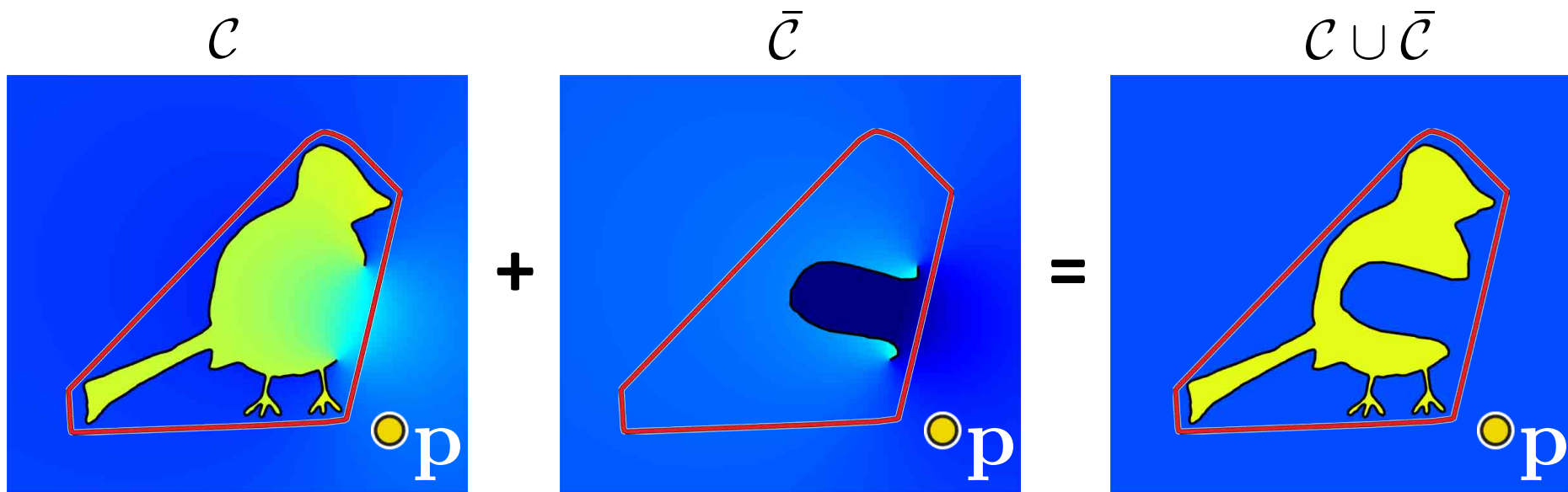
 \bar{c} 

=

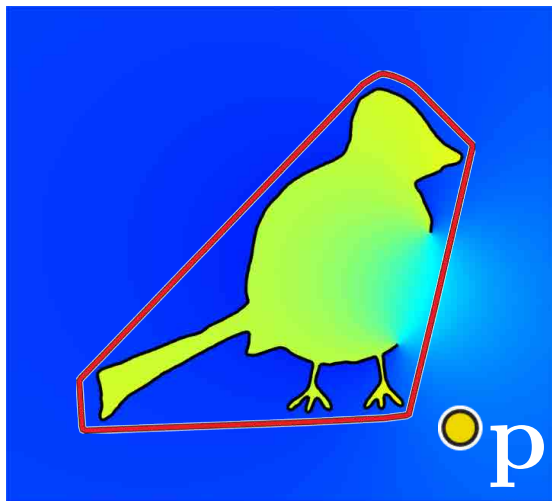
 $c \cup \bar{c}$ 

$$w_c(\mathbf{p}) = -w_{\bar{c}}(\mathbf{p})$$

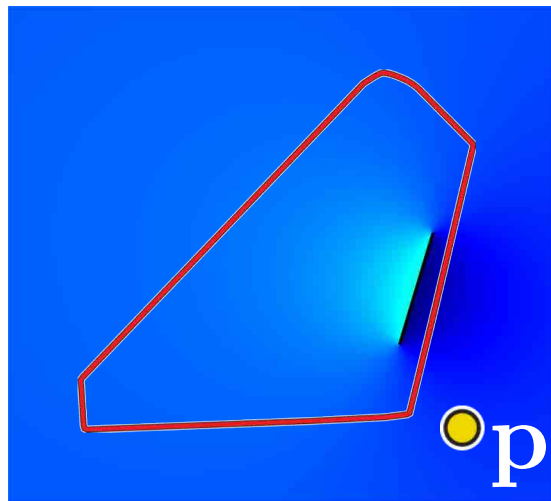
Exploit interesting fact for speedup



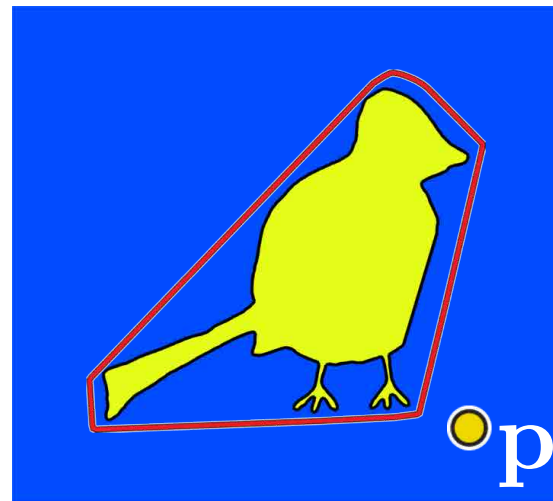
Exploit interesting fact for speedup

 c 

+

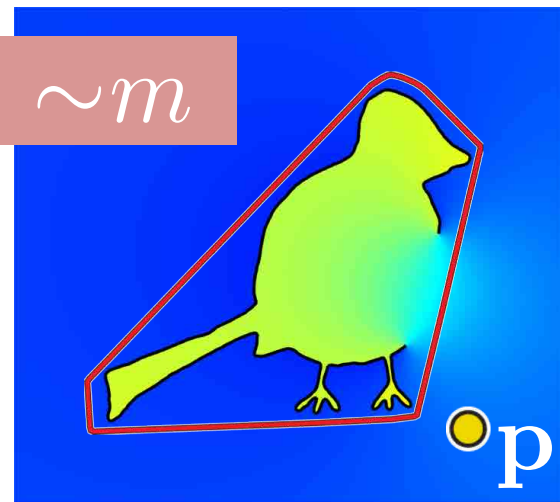
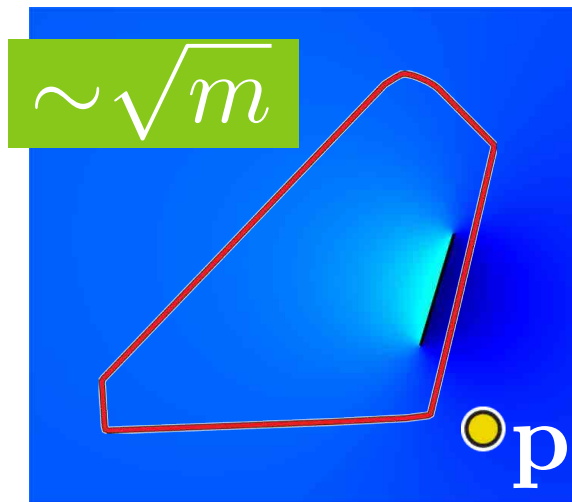
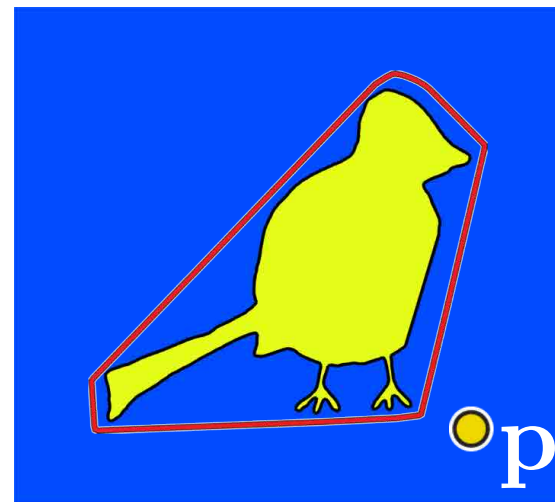
 \bar{c} 

=

 $c \cup \bar{c}$ 

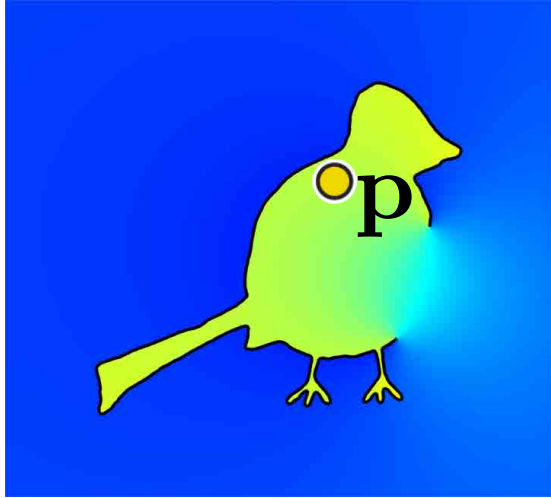
$$w_c(\mathbf{p}) = -w_{\bar{c}}(\mathbf{p})$$

Exploit interesting fact for speedup

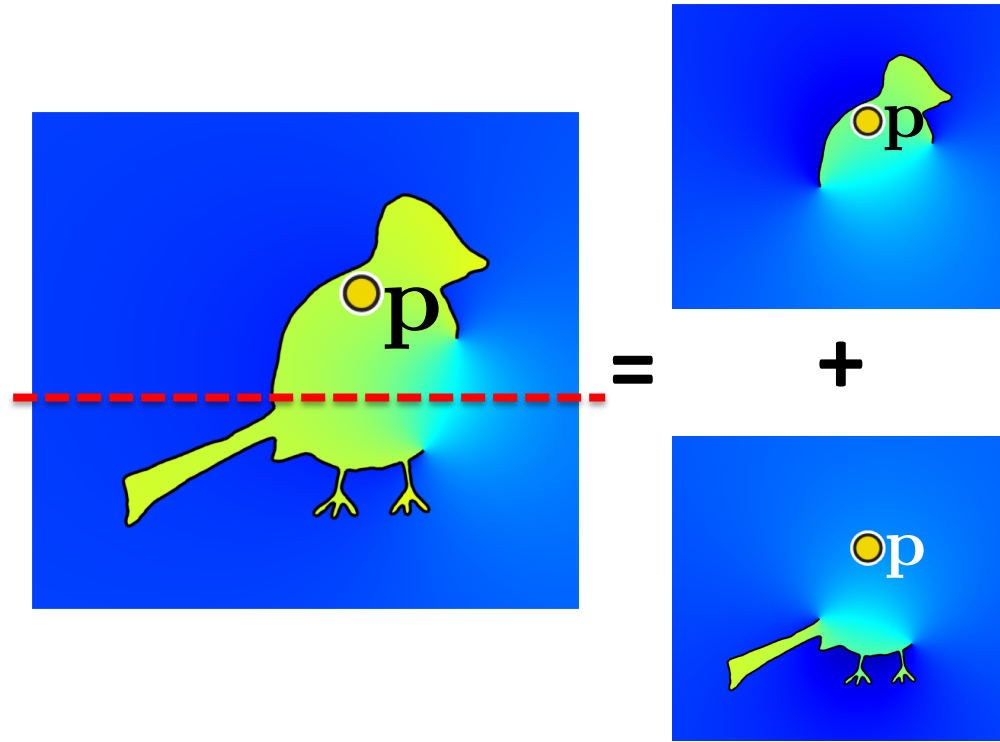
 c  $+$ \bar{c}  $=$ $c \cup \bar{c}$ 

$$w_c(\mathbf{p}) = -w_{\bar{c}}(\mathbf{p})$$

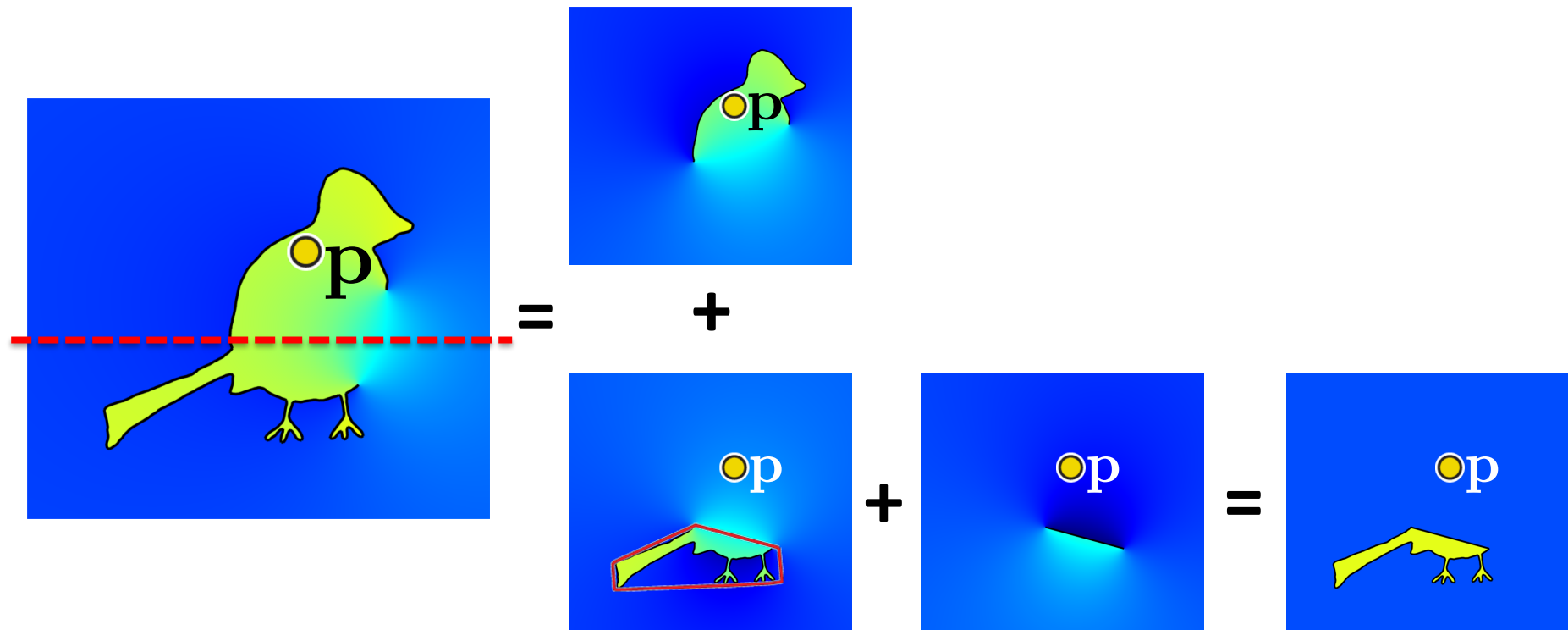
Divide and conquer!



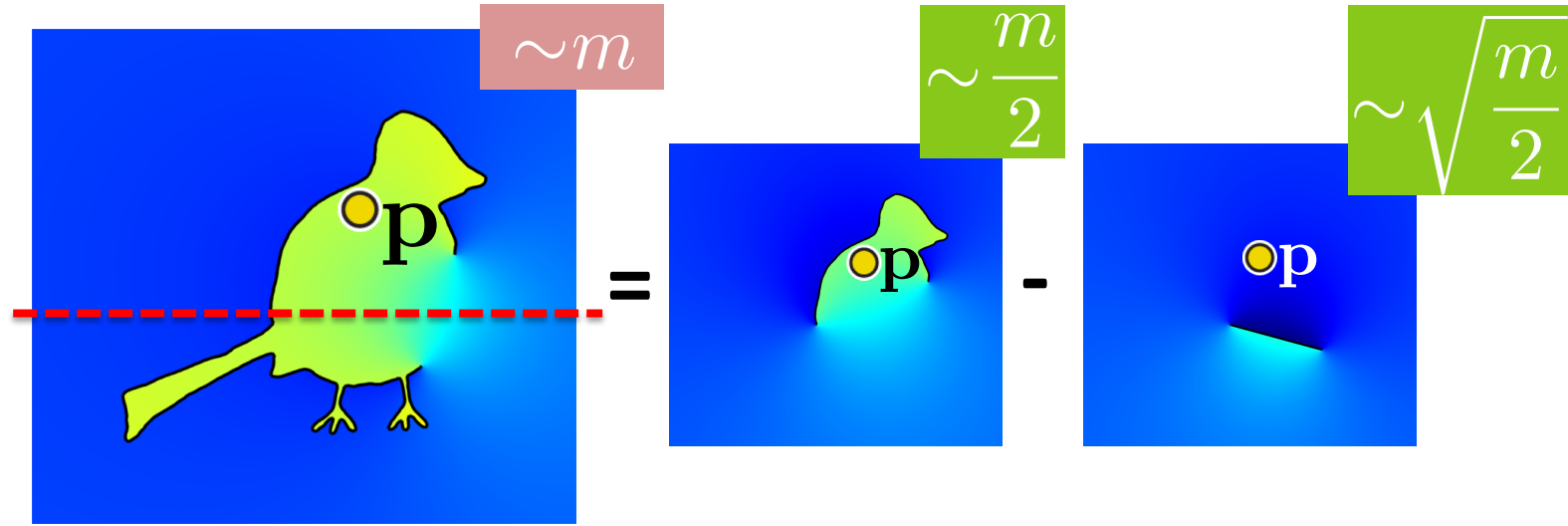
Divide and conquer!



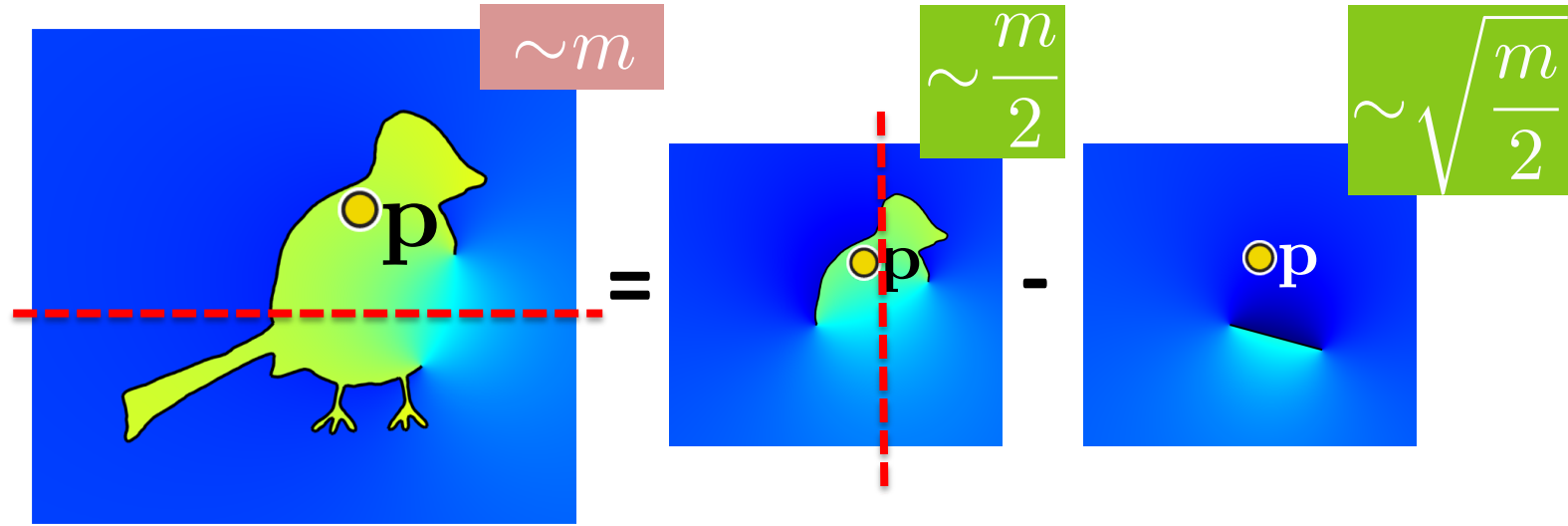
Divide and conquer!



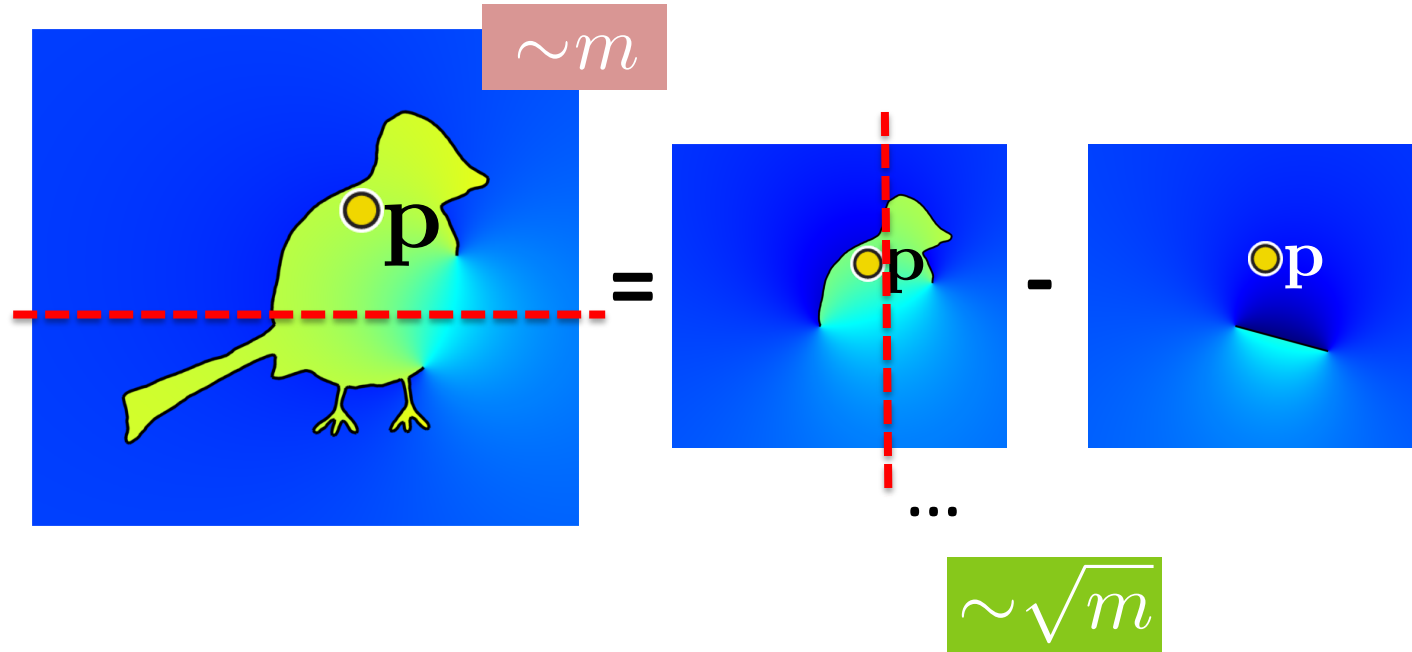
Divide and conquer!



Divide and conquer!

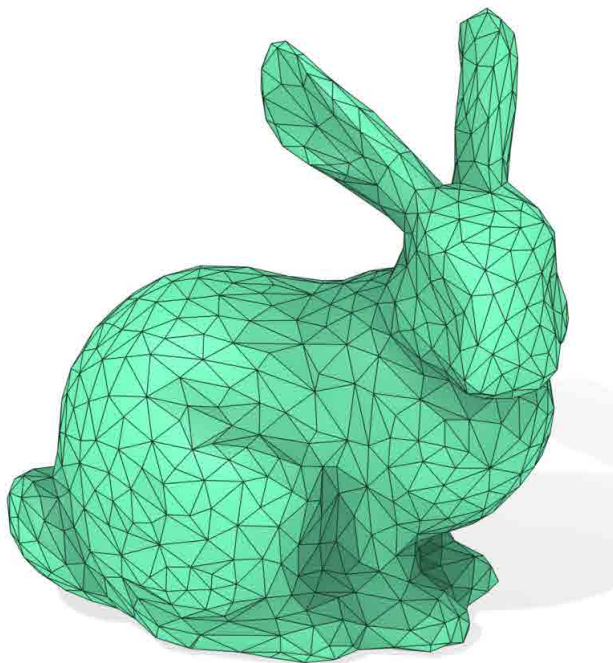


Divide and conquer!



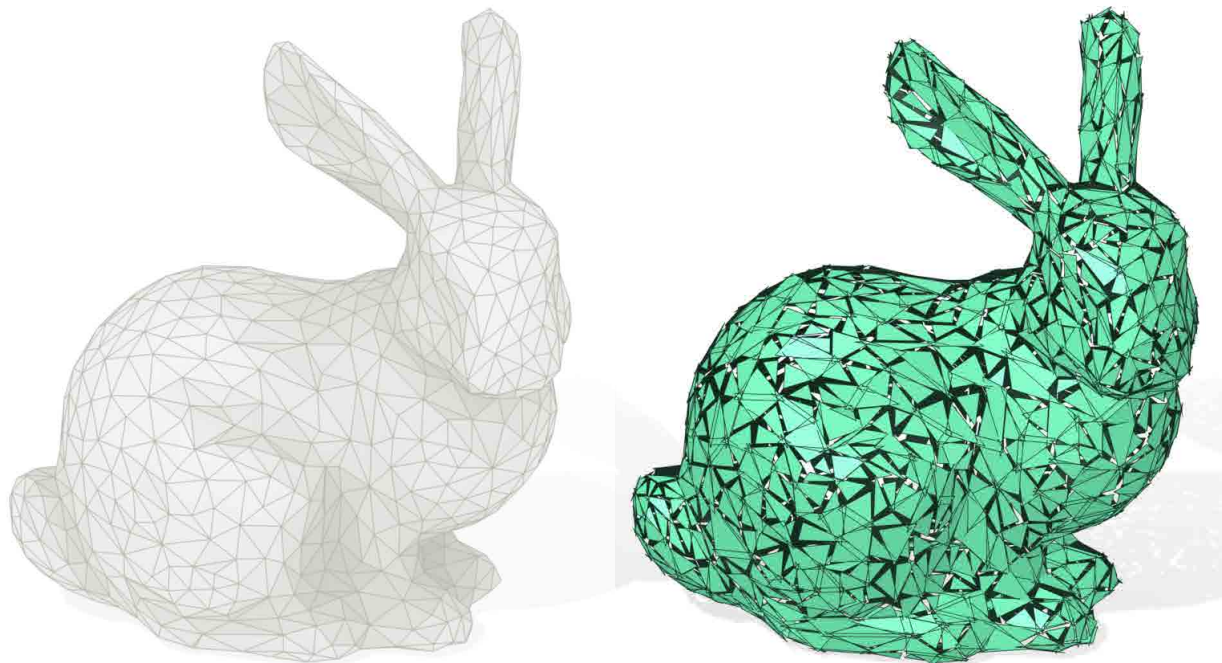
Trouble in paradise!

Trick depends on size of boundary being \ll size of mesh



For the imaginary “clean”
watertight mesh this is true

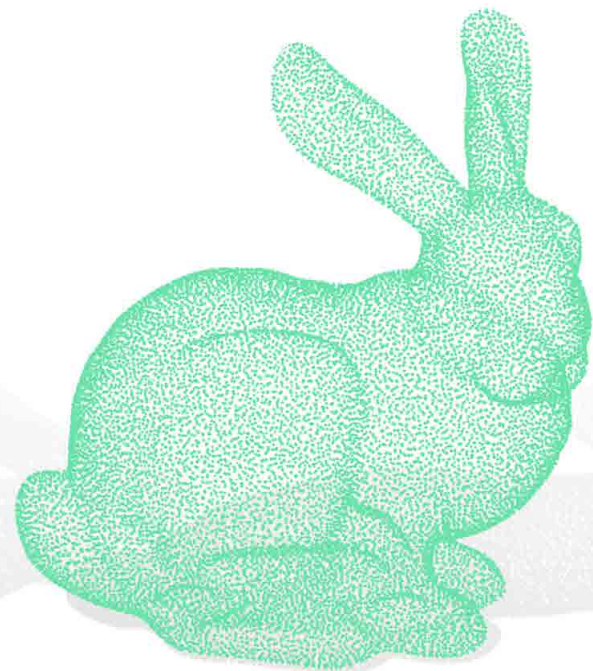
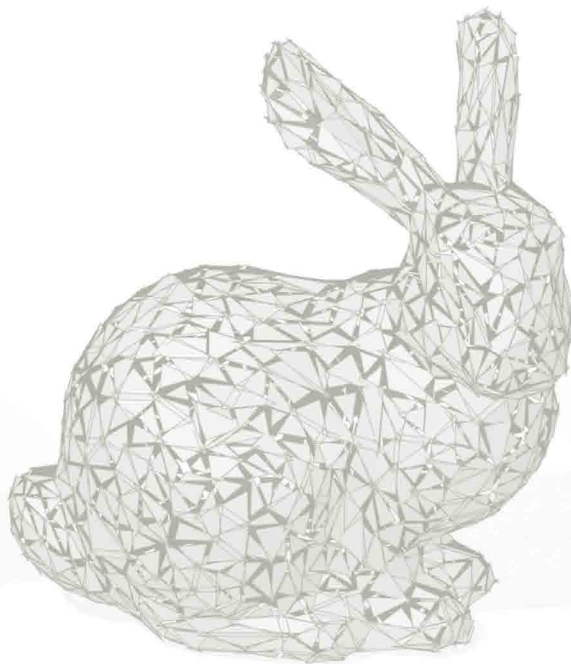
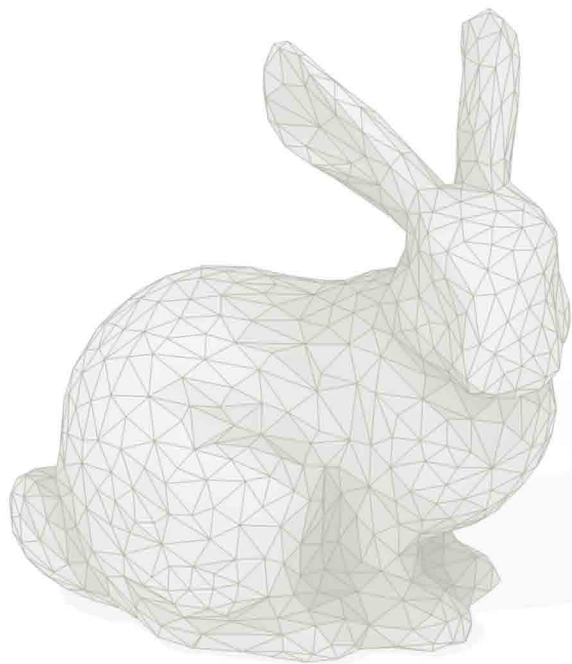
Trouble in paradise!



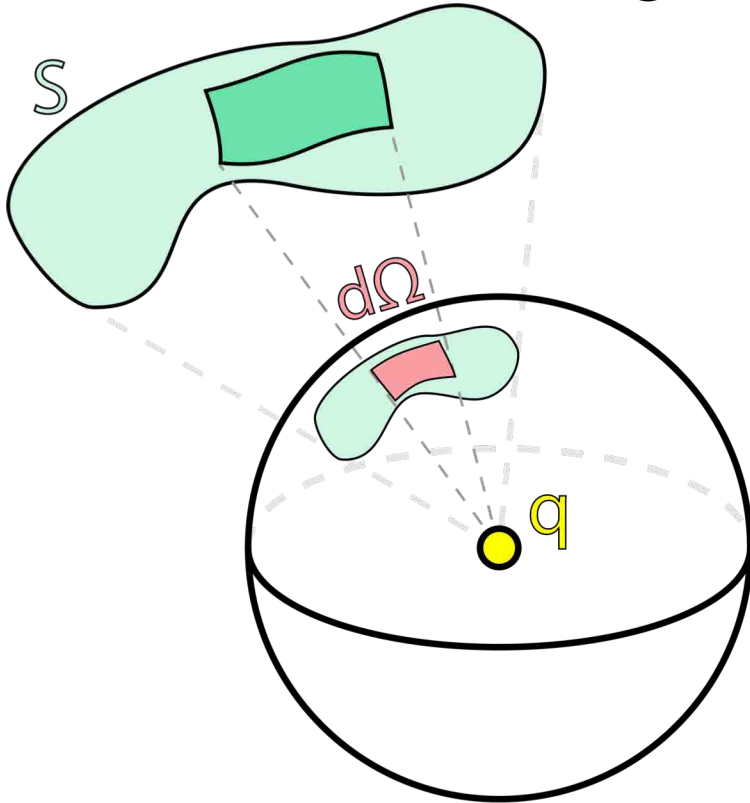
For disconnect “soup”,
in the worst case,
size of boundary \geq mesh

Trouble in paradise!

For point clouds,
there is *no connectivity*
“all boundary”

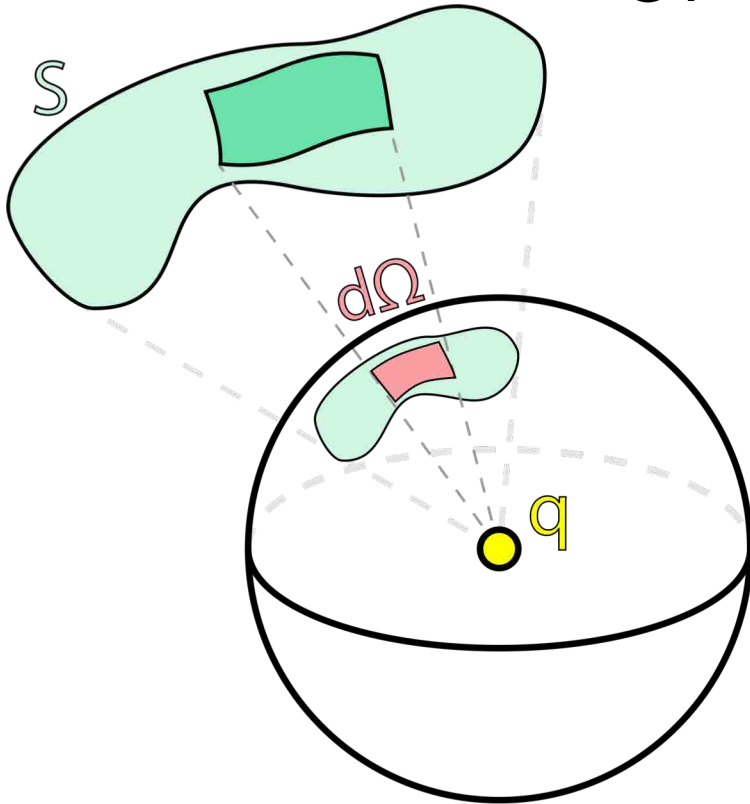


Winding number is sum
of solid angles in 3D



$$w_S(\mathbf{q}) = \frac{1}{4\pi} \int_S d\Omega$$

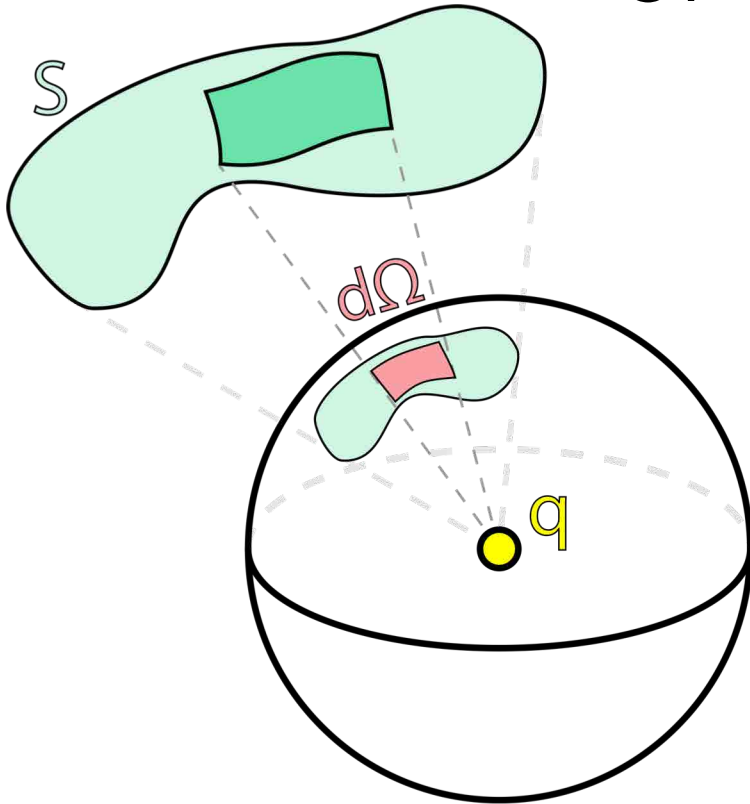
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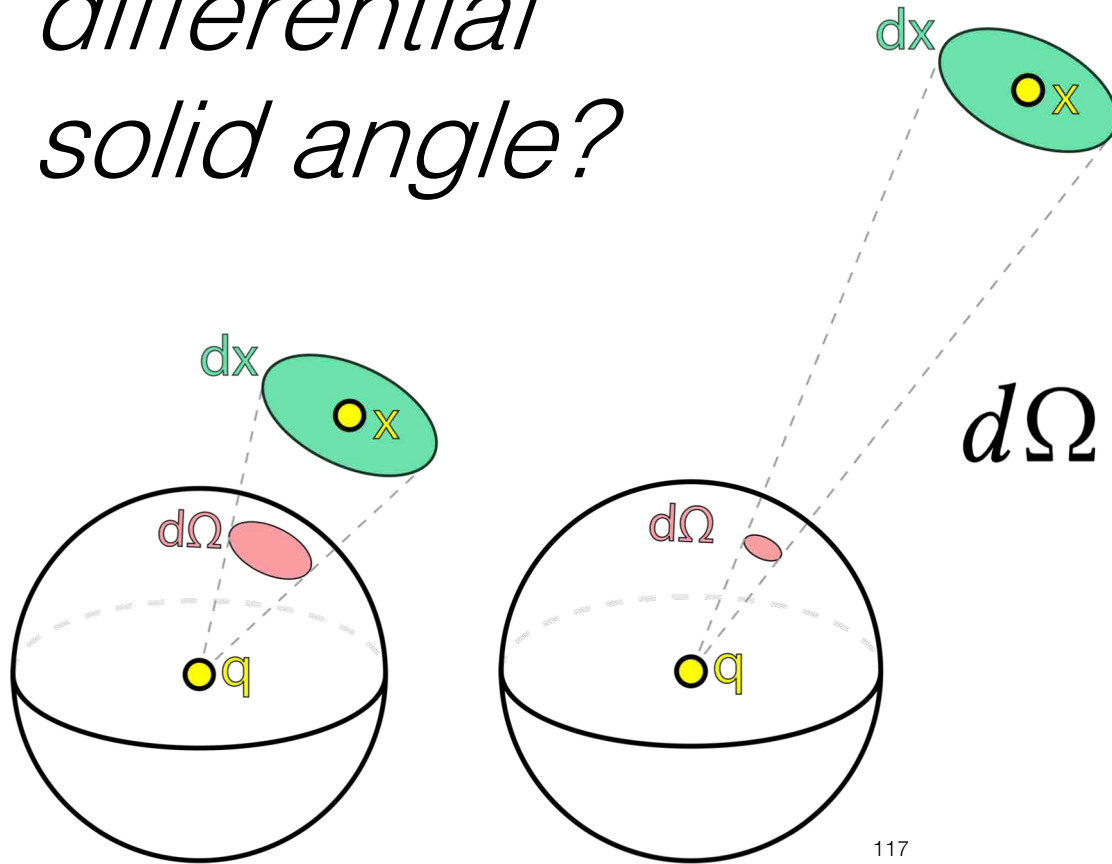
(surface area of unit sphere)

Winding number is sum of solid angles in 3D



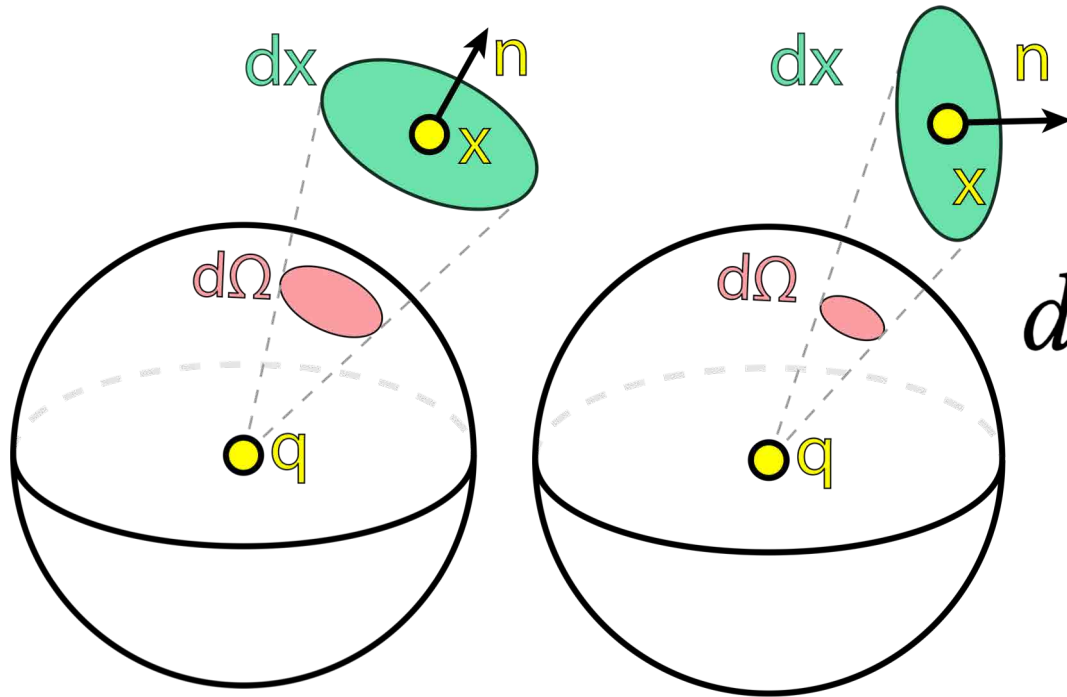
$$w_S(\mathbf{q}) = \frac{1}{4\pi} \int_S d\Omega$$

What can we say about this
differential
solid angle?



$$d\Omega \sim \frac{1}{\|\mathbf{x} - \mathbf{q}\|^2} d\mathbf{x}$$

What can we say about this
differential solid angle?



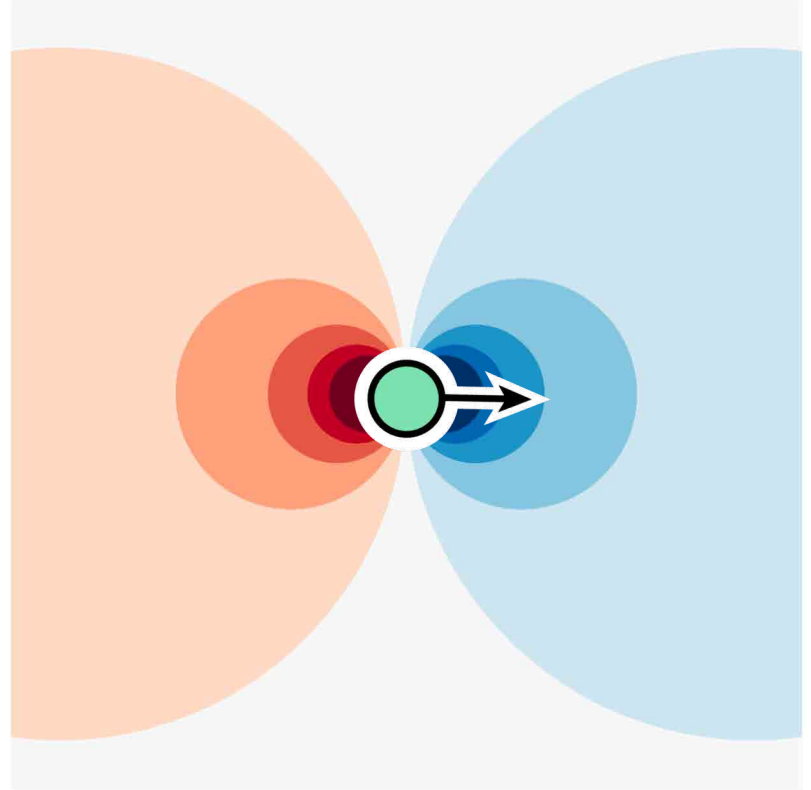
$$d\Omega \sim \mathbf{n} \cdot \frac{\mathbf{x} - \mathbf{q}}{\|\mathbf{x} - \mathbf{q}\|} d\mathbf{x}$$

differential solid angle is a dipole

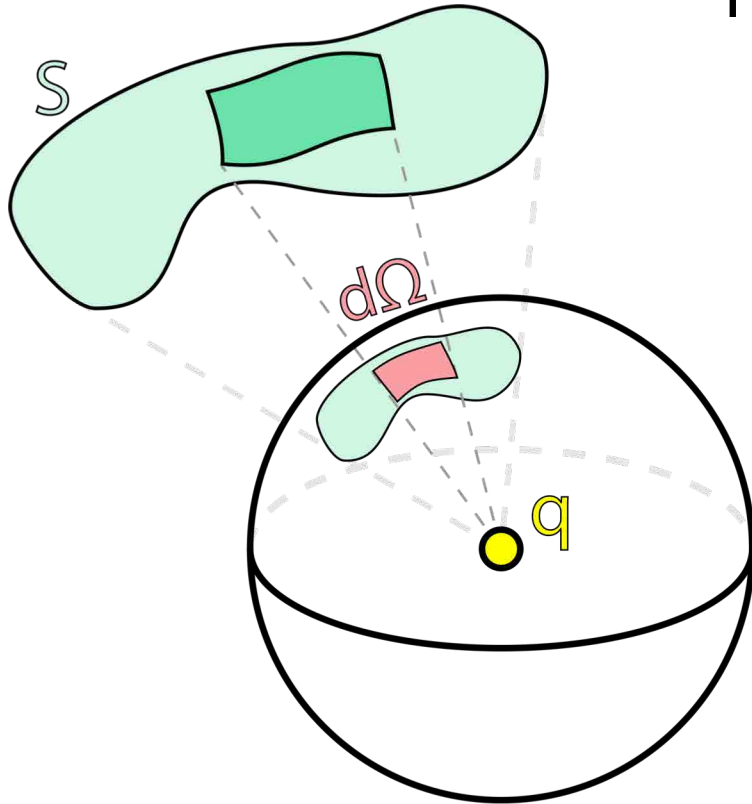
$$d\Omega \Big| = \frac{1}{\|\mathbf{x} - \mathbf{q}\|^2} \mathbf{n} \cdot \frac{\mathbf{x} - \mathbf{q}}{\|\mathbf{x} - \mathbf{q}\|} d\mathbf{x}$$

differential solid angle is a dipole

$$d\Omega = \frac{(\mathbf{x} - \mathbf{q}) \cdot \mathbf{n}}{\|\mathbf{x} - \mathbf{q}\|^3} d\mathbf{x}$$

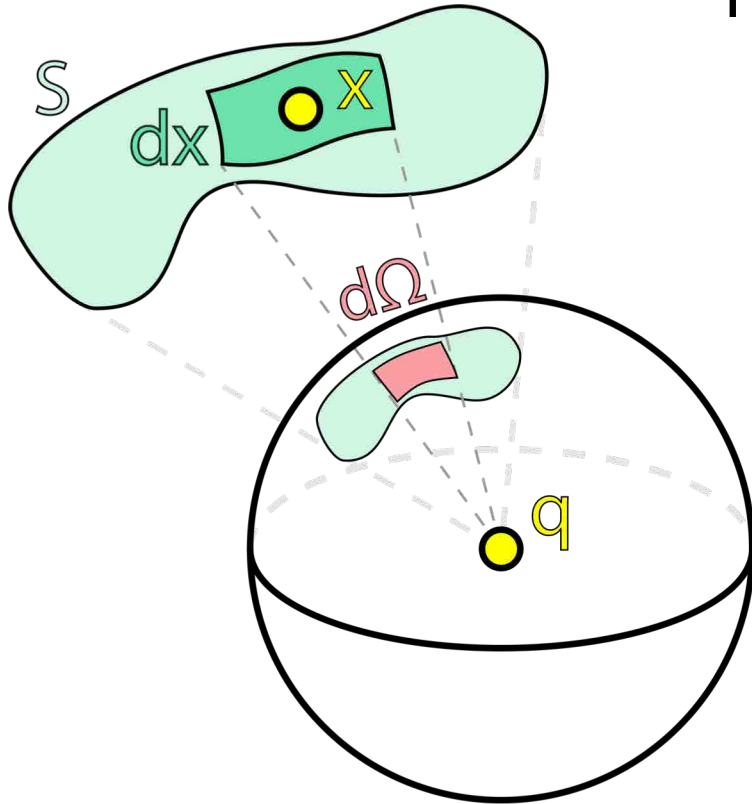


Winding number integrates dipoles on surface



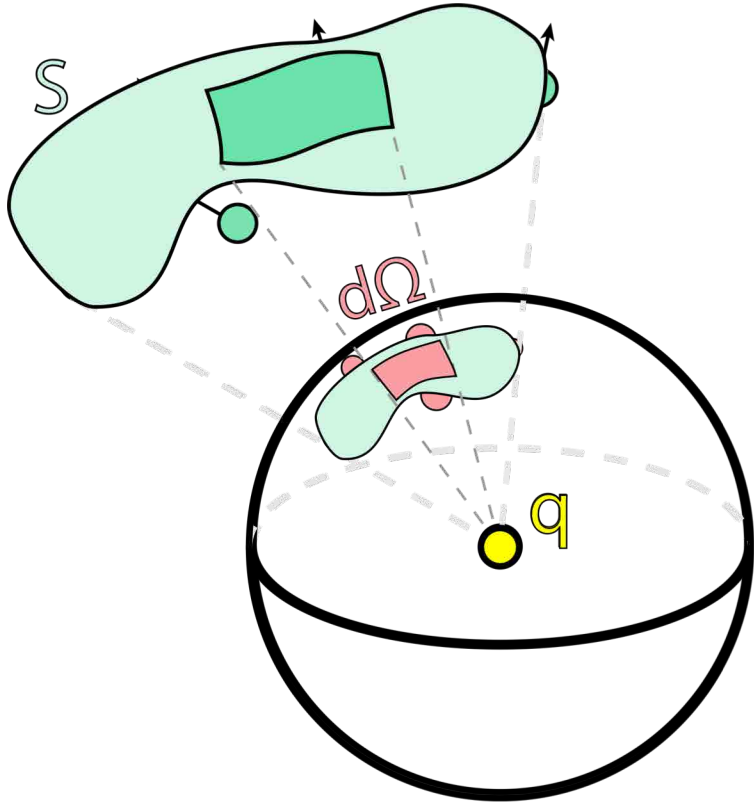
$$w_S(\mathbf{q}) = \frac{1}{4\pi} \int_S d\Omega$$

Winding number integrates
dipoles on surface



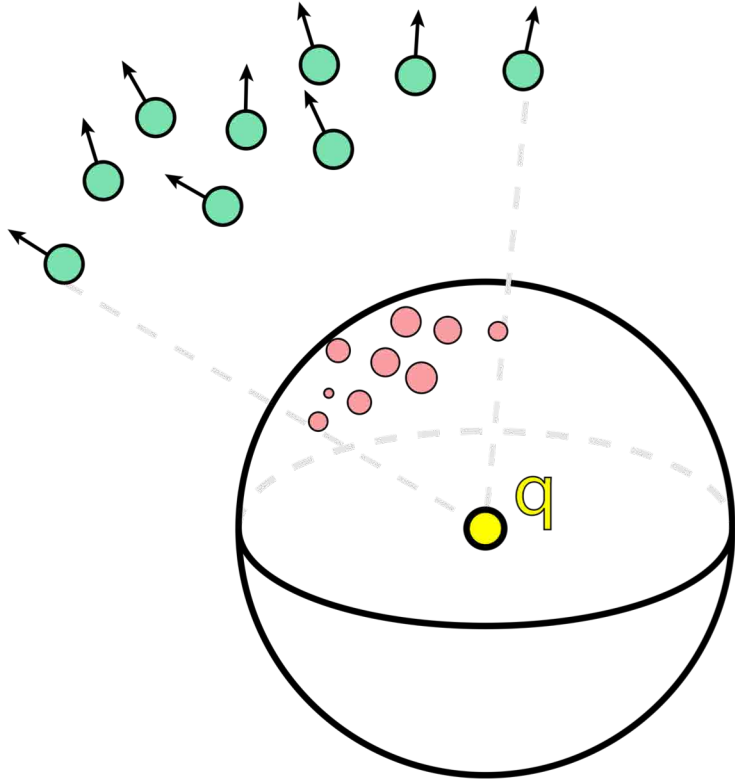
$$w_S(\mathbf{q}) = \frac{1}{4\pi} \int_S \frac{(\mathbf{x} - \mathbf{q}) \cdot \mathbf{n}}{\|\mathbf{x} - \mathbf{q}\|^3} d\mathbf{x}$$

Winding number integrates
dipoles on surface



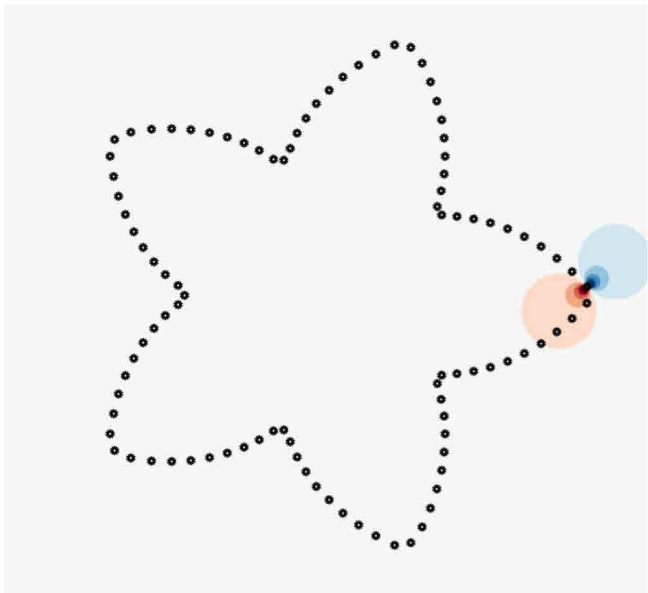
$$w_S(\mathbf{q}) = \frac{1}{4\pi} \int_S \frac{(\mathbf{x} - \mathbf{q}) \cdot \mathbf{n}}{\|\mathbf{x} - \mathbf{q}\|^3} d\mathbf{x}$$

Winding number sums discrete *cloud* of dipoles



$$\sum_i^n a_i \frac{(\mathbf{x}_i - \mathbf{q}) \cdot \mathbf{n}_i}{\|\mathbf{x}_i - \mathbf{q}\|^3}$$

area associated with i -th point



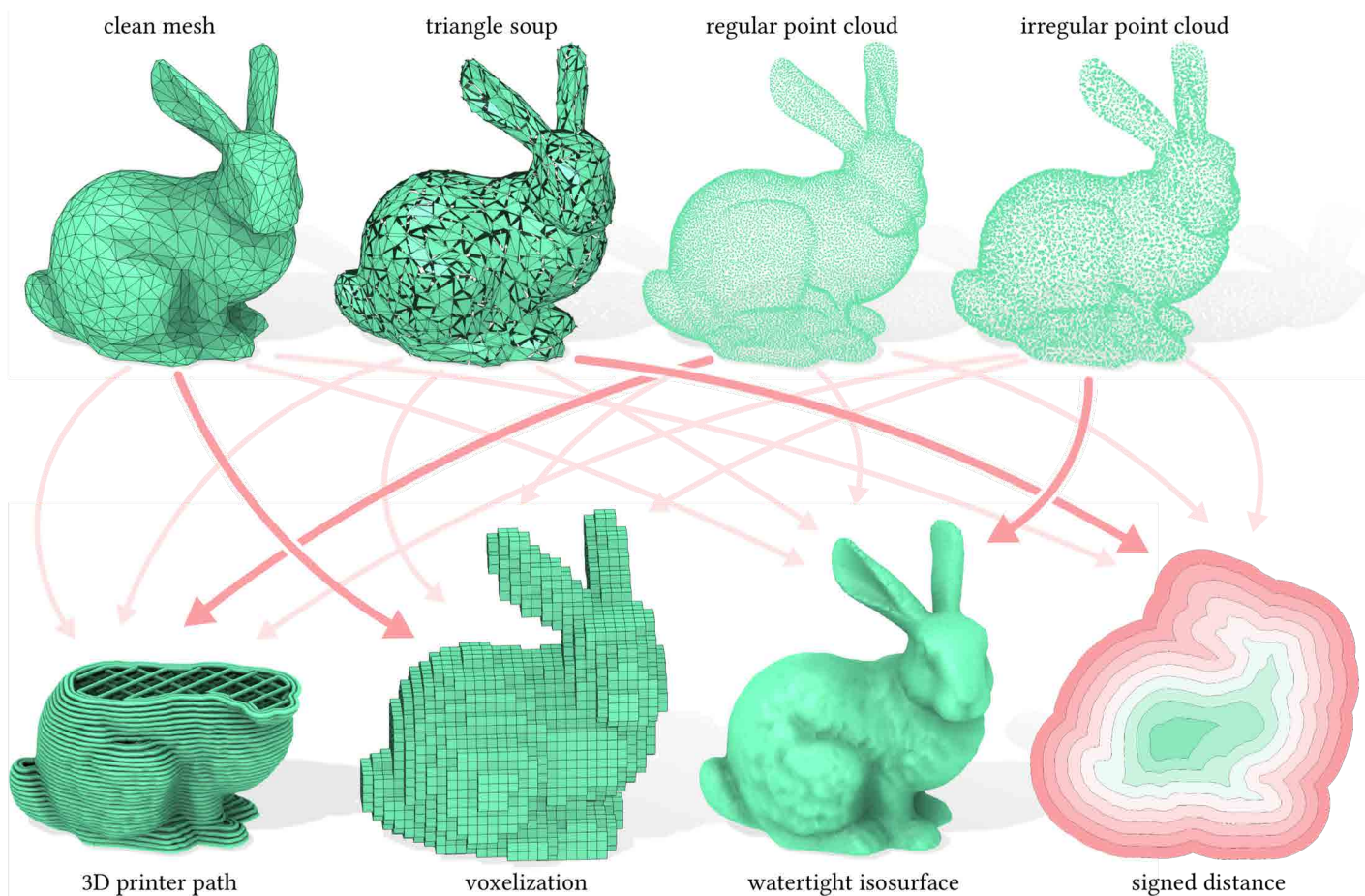
$$\Delta w = 0$$

Winding number for point clouds!



$$w_S(\mathbf{q}) = \frac{1}{4\pi} \sum_i^n a_i \frac{(\mathbf{x}_i - \mathbf{q}) \cdot \mathbf{n}_i}{\|\mathbf{x}_i - \mathbf{q}\|^3}$$

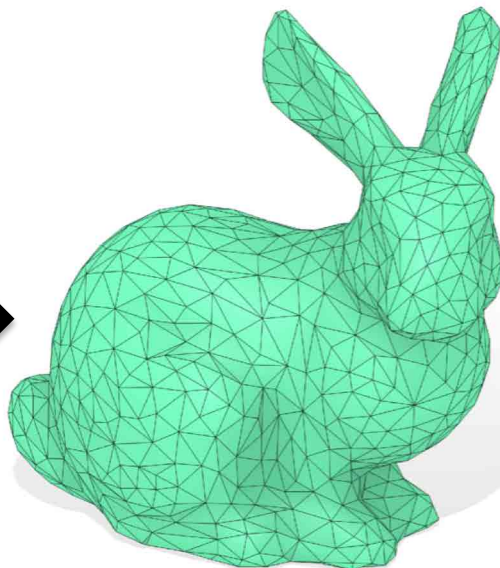
Winding number further generalizes to point clouds



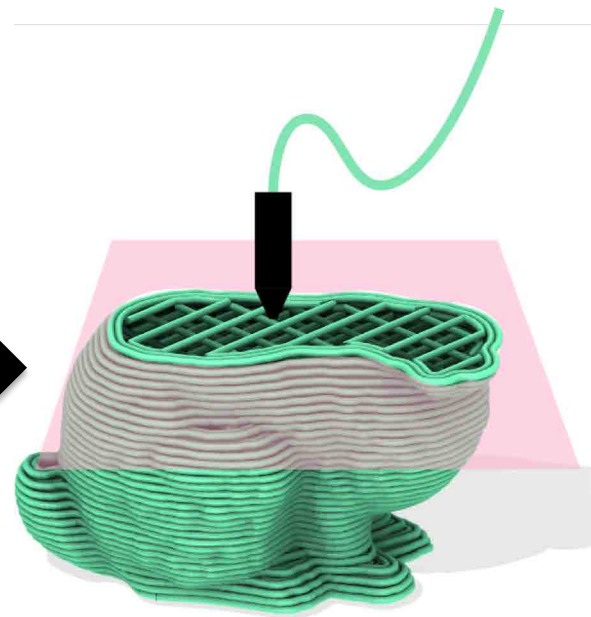
Typical 3D printing process requires .stl (triangle mesh) or 3D voxel image



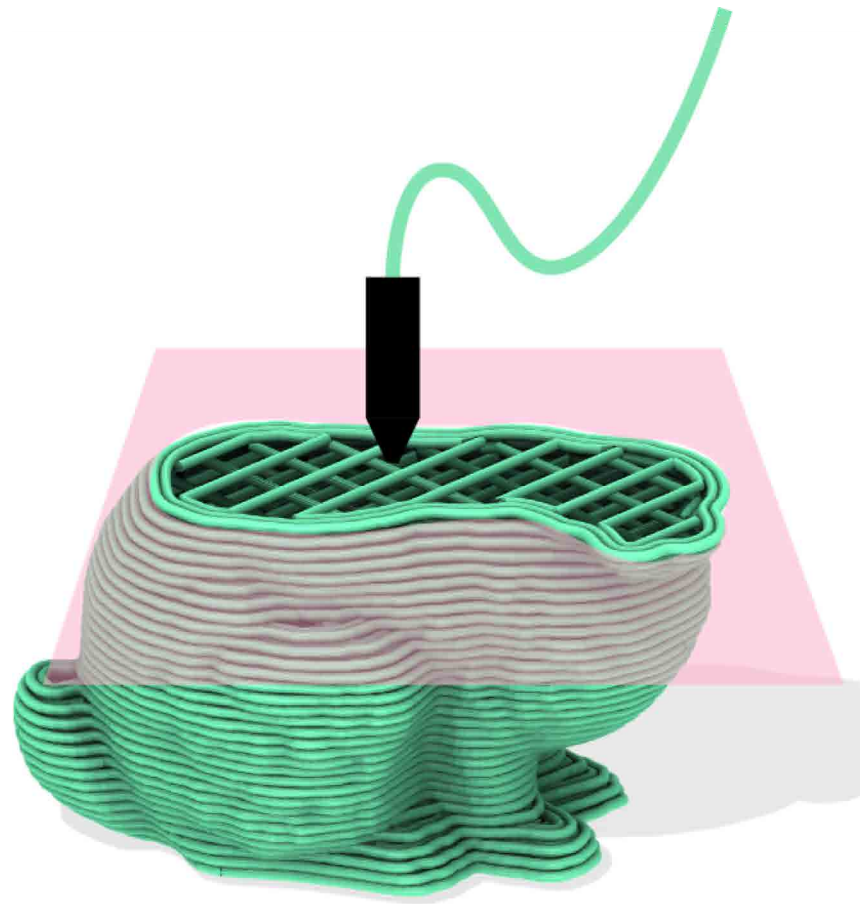
Point Cloud

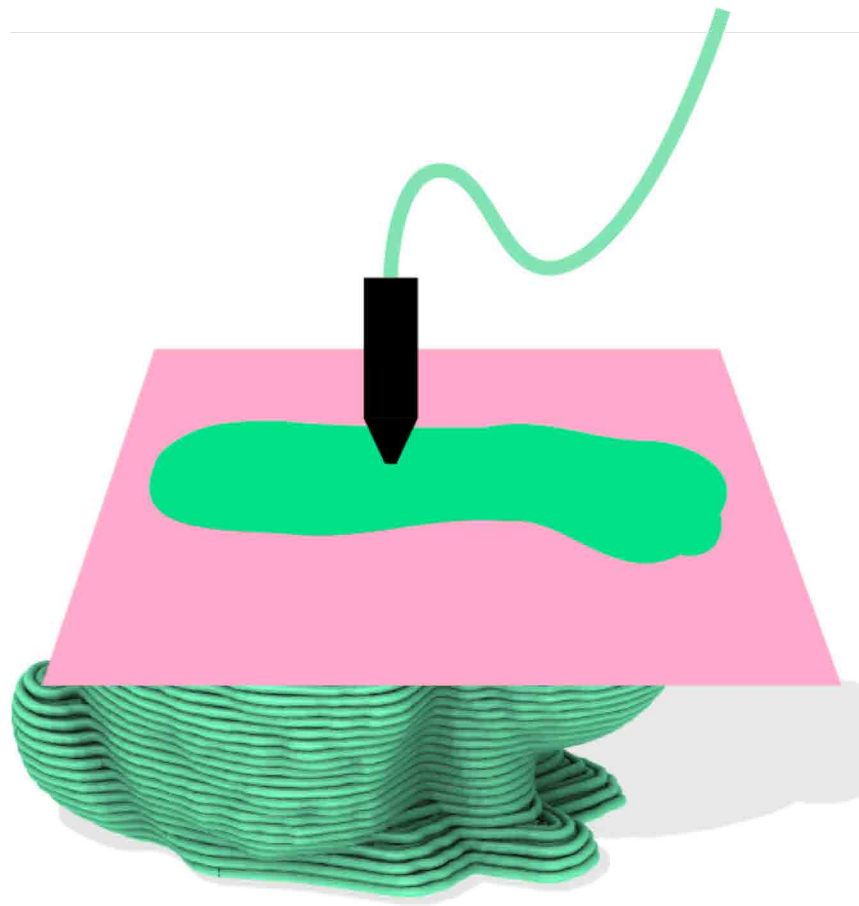


Surface Mesh (.stl)



Slice + Toolpath



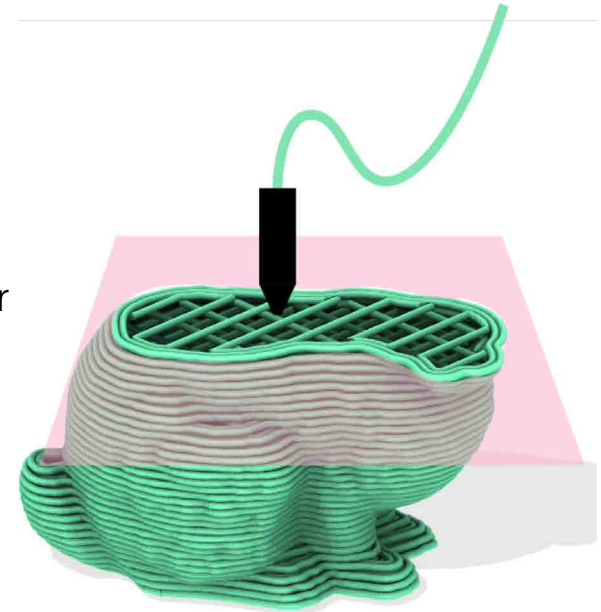


Winding number enables direct printing of point clouds



Point Cloud

Slice winding number



Toolpath

Winding number enables direct printing of point clouds

input point cloud



Winding number for point clouds!



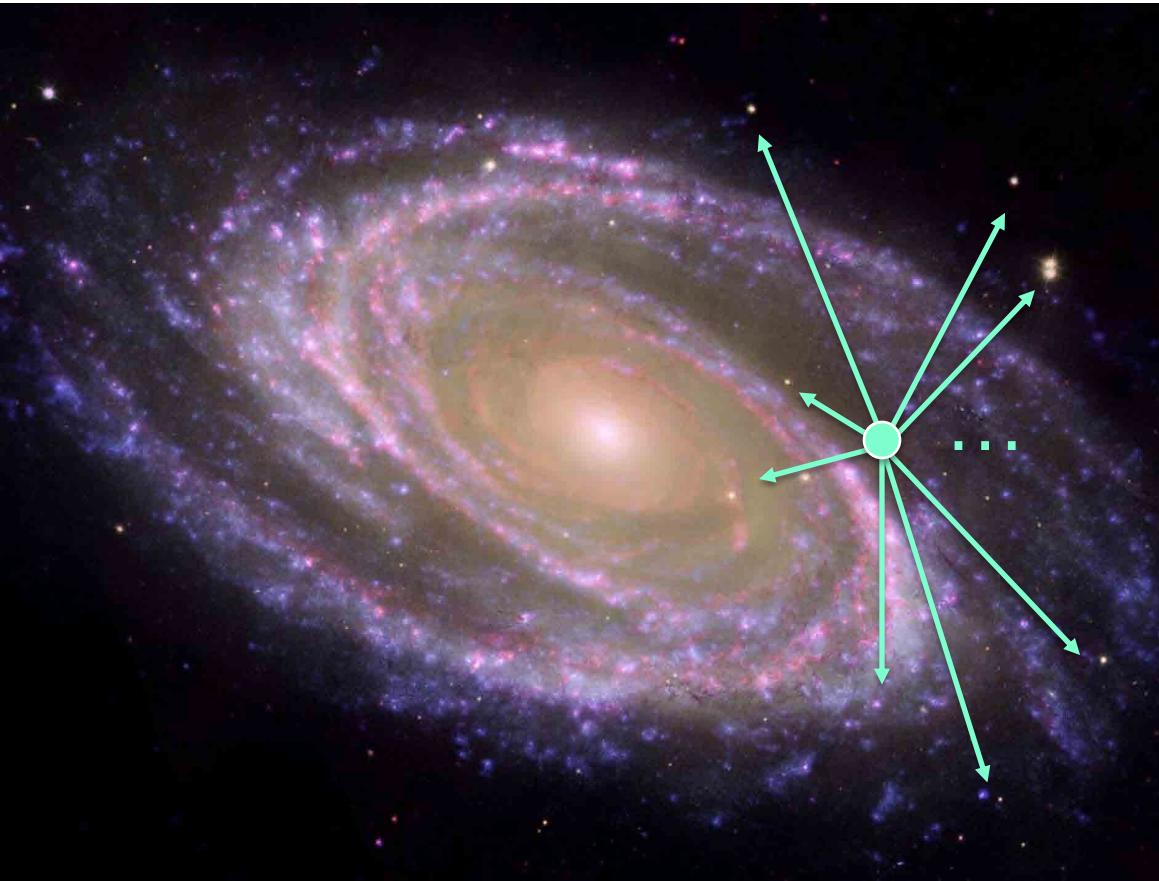
$$w_S(\mathbf{q}) = \frac{1}{4\pi} \sum_i^n a_i \frac{(\mathbf{x}_i - \mathbf{q}) \cdot \mathbf{n}_i}{\|\mathbf{x}_i - \mathbf{q}\|^3}$$

Evaluating winding number for
m queries over n points is still $O(mn)$ 🤔

Borrow insight from astrophysics:

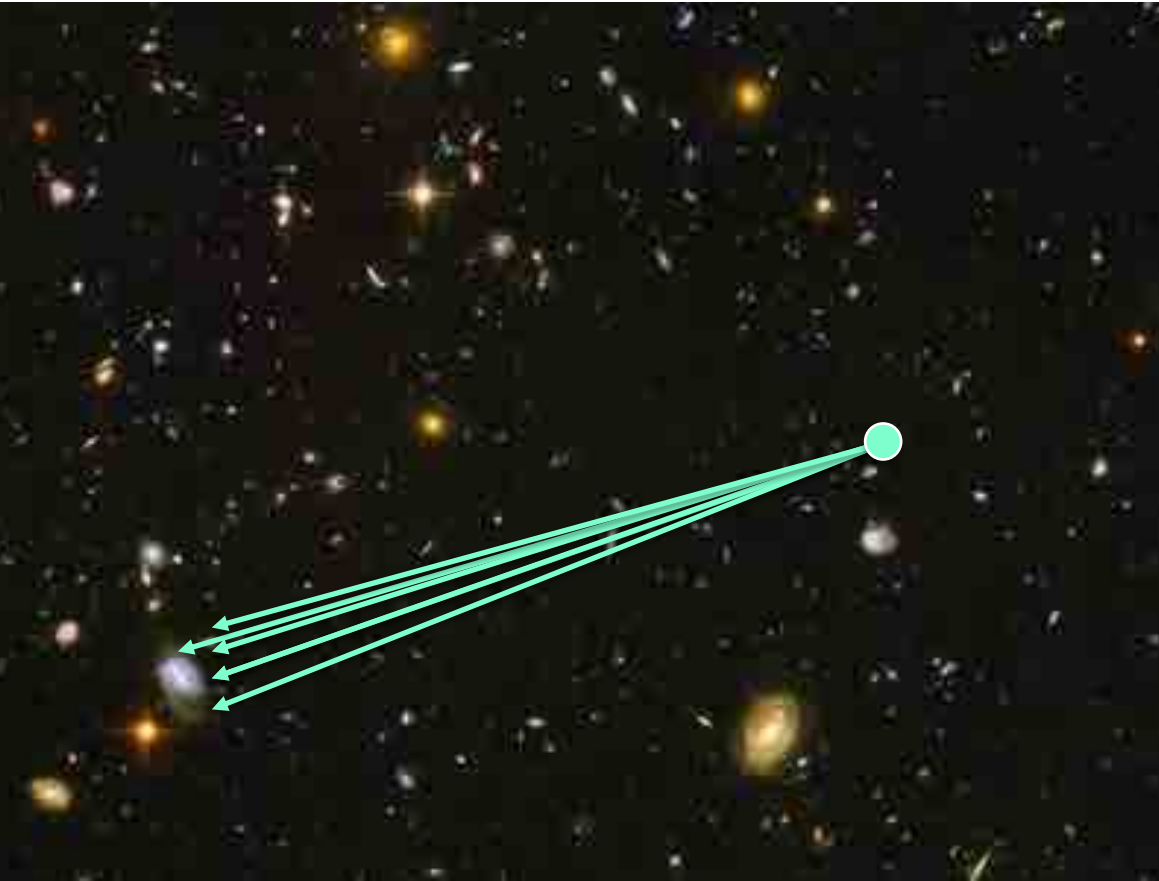


Borrow insight from astrophysics:



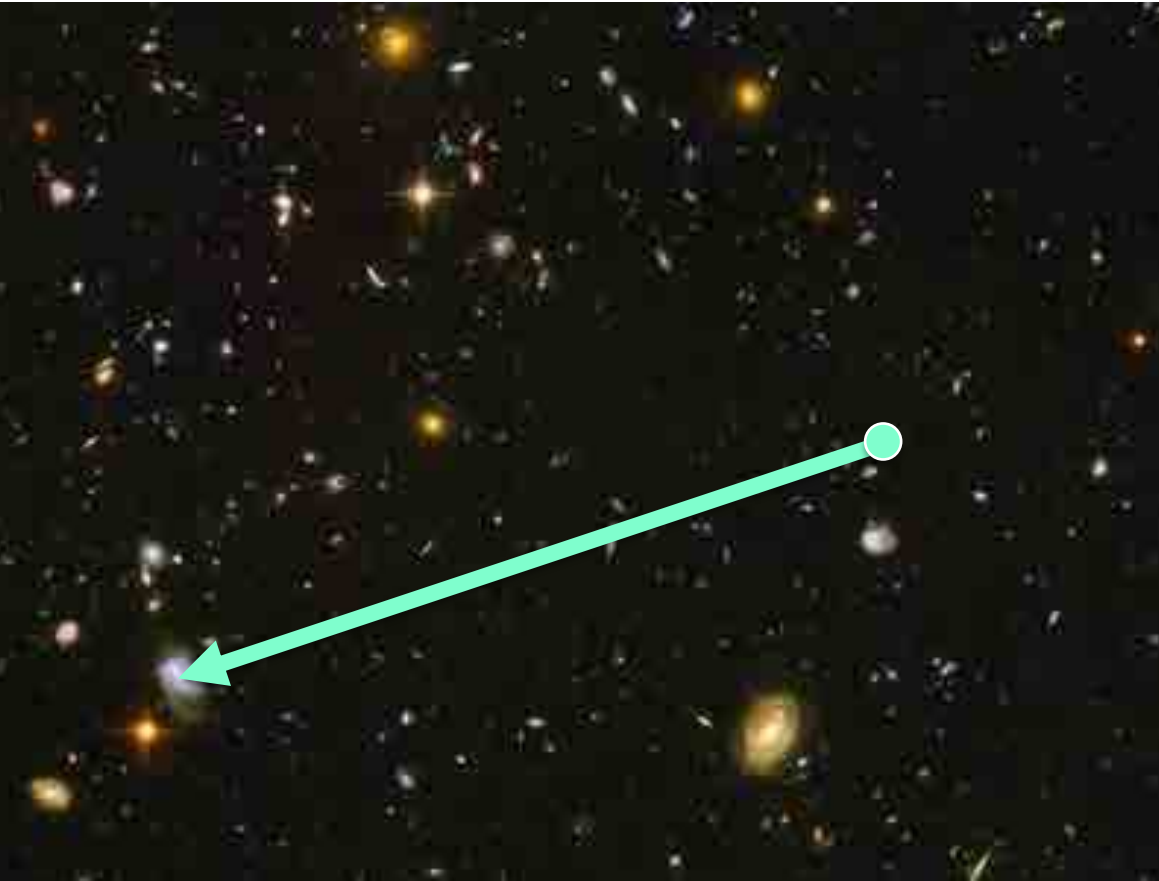
Gravitational forces of nearby bodies may be interesting...

Borrow insight from astrophysics:



... but force of many far
away objects is well
approximated by a single
(big) object

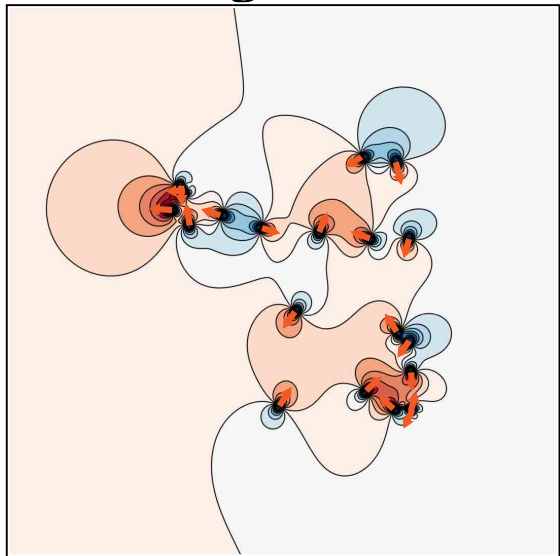
Borrow insight from astrophysics:



... but force of many far away objects is well approximated by a single (big) object

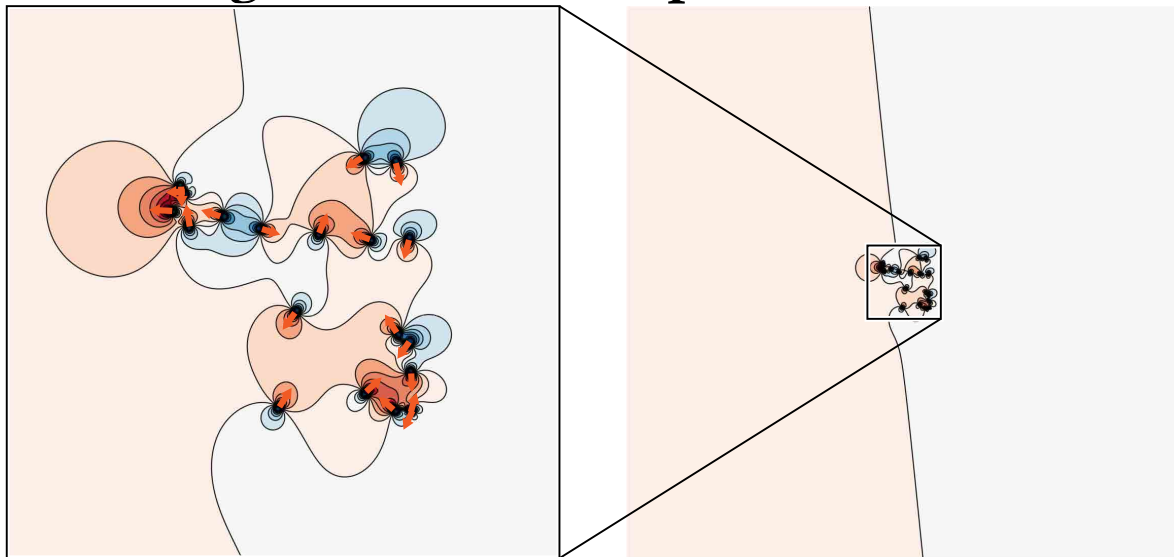
Winding number of many far away points
looks just like that of single point

Winding number of 20 points



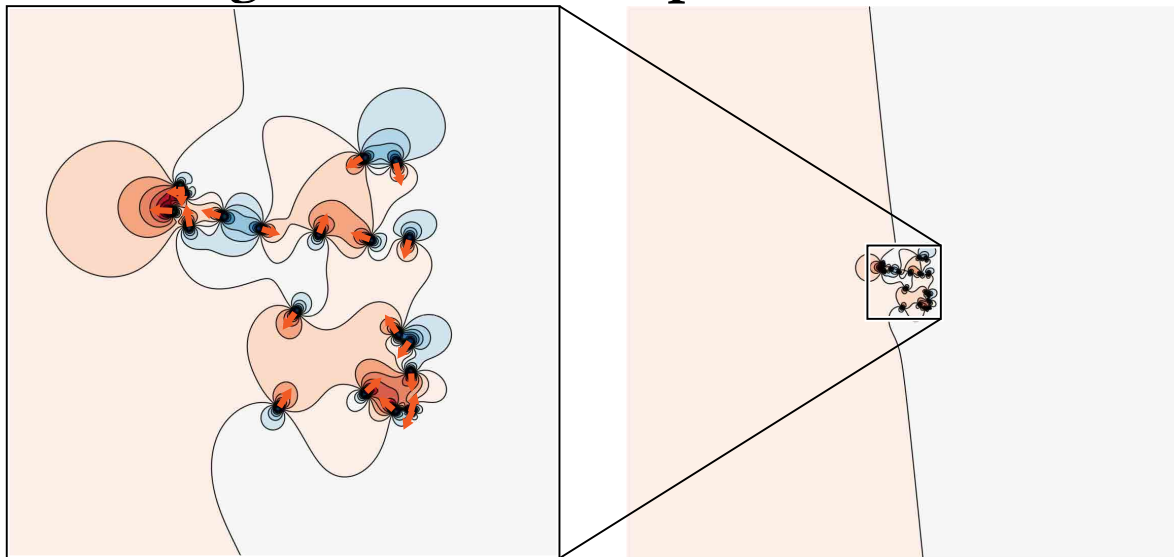
Winding number of many far away points
looks just like that of single point

Winding number of 20 points

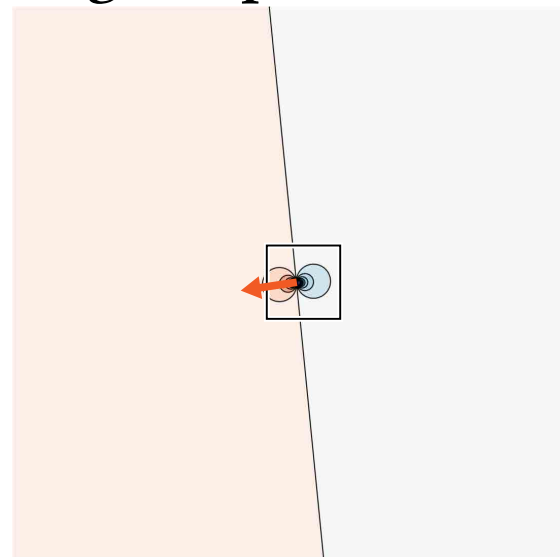


Winding number of many far away points looks just like that of single point

Winding number of 20 points



Single representative



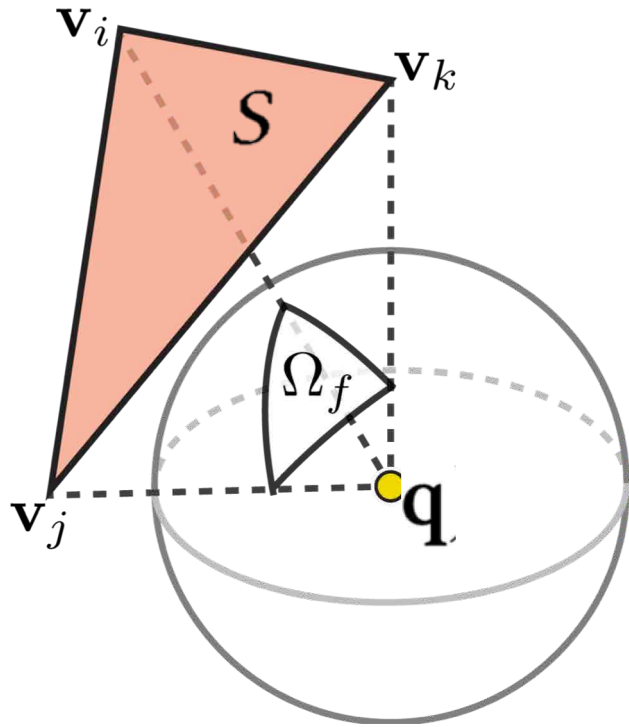
Precomputation:

Throw points into
bounding volume
hierarchy (e.g., octree)

Query evaluation:

Use cell representative
if far enough,
otherwise recursive
call on children

Immediately extends to triangles



$$w_S(\mathbf{q}) = \int_S \frac{(\mathbf{x} - \mathbf{q}) \cdot \hat{\mathbf{n}}}{4\pi \|\mathbf{x} - \mathbf{q}\|^3} dx$$

Just integrate point winding number over triangle...

... “representative point” of a bunch of triangles is :

\mathbf{q} : area-weighted barycenters

\mathbf{n} : area-weighted normals

a : total area

Higher-order accuracy is possible

$$w(\mathbf{q}) \approx \left(\sum_{t=1}^m \int_t \hat{\mathbf{n}}_t dA \right) \cdot \nabla G(\mathbf{q}, \tilde{\mathbf{p}}) \quad (18)$$

$$+ \left(\sum_{t=1}^m \int_t (\mathbf{x} - \tilde{\mathbf{p}}) \otimes \hat{\mathbf{n}}_t dA \right) \cdot \nabla^2 G(\mathbf{q}, \tilde{\mathbf{p}}) \quad (19)$$

$$+ \frac{1}{2} \left(\sum_{t=1}^m \int_t (\mathbf{x} - \tilde{\mathbf{p}}) \otimes ((\mathbf{x} - \tilde{\mathbf{p}}) \otimes \hat{\mathbf{n}}_t) dA \right) \cdot \nabla^3 G(\mathbf{q}, \tilde{\mathbf{p}}) \quad (20)$$

$$+ \frac{1}{2} \left(\sum_{t=1}^m \int_t (\mathbf{x} - \tilde{\mathbf{p}}) \otimes (\mathbf{x} - \tilde{\mathbf{p}}) \otimes \hat{\mathbf{n}}_t dA \right) \cdot \nabla^3 G(\mathbf{q}, \tilde{\mathbf{p}}) \quad (21)$$

$$+ \textit{higher order terms} =: w(\mathbf{q}). \quad (22)$$

Higher-order accuracy is possible

“Single representative” actually corresponds to first-order Taylor expansion...

We can take any order, see paper for up to 3rd order derivations.

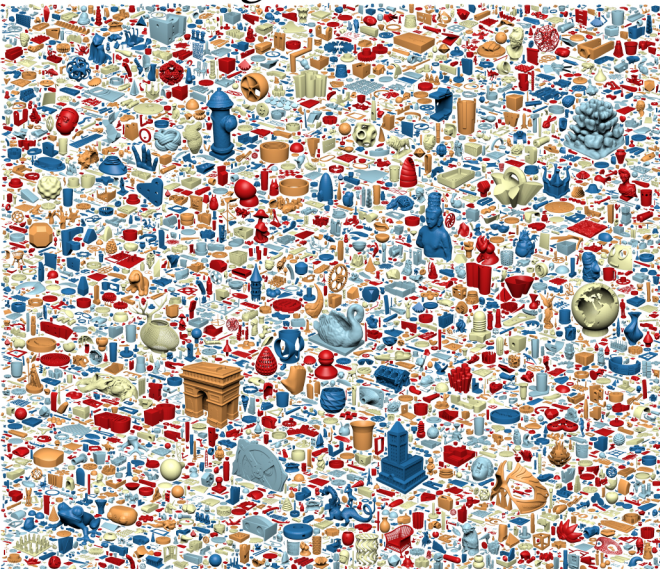
Higher-order accuracy is possible

“Single representative” actually corresponds to first-order Taylor expansion...

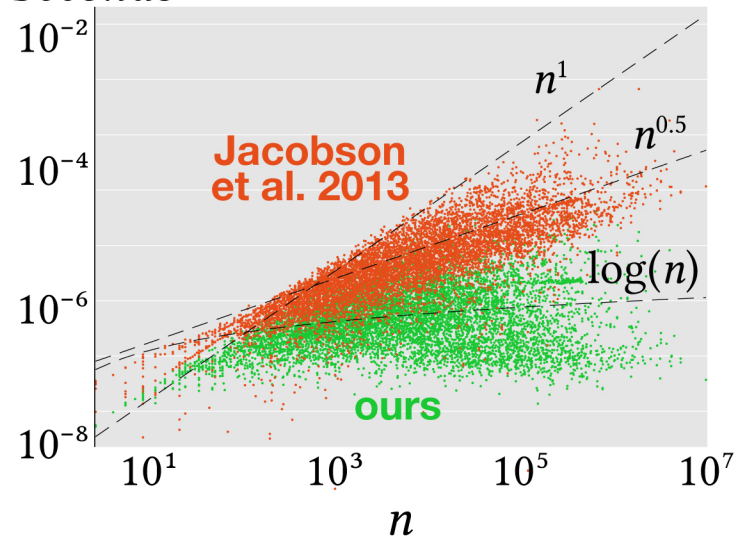
We can take any order, see paper for up to 3rd order derivations.

This time, really asymptotically faster

Thingi10k dataset

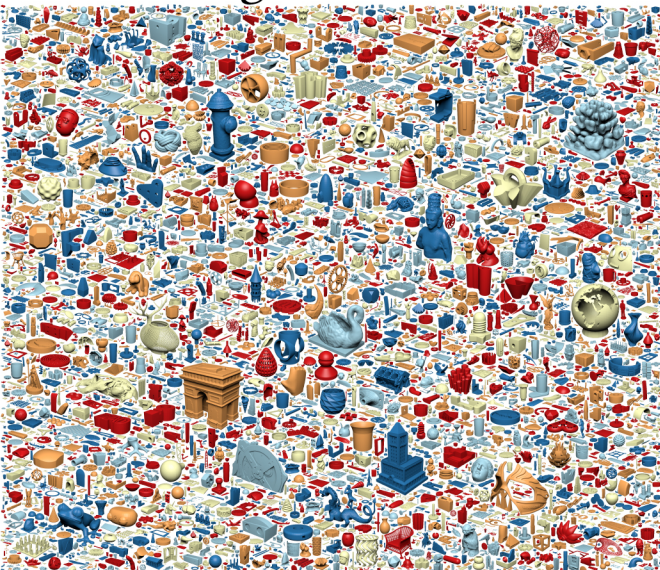


Average evaluation time,
Seconds

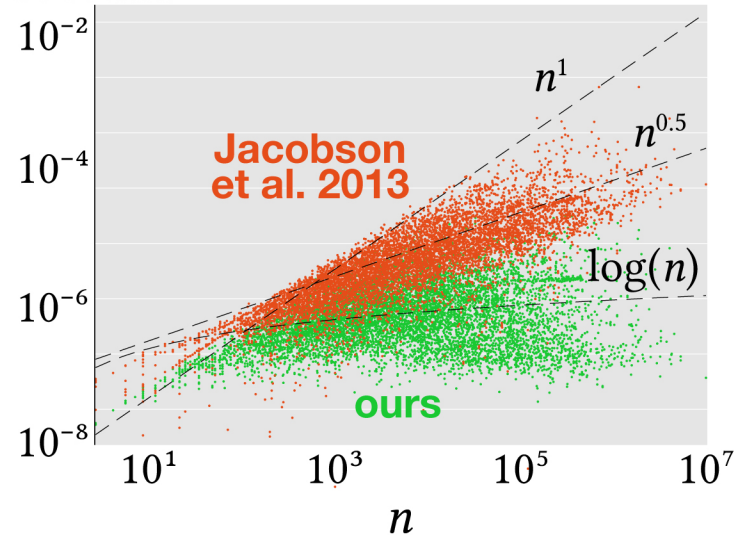


This time, really asymptotically faster

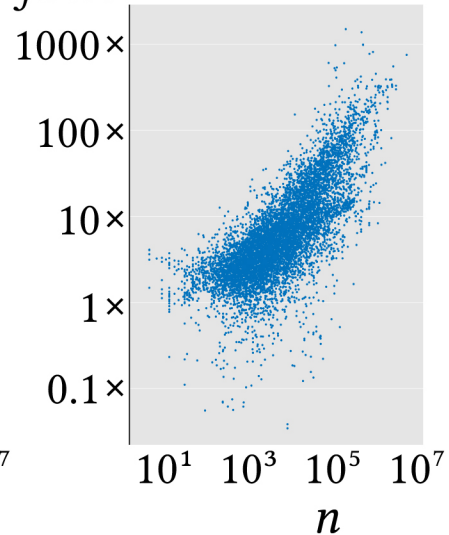
Thingi10k dataset



Average evaluation time,
Seconds



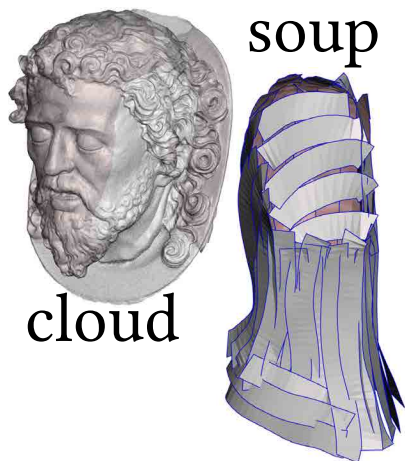
speedup
factor



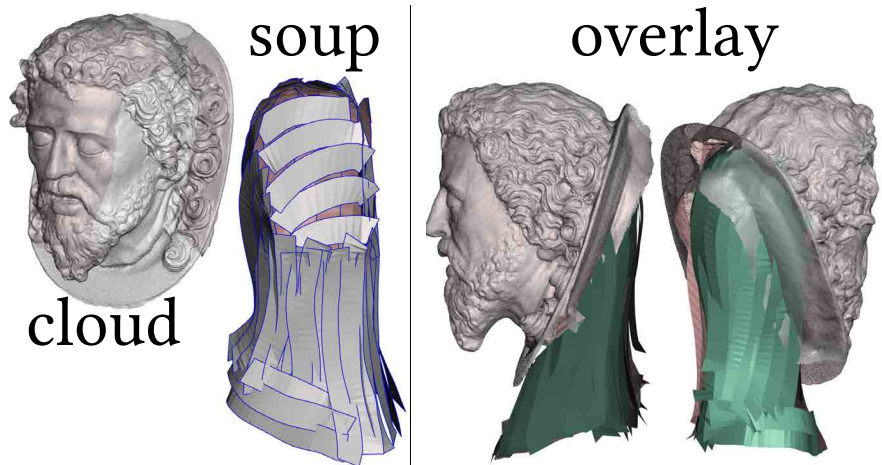
VR painters don't need/want to care about underlying data structures



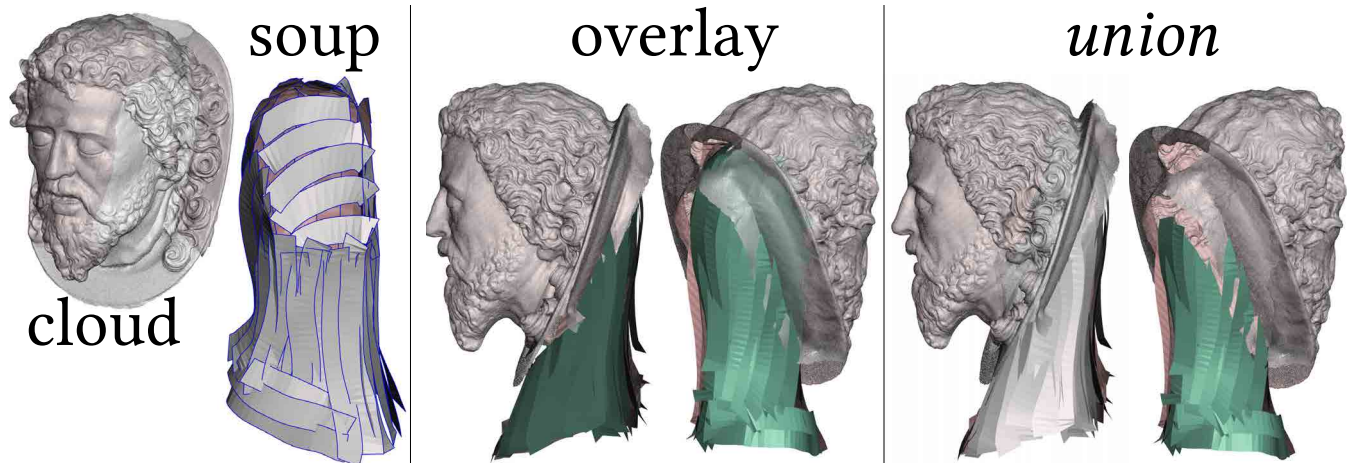
VR painters don't need/want to care about underlying data structures



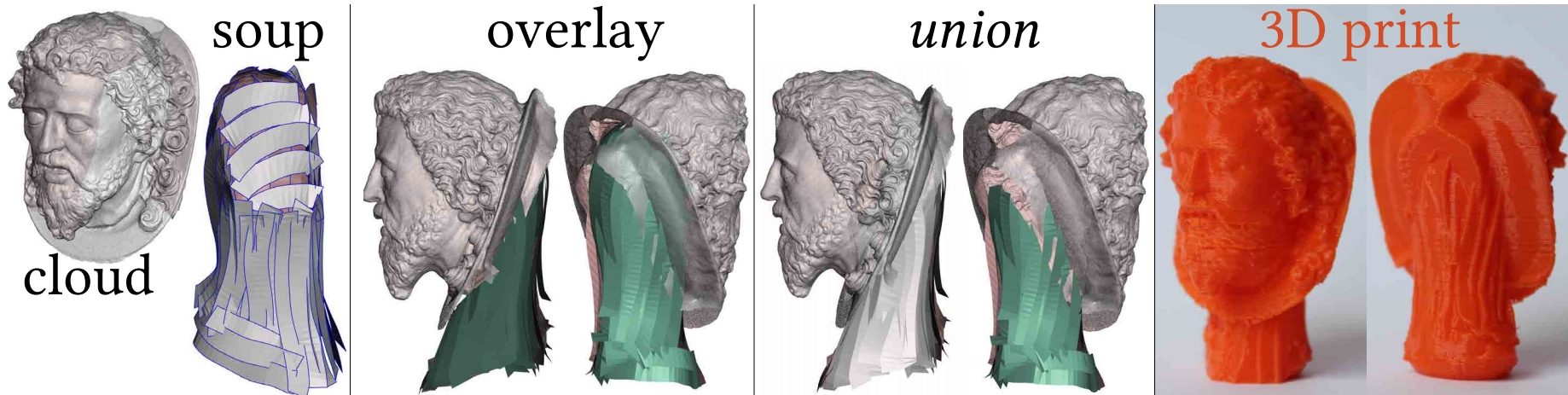
VR painters don't need/want to care about underlying data structures



VR painters don't need/want to care about underlying data structures



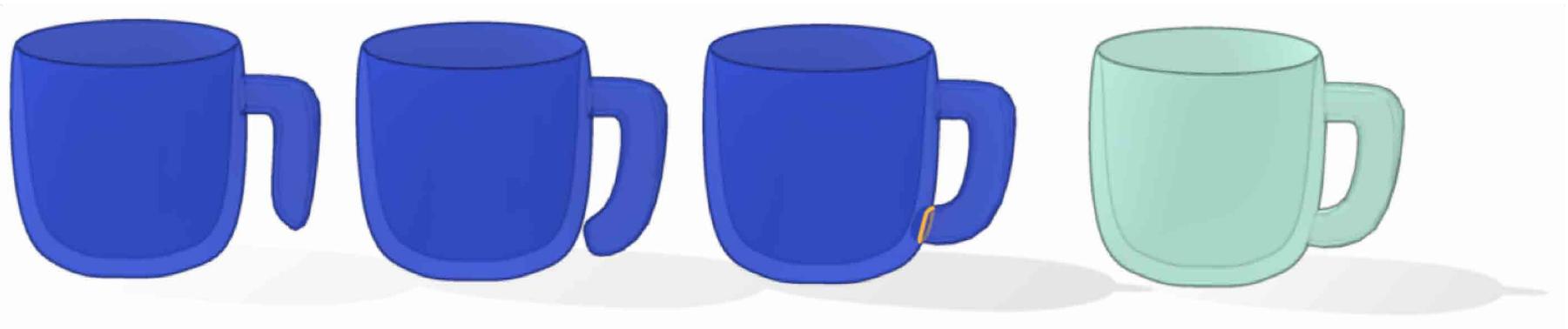
VR painters don't need/want to care about underlying data structures



“Artifacts” or “defects” are actually symptoms of friendly UIs



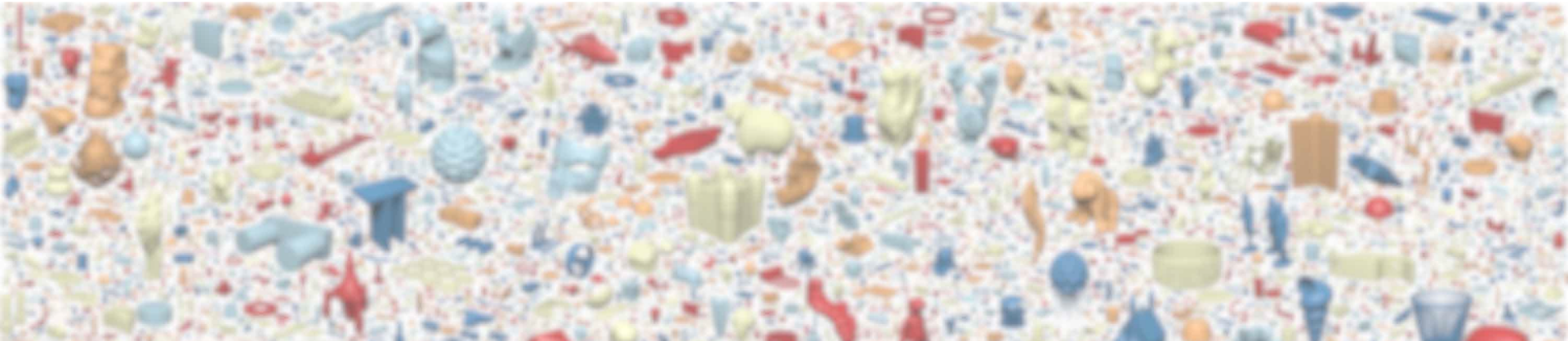
“Artifacts” or “defects” are actually symptoms of friendly UIs



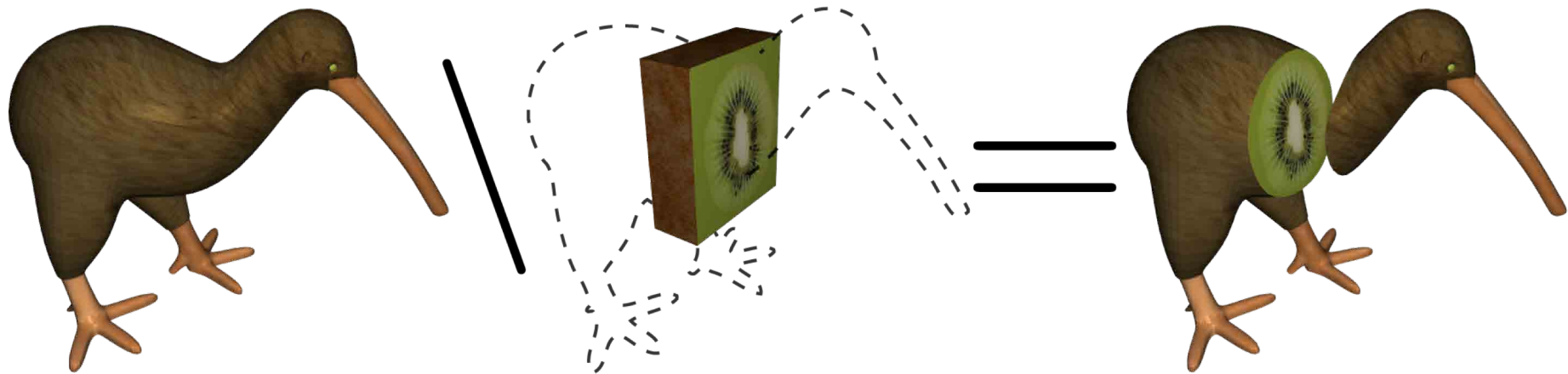
Generalized winding number, as a *concept*,
helps classify shapes

49% “clean” by standard metrics

86% piecewise-constant winding number



Novel Boolean algorithm accepts
all piecewise-constant winding number meshes



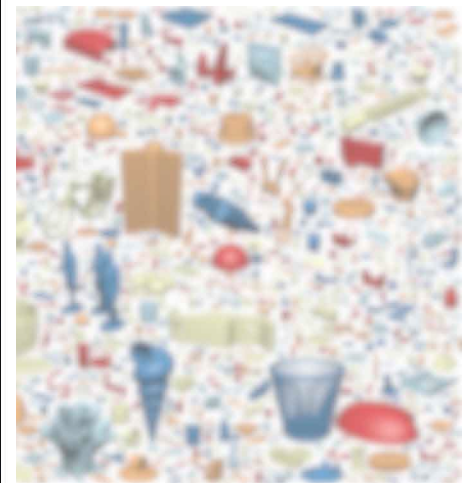


Sagrada Família, Gaudí (1852-1926)

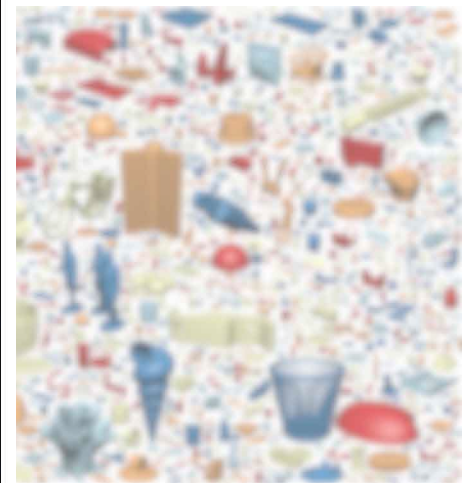
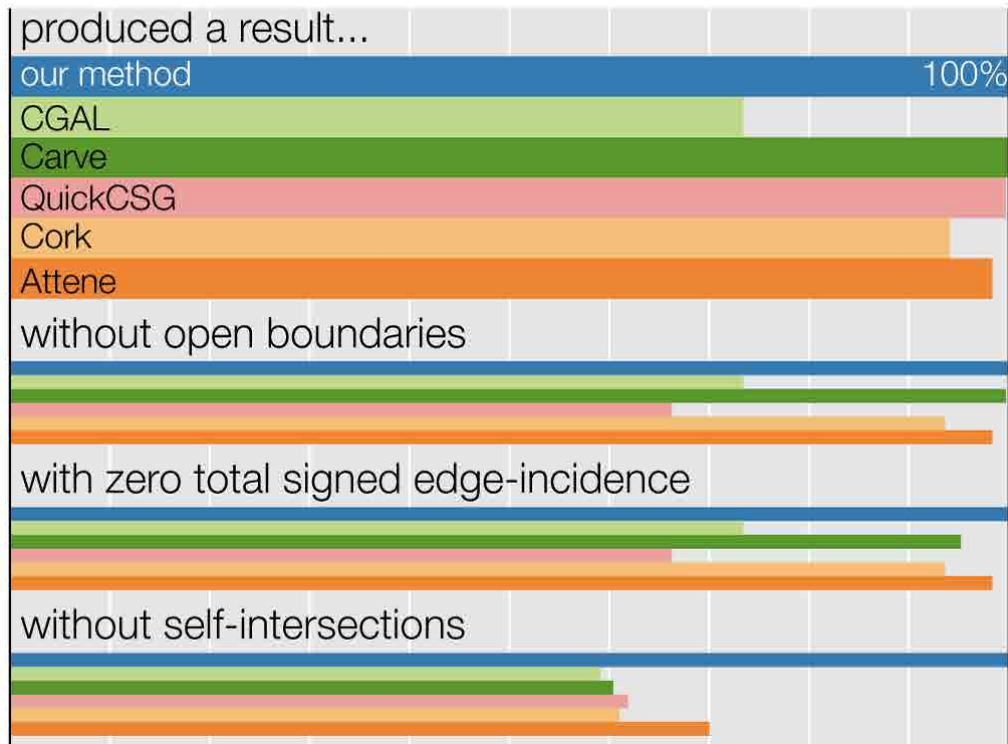
Careful preconditions, postconditions ensure robustness, validated empirically



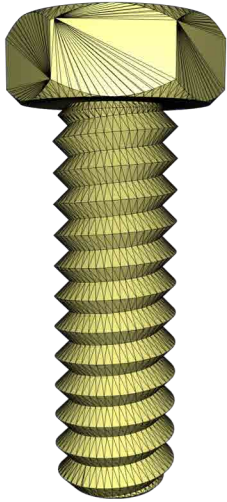
produced a result...									
our method	100%								
CGAL									
Carve									
QuickCSG									
Cork									
Attene									
without open boundaries									
with zero total signed edge-incidence									
without self-intersections									



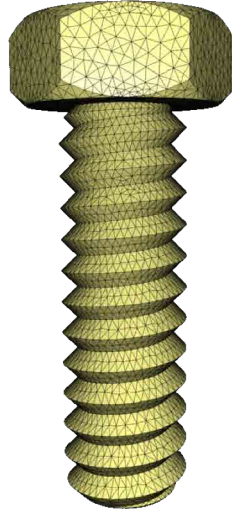
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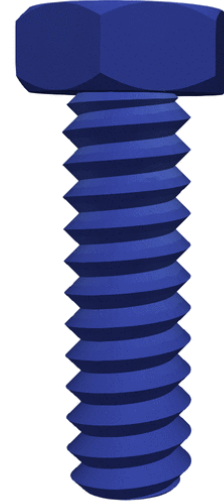
Simulation depends on tetrahedral meshing



Surface Representation
(From Thingi10k)

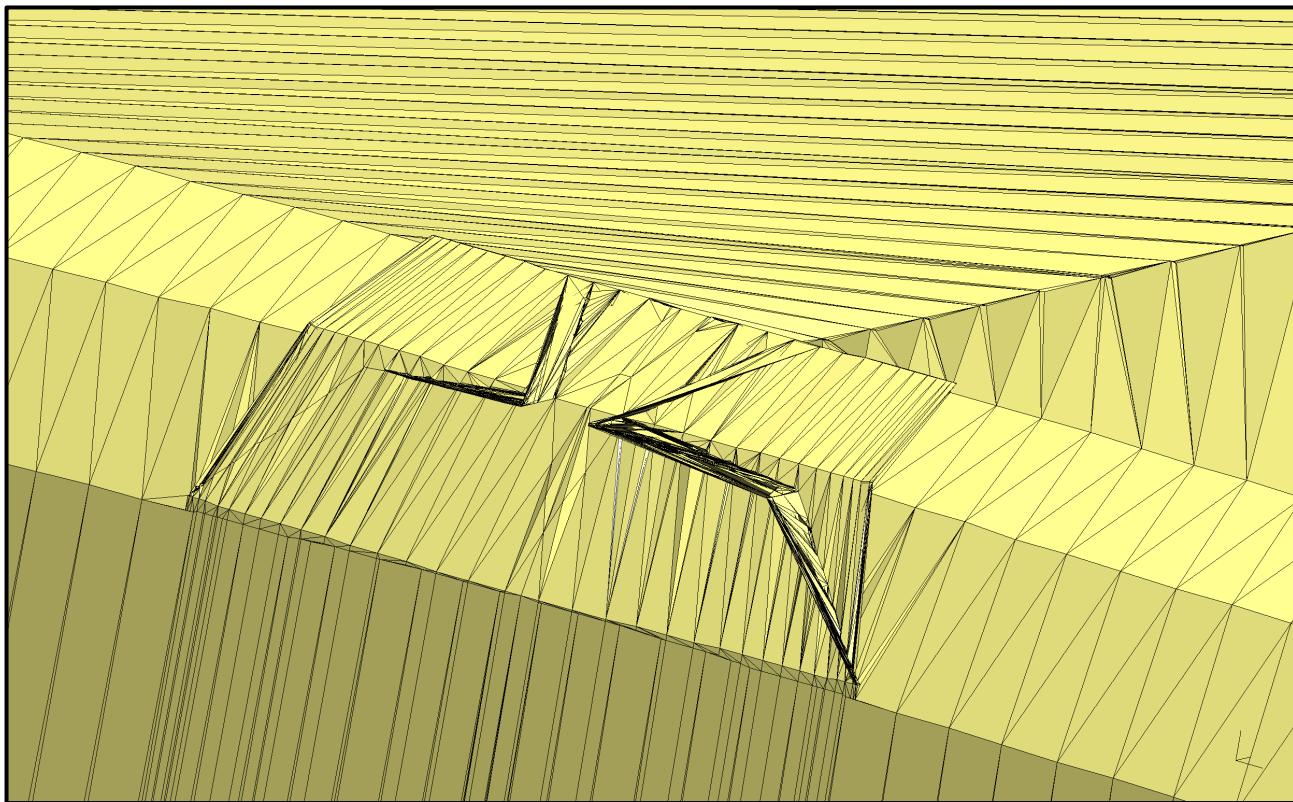


Volumetric Representation
(Generated by TetWild)



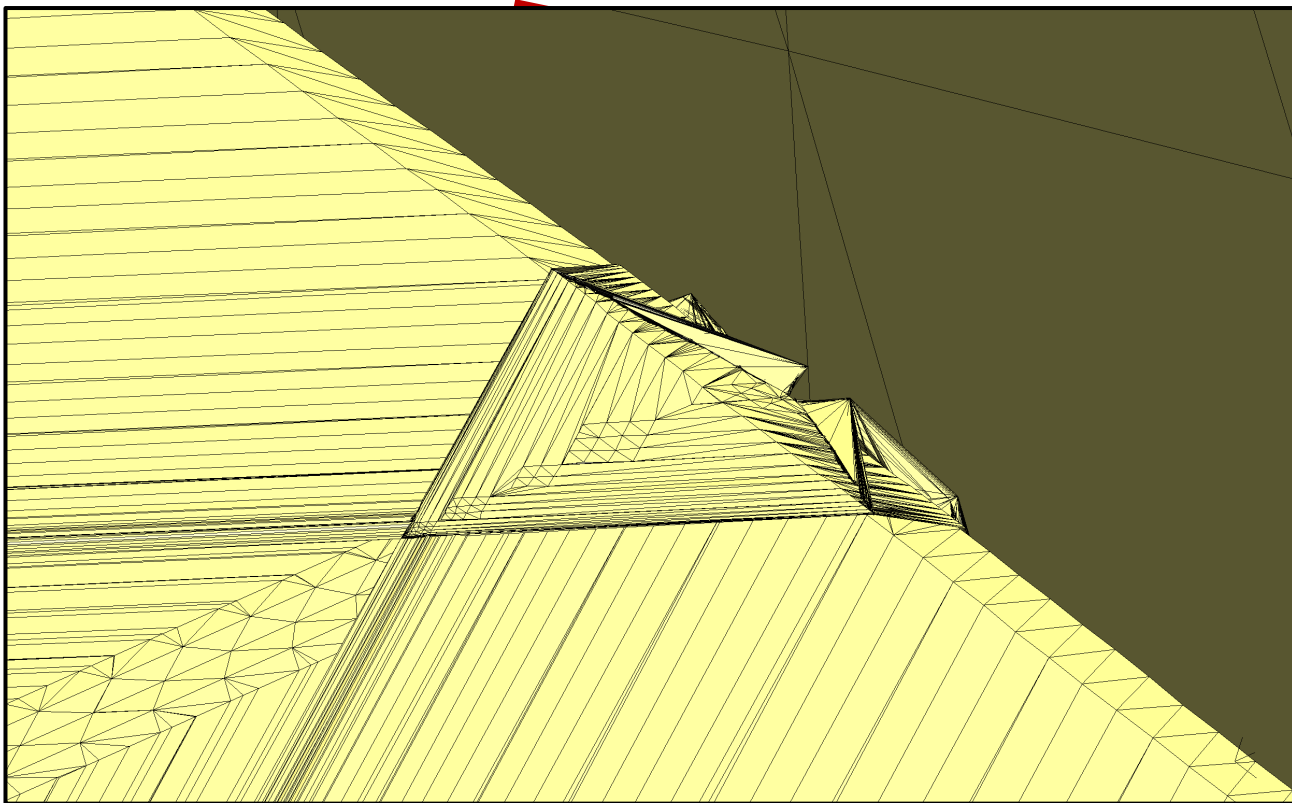
Physical Simulation

Why is tetrahedral meshing hard?



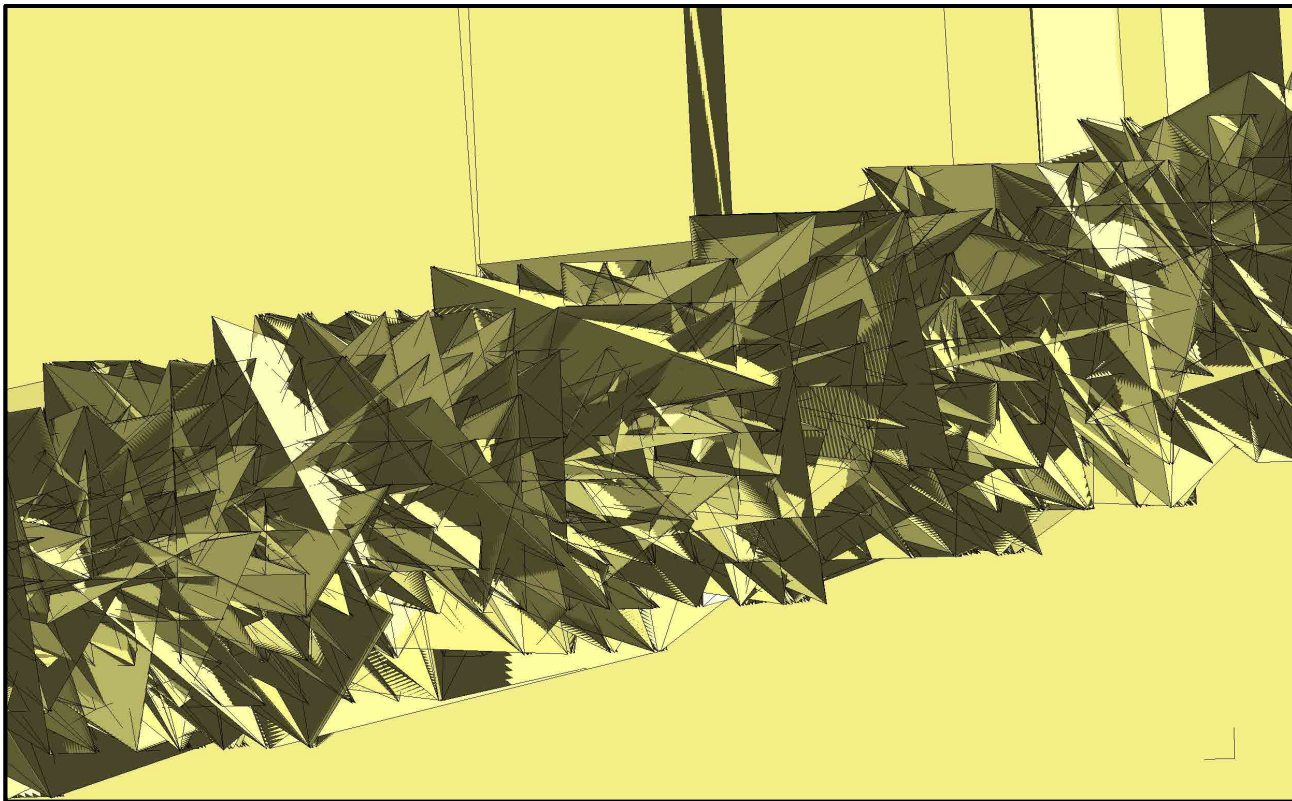
"Tetrahedral Meshing in the Wild" [Hu, Zhou, Gao, J., Zorin, Panozzo 2018]

Why is tetrahedral meshing hard?



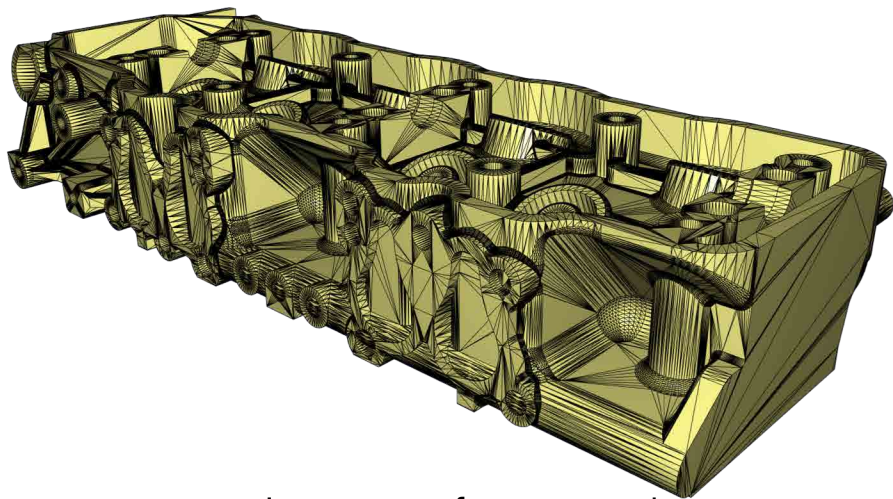
"Tetrahedral Meshing in the Wild" [Hu, Zhou, Gao, J., Zorin, Panozzo 2018]

Why is it a Hard Problem?



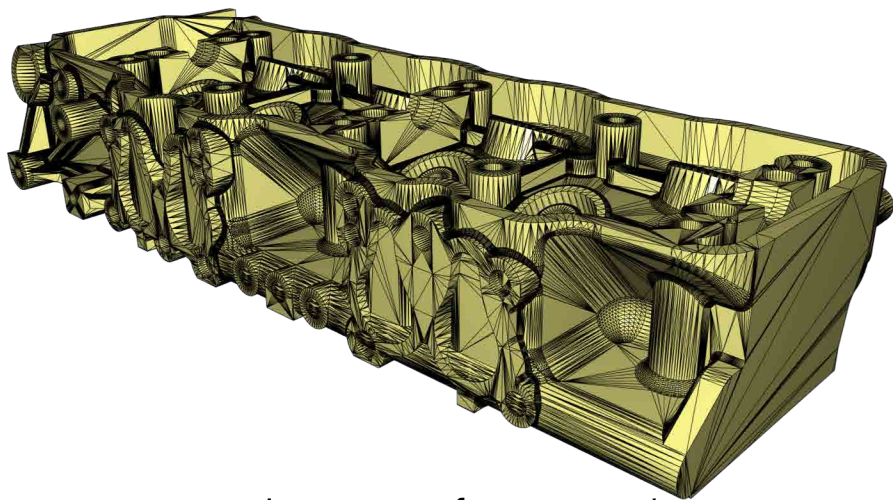
"Tetrahedral Meshing in the Wild" [Hu, Zhou, Gao, J., Zorin, Panozzo 2018]

Winding number and small *epsilon* protect against nasty faceted CAD models

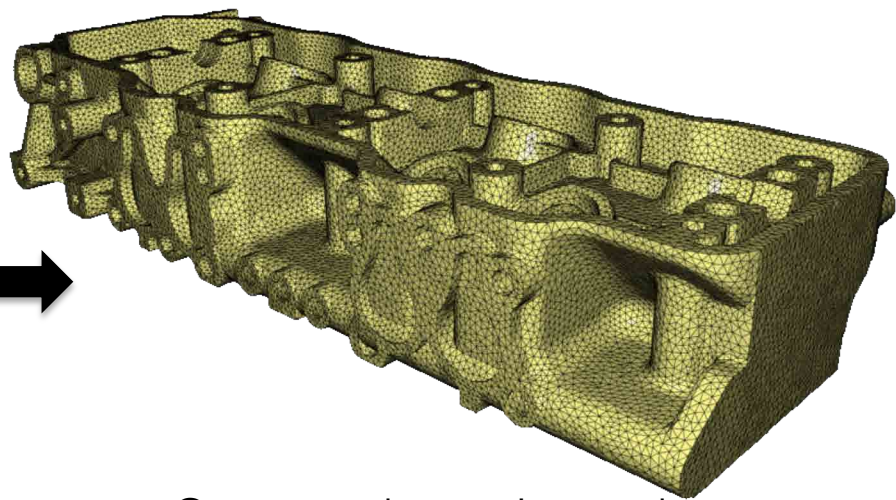


Input surface mesh

Winding number and small *epsilon* protect against nasty faceted CAD models

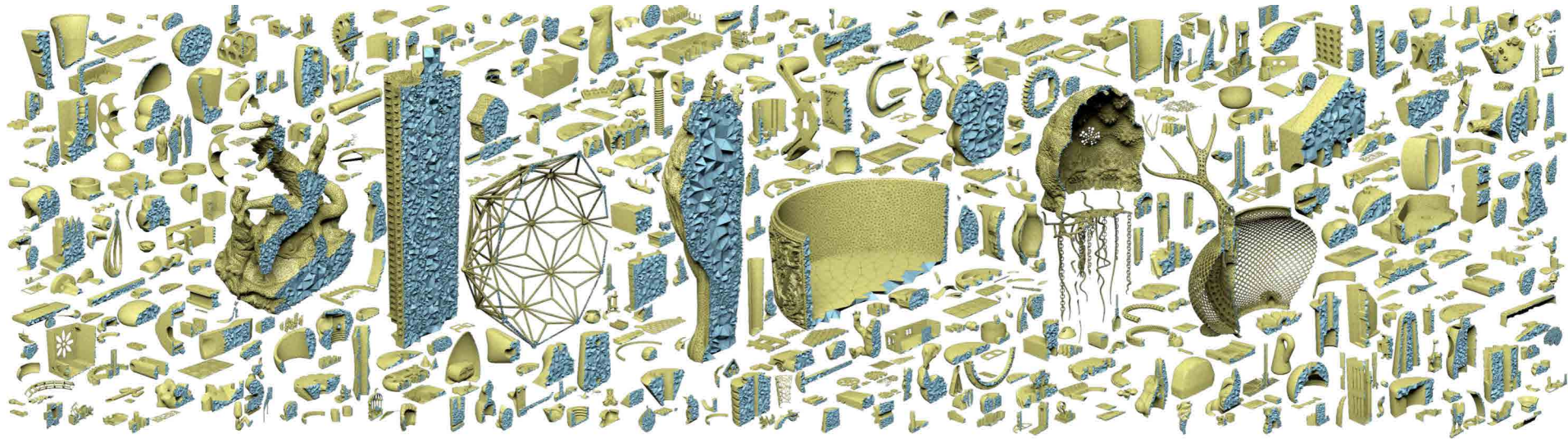


Input surface mesh



Output volumetric mesh

Validate in the wild on 10,000 models



mesh entire convex hull
conform to surface up to small *epsilon*
extract interior via generalized winding number

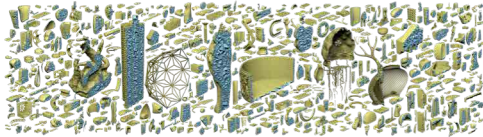
We're dedicated to open software...



libigl

c++ geometry processing library

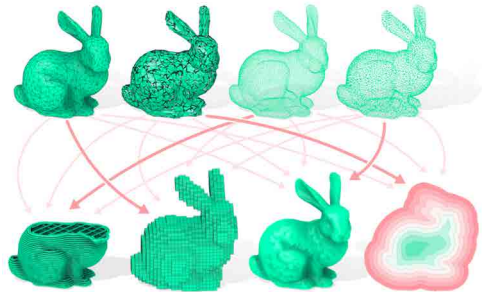
<https://github.com/libigl/libigl>



tetwild

c++ tetrahedral meshing

<https://github.com/Yixin-Hu/TetWild>

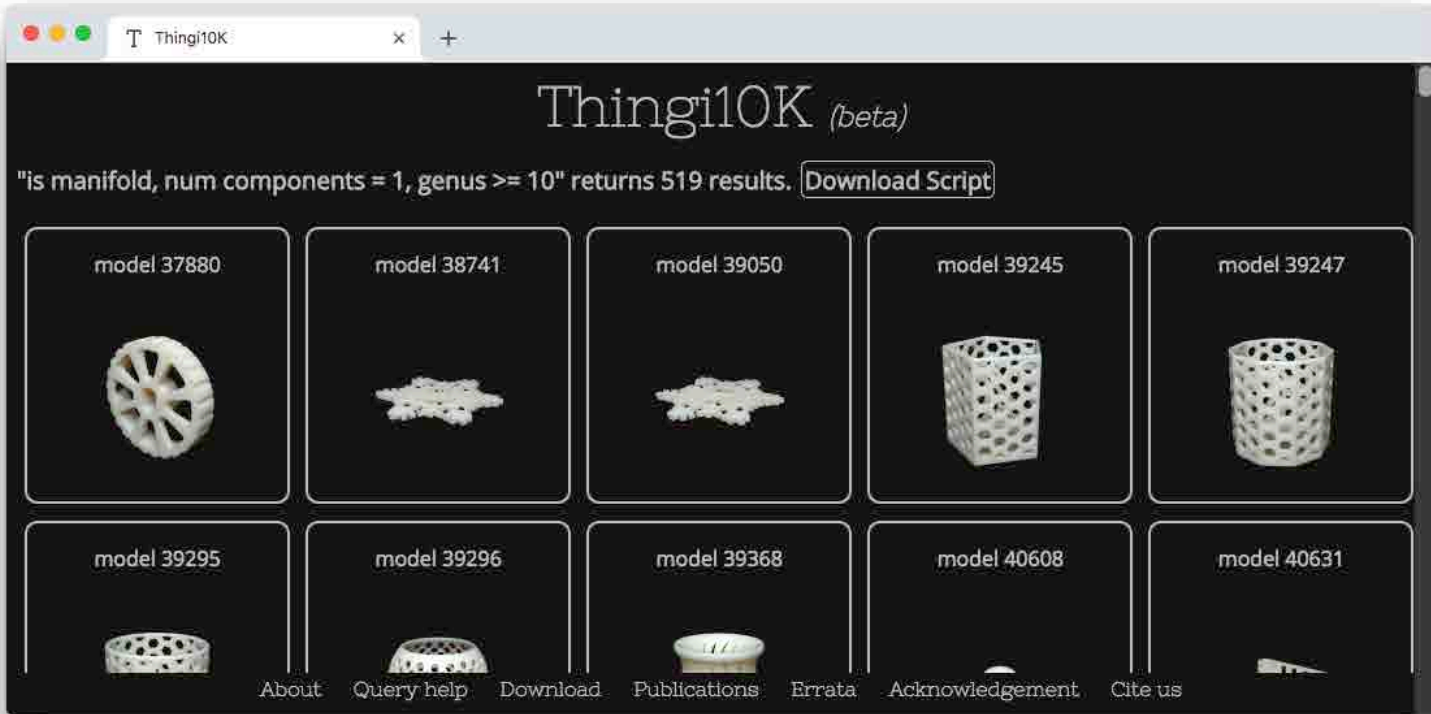


fast winding number

c++ robust inside/outside testing, voxelization

<https://github.com/GavinBarill/fast-winding-number-soups>

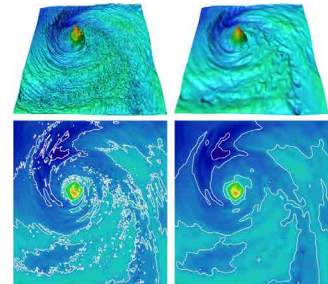
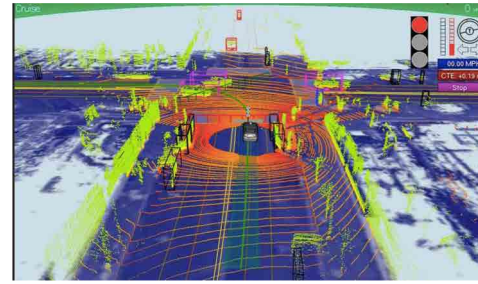
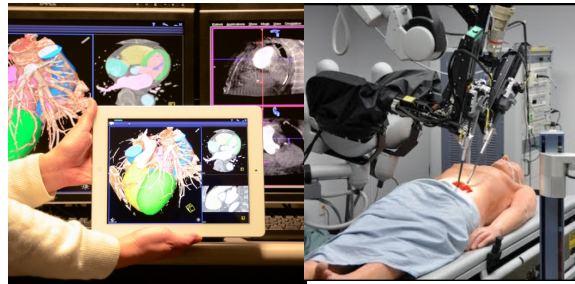
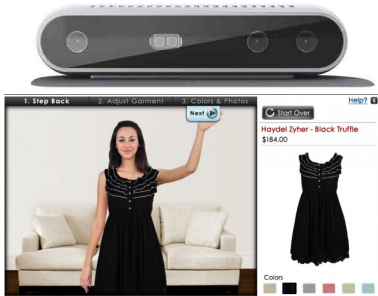
...and open data



thingi10k

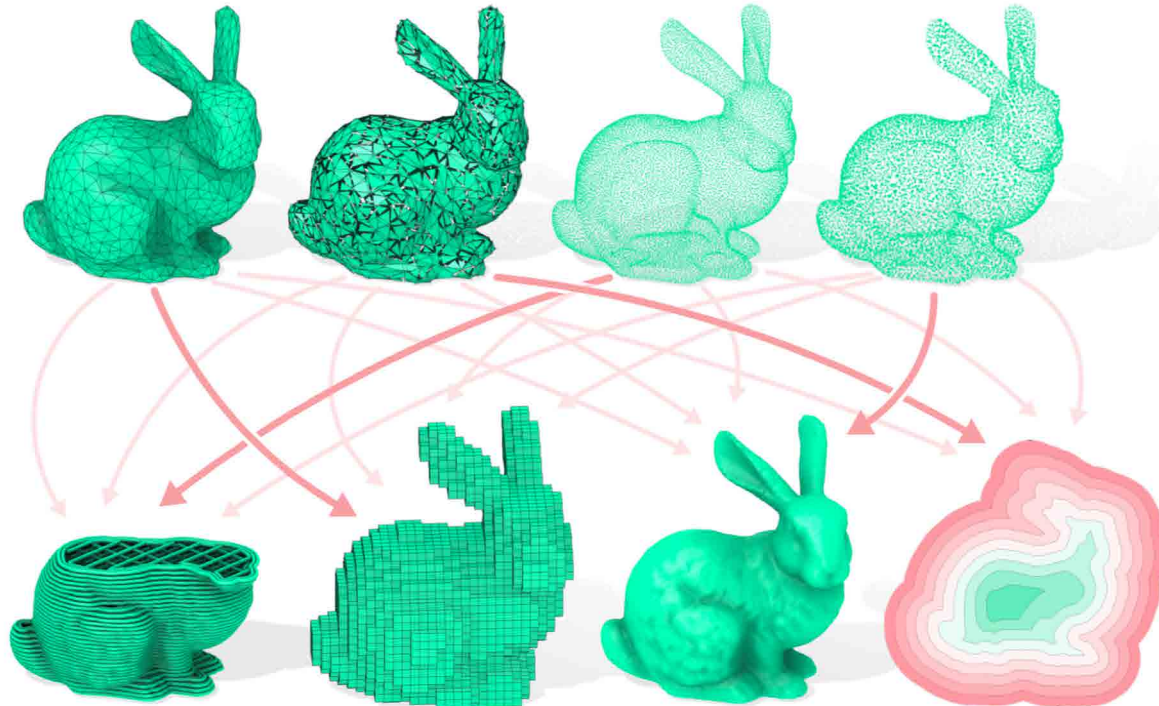
<http://ten-thousand-models.appspot.com>

Possibilities are everywhere



Concluding thoughts

Question common assumptions

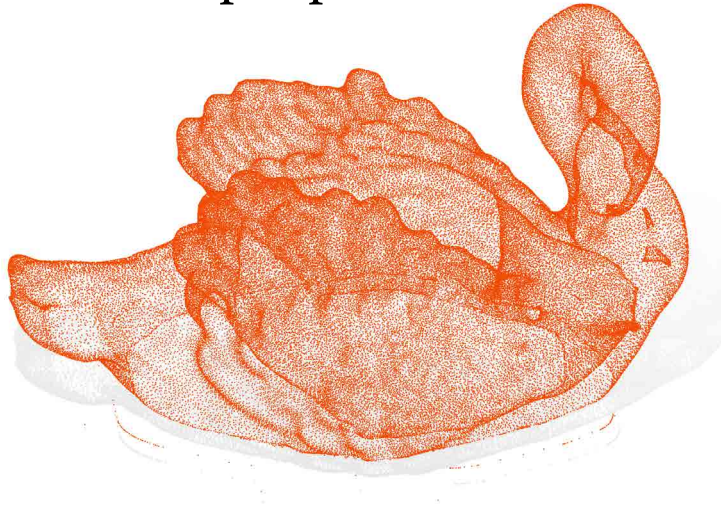


Concluding thoughts

Question common assumptions

Reach across entire pipeline

input point cloud



3D printed point cloud

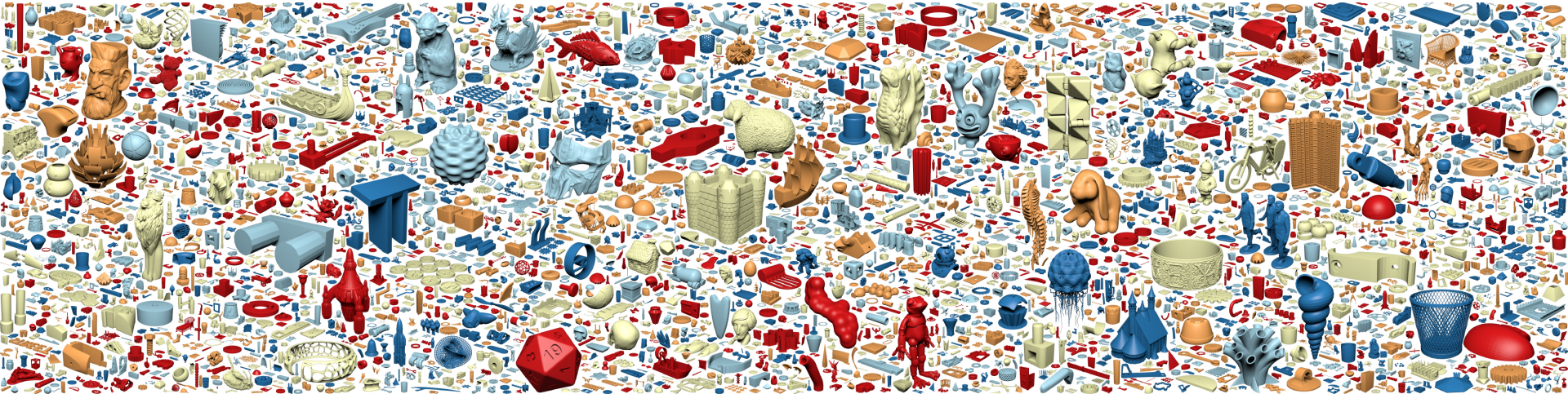


Concluding thoughts

Question common assumptions

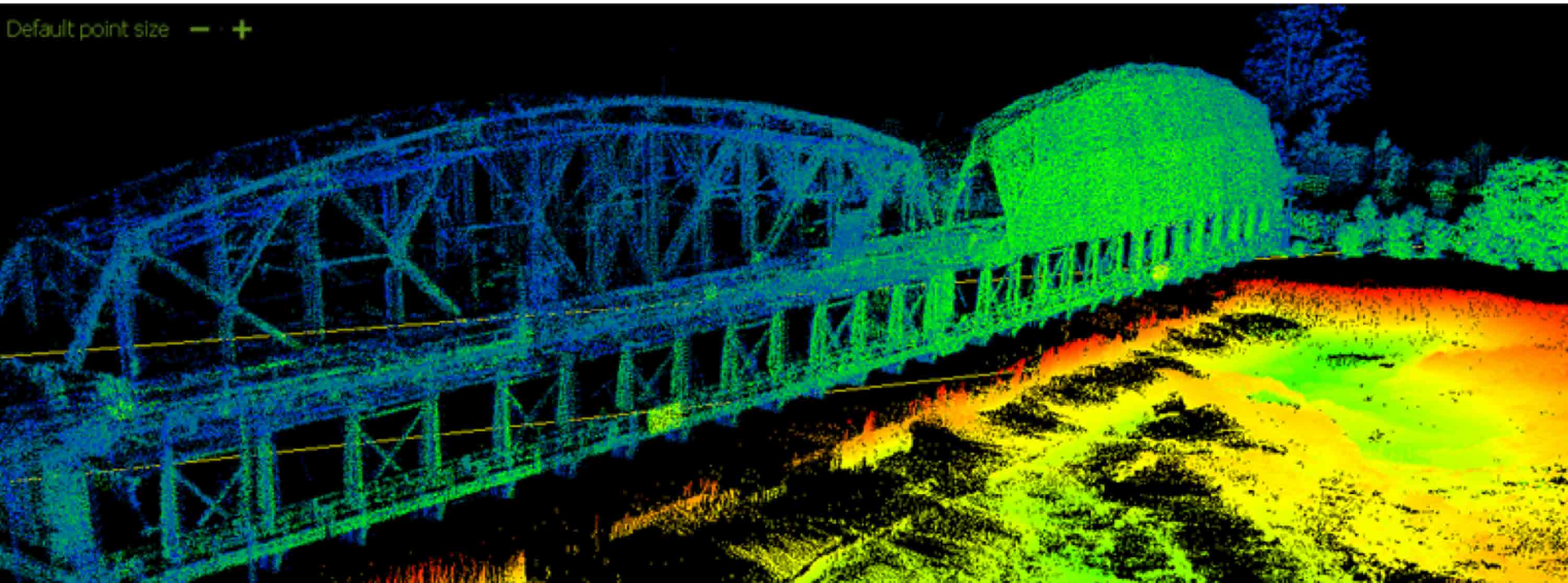
Reach across entire pipeline

Large-scale validation



Future directions

“Full stack” geometry processing

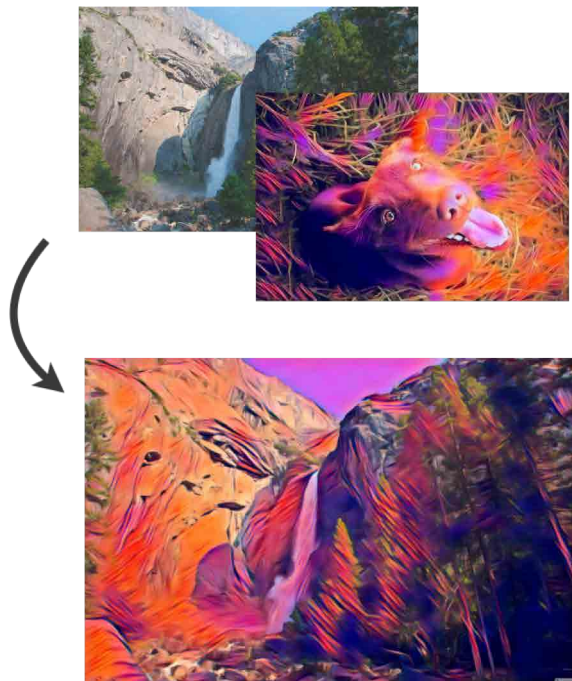


Future directions

Geometric learning *beyond* classification

Future directions

Geometric learning *beyond* classification



Future directions

Geometric learning *beyond* classification



Acknowledgments...

Chun-Liang Li, Daniele Panozzo, David Levin, Denis Zorin,
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Leonardo Sacht, Michael Tao, Mitchell Dembowski,
Neil Dickson, Olga Sorkine-Hornung, Qingnan Zhou,
Rinat Abdrashitov, Ryan Schmidt, Silvia Sellán, Xifeng Gao,
Yixin Hu, Yuming Ma

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Autodesk, Adobe, MESH Consultants

Geometry Processing in the Wild

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Canada Research Chair

University of Toronto

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