Human Motion Analysis
Lecture 10: Physics

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Materials used for this lecture

- The physics slides come from the fantastic tutorial that M. Brubaker, L. Sigal and D. Fleet gave at ICCV 2009.

Contents of today's lecture?

We will look into generative approaches to pose estimation. We will focus on:

- physical priors

The problem of human pose estimation

- The goal is given an image $\mathbf{I}$ to estimate the 3D location and orientation of the body parts $\mathbf{y}$.

Pose estimation

- **Generative approaches:** focus on modeling

$$p(\phi|\mathbf{I}) = \frac{p(\mathbf{I}|\phi)p(\phi)}{p(\mathbf{I})}$$

- **Discriminative approaches:** focus on modeling directly

$$p(\phi|\mathbf{I})$$

Today we will talk about generative approaches.
Later in the class we will cover discriminative approaches.

Generative approaches

Generative approach models

$$p(\phi|\mathbf{I}) = \frac{p(\mathbf{I}|\phi)p(\phi)}{p(\mathbf{I})}$$

Types of generative approaches:

- **Bayesian approaches:** focus on approximating $p(\phi|\mathbf{I})$, usually via sampling (e.g., particle filter).
- **Optimization or energy-based techniques:** focus on computing the MAP or ML estimate of $p(\phi|\mathbf{I})$.

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Common to all of them is the need to model

- **Image likelihood:** $p(\mathbf{I}|\phi)$
- **Priors:** $p(\phi)$

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In general $p(\mathbf{I})$ is assumed constant and ignored. The different trackers then depend on the different modeling choices and optimization procedures.

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Priors $p(\phi)$

- Joint limits
- Shape priors
- Pose priors
- Dynamical priors
- Physics

Why physics-based models?

Three main reasons:

- Ensure physically plausible tracking results
- Reduce dependence of models on mocap (generalization)
- Incorporate interactions into tracking formulation

Physics-based models in several related fields provide ideas for models suitable for human pose tracking and analysis in vision.

Robotics

- Control of machines that interact with unpredictable environments.

Figure: (Brubaker et al. 09)

Humanoid Robotics

- Machines that move like people and perform everyday tasks:

Figure: Honda's Asimo Robot (Brubaker et al. 09)

Humanoid Robotics

- Robots typically have different mass and inertial parameters, torque limits, stability criteria, etc, but approaches to control may be useful

Figure: HRP2 performing a Japanese folk dance (Nakaoka et al. 2004)

- Study of human morphology, kinematics and kinetics.
- Knowledge of musculature, kinematics and kinetics of locomotion,balance recovery etc. But the neural basis for motor control and the principles underlying human motion less well understood.

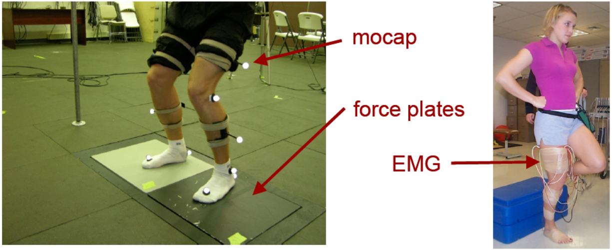

Figure: (Brubaker et al. 09)

Computer graphics: Space-time optimization

- Search for physically plausible motions satisfying user-specified constraints (e.g., foot placements) often while minimizing energy

Figure: [K. Liu and Z. Popovic, Siggraph 2005]

Computer graphics: Controllers

- Compact representations for repetitive motions that are reactive and robust to environmental perturbation

Figure: [Muico et al., Siggraph 2009]

Computer graphics: Interactions

- Constrained space-time optimization techniques for handling dynamic interactions with the environment (batch or on-line)

Figure: [Jain et al., TOG 2009]

Physics-based models in computer vision[

- Dynamics: upper body only, no contact
- Observations: 3D markers, stereo cameras
- Inference: Kalman filter

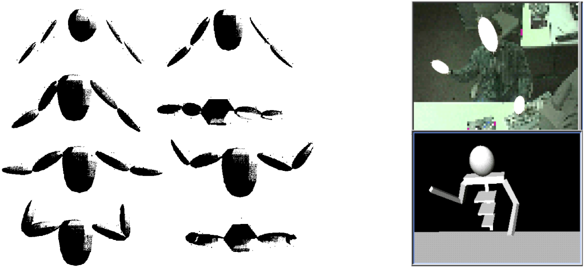

Figure: (left) Metaxas and Terzopoulos,93, (right) (Wren and Pentland 98)

Physics-based models for pose tracking

- Low-dimensional, biomechanically inspired models of locomotion

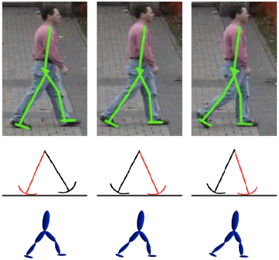

Figure: (Brubaker et al. 07, 08)

Physics-based models for pose tracking

- Mocap-driven full-body control for pose tracking

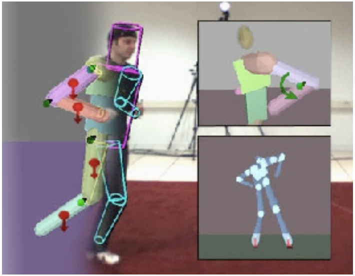

Figure: (Vondrak et al. 08)

Classical and rigid body mechanics

- We will focus in this class in the rigid body mechanics

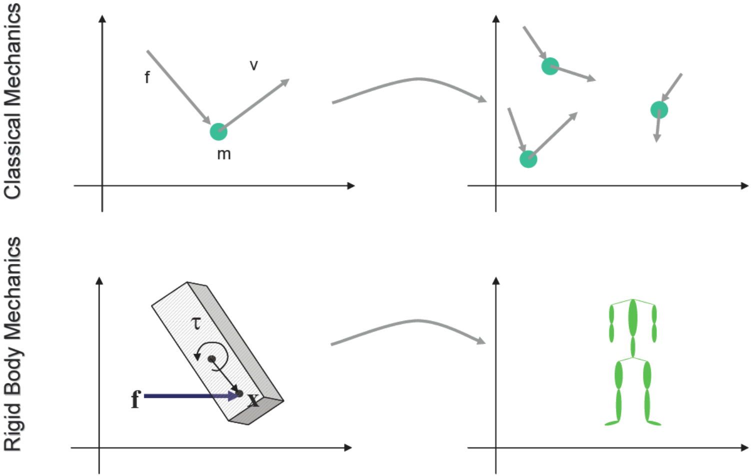

Figure: (Brubaker et al. 09)

Pose of a rigid body

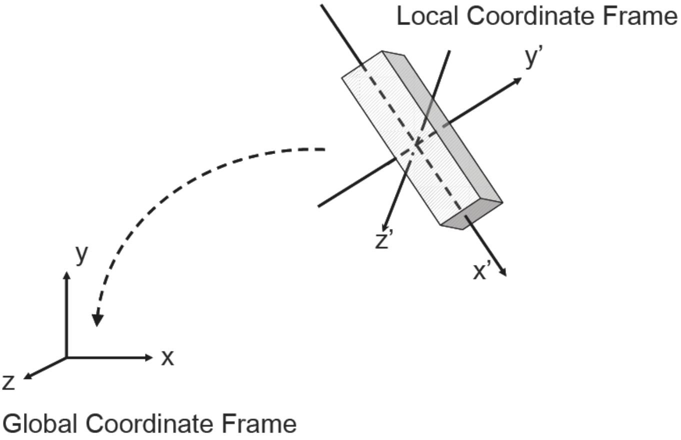

Figure: (Brubaker et al. 09)

Pose of a rigid body

- Pose is the rigid transformation from a local coordinate frame to a global coordinate frame

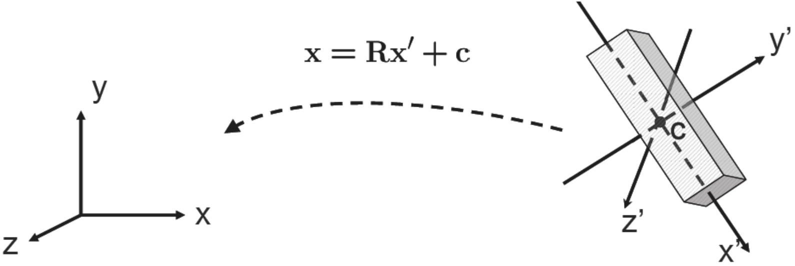

Figure: (Brubaker et al. 09)

Rigid-Body Mechanics: Velocity

- Mechanics model **motion**: linear and angular velocity

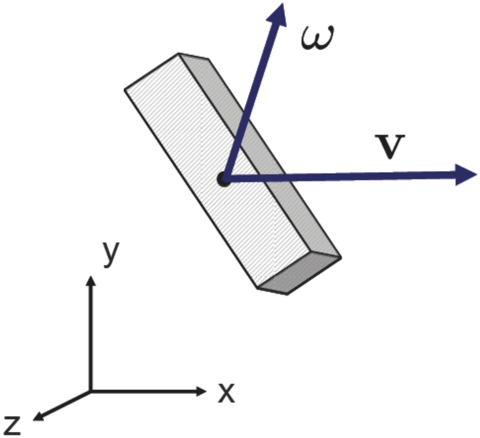

Figure: (Brubaker et al. 09)

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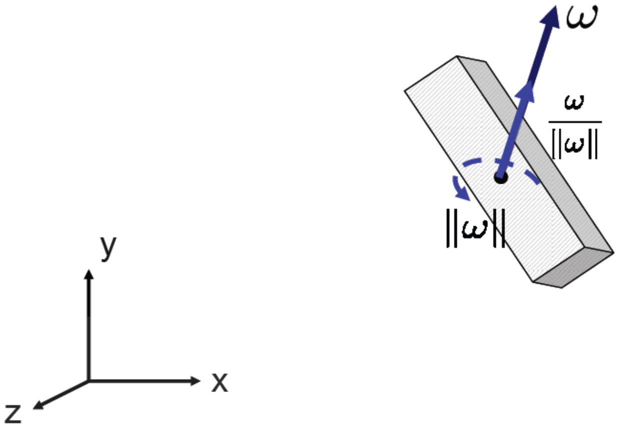


Figure: (Brubaker et al. 09)

Rigid-Body Mechanics: Mass

- **Mass** is defined in terms of a mass density function $\rho(\mathbf{x})$
- **Total mass**: zero-order moment

$$m = \int \rho(\mathbf{x}) d\mathbf{x}$$

- **Center of mass**: proportional to the first-order moment

$$\mathbf{c} = m^{-1} \int \mathbf{x} \rho(\mathbf{x}) d\mathbf{x}$$

Rigid-Body Mechanics: Momentum and Inertia

- **Linear momentum:**

$$\mathbf{p} = m\mathbf{v}$$

  with m the mass and $\mathbf{v}$ the linear velocity.

- **Angular momentum:** depends on distributionof mass

$$\ell = \mathbf{I}\boldsymbol{\omega}$$

  with $\mathbf{I}$ the inertia tensor and $\boldsymbol{\omega}$ the angular velocity

Rigid-Body Mechanics: Inertia

- The **Inertia Tensor** about the Center of Mass is a real, symmetric matrix summarizing the distribution of mass about the center of mass

$$\mathbf{I} = \begin{pmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{pmatrix}$$

  with diagonal entries called the **moments of inertia**, and off diagonals call the **products of inertia**.

- It can be computed as a second moment of the mass density function

$$\mathbf{I} = \int \left(||\mathbf{r}||^2 \mathbf{E}_{3\times3} - \mathbf{r}\mathbf{r}^T \right) \rho(\mathbf{x}) d\mathbf{x}$$

  with

$$\mathbf{r} = \mathbf{x} - \mathbf{c}$$

  and $\mathbf{E}_{3\times3}$ the identity matrix.

Rigid-Body Mechanics: Inertia in local coordinates

- The Inertia Tensor about the Center of Mass in the local coordinate frame is defined by the eigenvectors of the inertia tensor $\mathbf{I}$

$$\mathbf{I}' = \begin{pmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{pmatrix}$$

- The diagonals are the **principal moments of inertia** and the corresponding axes are the **principal axes of inertia**.
- When the coordinate system is rotated

$$\mathbf{I} = \mathbf{R}\mathbf{I}'\mathbf{R}^T$$

- We only need to compute the integral once, then we can just rotate it.

Pose of a rigid body

- The pose of a rigid body specifies the rigid transformation from the body frame to the world frame
- The body frame has its origin at the center of mass and its axes aligned with the principal axes of inertia

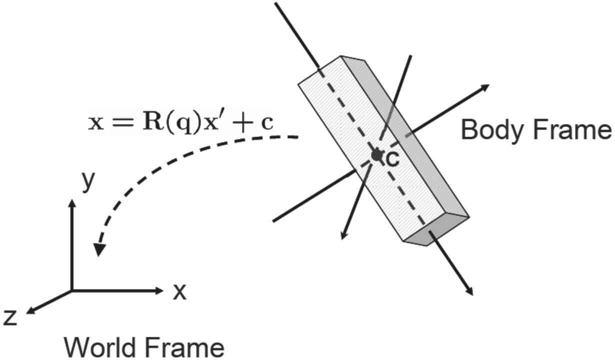


Figure: (Brubaker et al. 09)

Rigid-Body Mechanics: Equations of Motion

- **Newton's Second Law of Motion:** In a static frame of reference, the time derivative of momentum is force

$$\mathbf{f} = \dot{\mathbf{p}} = m\dot{\mathbf{v}}$$

- The inertia tensor is however not constant in a static frame

$$\tau = \dot{\ell} = \mathbf{I}\dot{\boldsymbol{\omega}} + \dot{\mathbf{I}}\boldsymbol{\omega}$$

- The Newton-Euler equations are for motion measured in a motionless frame of reference (i.e., the world frame).
- It is useful to consider rotational motion in the body frame

$$\mathbf{I}'\dot{\boldsymbol{\omega}}' = \tau' - \boldsymbol{\omega}' \times (\mathbf{I}'\boldsymbol{\omega}')$$

- Since the inertia tensor is constant and diagonal this equation is easier to work with.

Rigid-Body Mechanics: Force and torque

- Forces (torques) in equations of motion are applied on (about) the center of mass of the object.
- Force applied at a point results in both a force on the center of mass and a torque about the center of mass
- Force is linear, so the net of all forces on a body can be summarized by a force on the center of mass and a torque about the center of mass

$$\tau = \mathbf{f} \times (\mathbf{x} - \mathbf{c})$$

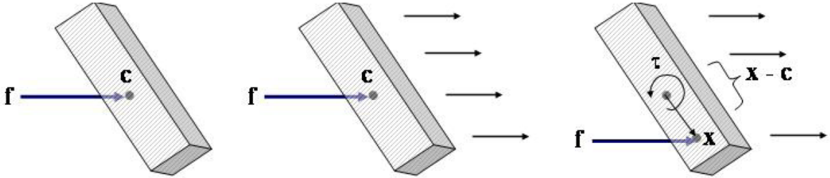


Figure: (Brubaker et al. 09)

Rigid-Body Mechanics: angular velocity and quaternions

- Relating Angular Velocity to Quaternions

$$
\begin{aligned}
\dot{\mathbf{q}} &= \frac{1}{2}\begin{pmatrix} 0 \\ \boldsymbol{\omega} \end{pmatrix} \circ \mathbf{q} \\
&= \frac{1}{2}\mathbf{q} \circ \begin{pmatrix} 0 \\ \boldsymbol{\omega}' \end{pmatrix}
\end{aligned}
$$

with $\mathbf{q}$ a quaternion, $\boldsymbol{\omega} = \mathbf{I}^{-1}\ell$, $\boldsymbol{\omega}' = \mathbf{I}^{-1}\ell'$ and where the product is defined as

$$
\mathbf{q}_0 \circ \mathbf{q}_1 = \begin{pmatrix} v_0 v_1 - \mathbf{u}_0 \mathbf{u}_1 \\ v_0 \mathbf{u}_1 + v_1 \mathbf{u}_0 + \mathbf{u}_0 \times \mathbf{u}_1 \end{pmatrix}
$$

with quaternions

$$
\mathbf{q}_0 = \begin{pmatrix} v_0 \\ \mathbf{u}_0 \end{pmatrix} \qquad \mathbf{q}_1 = \begin{pmatrix} v_1 \\ \mathbf{u}_1 \end{pmatrix}
$$

Rigid-Body Mechanics: Simulation

- To simulate, we define the state vector and it's derivatives

$$\phi = \begin{pmatrix} \mathbf{c} \\ \mathbf{q} \\ \mathbf{v} \\ \omega' \end{pmatrix} \qquad \dot{\phi} = \begin{pmatrix} \mathbf{v} \\ \frac{1}{2}\mathbf{q} \circ \begin{pmatrix} 0 \\ \omega' \end{pmatrix} \\ m^{-1}\mathbf{f} \\ \mathbf{I}'^{-1}(\tau' - \omega' \times (\mathbf{I}'\omega')) \end{pmatrix}$$

- Simulation is done by integration, for example by using a first order

$$\phi(t + \Delta t) = \phi(t) + \Delta t \dot{\phi}(t)$$

- This is simple, fast and easy to implement but inaccurate.
- Special care has to be taken with $\mathbf{q}$ so that $||\mathbf{q}||_2 = 1$. Use a projection step.

Constrained Dynamics

Sources of Constraints:

- distance and orientation between parts (e.g., joints)
- parameter constraints (e.g., unit norm quaternions)
- limited range of motion (e.g., joint limits)
- interpenetration constraints (e.g., ground contact, self intersection)

Types of explicit constraints

$$e(\mathbf{x}) = 0$$

$$e(\mathbf{x}) \leq 0$$

We will look into equality constraints.

Constrained Dynamics: Explicit Constraints

- State $\mathbf{x}$ is the position of the ball
- Constraint is similar to the constraints in quaternions

$$e(\mathbf{x}) = \mathbf{x}^T\mathbf{x} - 1 = 0$$

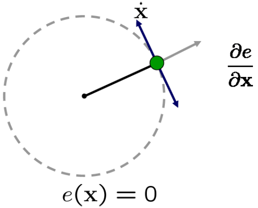
- Admissible state derivatives

$$\dot{e}(\mathbf{x}) = \frac{\partial e}{\partial \mathbf{x}}\dot{\mathbf{x}} = 0 \qquad \ddot{e}(\mathbf{x}) = \frac{\partial \dot{e}}{\partial \mathbf{x}}\dot{\mathbf{x}} + \frac{\partial e}{\partial \mathbf{x}}\ddot{\mathbf{x}} = 0$$

- Equations of motion

$$m\ddot{\mathbf{x}} = \mathbf{f} + \mathbf{f}_c$$

with $\mathbf{f}$ the external forces and $\mathbf{f}_c$ the constraint forces.



Constrained Dynamics: Principle of Virtual Work

- Constraint forces must do no work for every admissible velocity

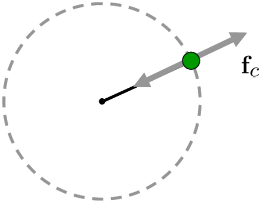
$$\delta W = \mathbf{f}_c^T \dot{\mathbf{x}} = 0 \quad \forall \dot{\mathbf{x}}, \quad \frac{\partial e}{\partial \mathbf{x}} \dot{\mathbf{x}} = 0$$

- Constraint force is proportional to the constraint gradient

$$\mathbf{f}_c = \lambda \frac{\partial e}{\partial \mathbf{x}}$$

- Adding constraint on accelerations gives the augmented equations of motion

$$\begin{pmatrix} m\mathbf{E} & \frac{\partial e}{\partial \mathbf{x}}^T \\ \frac{\partial e}{\partial \mathbf{x}} & 0 \end{pmatrix} \begin{pmatrix} \ddot{\mathbf{x}} \\ \lambda \end{pmatrix} = \begin{pmatrix} \mathbf{f} \\ -\frac{\partial \dot{e}}{\partial \mathbf{x}} \dot{\mathbf{x}} \end{pmatrix}$$



Constrained Dynamics: General constraints

- For a system with equations of motion

$$\mathbf{M}(\mathbf{x})\ddot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \dot{\mathbf{x}}) + \mathbf{f}_c$$

- With N constraints

$$\mathbf{e}(\mathbf{x}) = \begin{pmatrix} e_1(\mathbf{x}) \\ \cdots \\ e_N(\mathbf{x}) \end{pmatrix}$$

the augmented equations of motion are

$$\begin{pmatrix} \mathbf{M}(\mathbf{x}) & \frac{\partial \mathbf{e}}{\partial \mathbf{x}}^T \\ \frac{\partial \mathbf{e}}{\partial \mathbf{x}} & 0 \end{pmatrix} \begin{pmatrix} \ddot{\mathbf{x}} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} \mathbf{f}(\mathbf{x}, \dot{\mathbf{x}}) \\ -\frac{\partial \dot{\mathbf{e}}}{\partial \mathbf{x}} \dot{\mathbf{x}} \end{pmatrix}$$

- E.g., the quaternion equation of motion

$$\begin{pmatrix} 4\mathbf{Q}\mathbf{J}\mathbf{Q}^T & 2\mathbf{q} \\ 2\mathbf{q}^T & 0 \end{pmatrix} \begin{pmatrix} \ddot{\mathbf{q}} \\ \lambda \end{pmatrix} = \begin{pmatrix} 2\mathbf{Q} \begin{pmatrix} 0 \\ \tau' \end{pmatrix} + 8\mathbf{Q}\mathbf{J}\mathbf{Q}^T\mathbf{q} \\ -2||\mathbf{q}||_2^2 \end{pmatrix}$$

where $\mathbf{J} = \begin{pmatrix} 0 & 0 \\ 0 & \mathbf{I}' \end{pmatrix}$

Constrained Dynamics: General constraints

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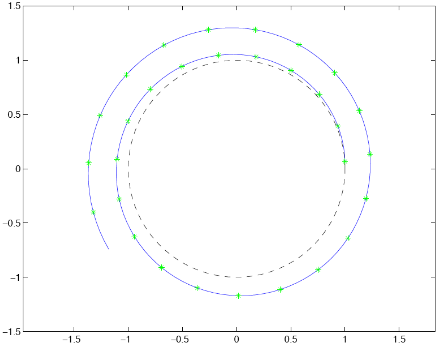
Problems with explicit constraint enforcement

- When integrated numerically, constraints drift
- Dimensionality of state is much larger than effective dimension of constrained space
- Bad for articulation constraints



Constrained Dynamics: Generalized Coordinates

- A set of coordinates $\mathbf{u}$ which exactly specify the state of a constrained system $\mathbf{x}(\mathbf{u})$
- Admissible velocities and accelerations

$$\begin{aligned}\dot{\mathbf{x}} &= \frac{\partial \mathbf{x}}{\partial t}\frac{\partial \mathbf{u}}{\partial \mathbf{u}} = \frac{\partial \mathbf{x}}{\partial \mathbf{u}}\dot{\mathbf{u}} = \mathbf{T}(\mathbf{u})\dot{\mathbf{u}} \\ \ddot{\mathbf{x}} &= \mathbf{T}(\mathbf{u})\ddot{\mathbf{u}} + \dot{\mathbf{T}}(\mathbf{u},\dot{\mathbf{u}})\dot{\mathbf{u}}\end{aligned}$$

- Equations of motion

$$\mathbf{M}(\mathbf{x})\ddot{\mathbf{x}} = \mathbf{f}(\mathbf{x},\dot{\mathbf{x}}) + \mathbf{f}_c$$

Principle of Virtual Work for Generalized Coordinates

- Constraint forces must do no work for every admissible velocity

$$\begin{aligned} \delta W &= \mathbf{f}_c^T \dot{\mathbf{x}} = 0 \\ &= \mathbf{f}_c^T \mathbf{T}(\mathbf{u}) \dot{\mathbf{u}} = 0 \qquad \forall \dot{\mathbf{u}} \\ &\Longrightarrow \mathbf{f}_c^T \mathbf{T}(\mathbf{u}) = 0 \end{aligned}$$

- Premultiply equations of motion

$$\mathbf{T}(\mathbf{u})^T \mathbf{M}(\mathbf{x}) \ddot{\mathbf{x}} = \mathbf{T}(\mathbf{u})^T \mathbf{f}(\mathbf{x}, \dot{\mathbf{x}}) + \mathbf{T}(\mathbf{u})^T \mathbf{f}_c = \mathbf{T}(\mathbf{u})^T \mathbf{f}(\mathbf{x}, \dot{\mathbf{x}})$$

- Substitute in the generalized coordinates

$$\mathbf{T}(\mathbf{u})^T \mathbf{M}(\mathbf{x}) \mathbf{T}(\mathbf{u}) \ddot{\mathbf{u}} = \mathbf{T}(\mathbf{u})^T \left(\mathbf{f}(\mathbf{u}, \dot{\mathbf{u}}) - \mathbf{M}(\mathbf{u}) \dot{\mathbf{T}}(\mathbf{u}, \dot{\mathbf{u}}) \dot{\mathbf{u}} \right)$$

- More compactly

$$\mathcal{M}(\mathbf{u}) \ddot{\mathbf{u}} = \mathbf{T}(\mathbf{u})^T \mathbf{f}(\mathbf{u}, \dot{\mathbf{u}}) + g(\mathbf{u}, \dot{\mathbf{u}})$$

Adding more constraints

- Other explicit constraints can still be used with generalized coordinates

$$\begin{pmatrix} \mathcal{M}(\mathbf{u}) & \frac{\partial \mathbf{e}}{\partial \mathbf{u}}^T \\ \frac{\partial \mathbf{e}}{\partial \mathbf{u}} & 0 \end{pmatrix} \begin{pmatrix} \ddot{\mathbf{u}} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} \mathbf{T}(\mathbf{u})^T \mathbf{f}(\mathbf{u}, \dot{\mathbf{u}}) + g(\mathbf{u}, \dot{\mathbf{u}}) \\ -\frac{\partial \dot{\mathbf{e}}}{\partial \mathbf{u}} \dot{\mathbf{u}} \end{pmatrix}$$

- This is particularly useful for quaternions and transient attachment constraints

Dynamics of Articulated Rigid Bodies

Equations of Motion for Articulated Rigid Bodies

1. Define parts
2. Define generalized coordinates
3. Define other constraints
4. Differentiate kinematic transformation and constraints

Defining parts

For each part i we need to define

- Mass m_i
- Inertia tensor in the body frame $\mathbf{I}'_i$
- Body frame (i.e., center of mass $\mathbf{c}_i$ and principal axes)

The biomechanics literature provides this information

Defining generalize coordinates and constraints

To define the generalized constraints

- Select a root node (e.g., the pelvis)
- Define the joints of the body
- Generalized coordinates $\mathbf{u}$ are: the pose of the root node plus joint angles
- Define the kinematic transform $\mathbf{x}(\mathbf{u})$ from the generalized coordinates $\mathbf{u}$ to the pose of each body part

Define Constraints:

- Quaternion norm constraint for joint angles
- Attachment constraints (e.g., hands or feet)

Differentiate constraints and transformations

- Compute the partial derivatives

$$\begin{aligned} \mathbf{T}(\mathbf{u}) &= \frac{\partial \mathbf{x}}{\partial \mathbf{u}} \\ \dot{\mathbf{T}}(\mathbf{u}, \dot{\mathbf{u}}) &= \sum_i \frac{\partial^2 \mathbf{x}}{\partial \mathbf{u} \partial u_i} \dot{u}_i \end{aligned} \qquad \begin{aligned} &\frac{\partial \mathbf{e}}{\partial \mathbf{u}} \\ \frac{\partial \dot{\mathbf{e}}}{\partial \mathbf{u}} &= \sum_i \frac{\partial^2 \mathbf{x}}{\partial \mathbf{u} \partial u_i} \dot{u}_i \end{aligned}$$

- Now we can do simulation!
- Brubaker et al. 09 provide code to do this simulation.

Dynamics and sources of force

- Kinematics and anthropometrics describe how the structure and geometry of the body are attached.
- Dynamics uses both to describe how the body responds to applied forces.
- Motion is the result of momentum and, more importantly, the forces acting on the system
- Sources of force:
  - **Gravity:** the force of the Earth on other bodies, is not a force in that its effect on an object does not depend on the mass of an object.
  - **Skeletal muscles** are tissues which connect bones and are able to voluntarily contract and relax, inducing forces between parts of the body and, therefore, producing motion.
  - **Contact:** Ground contact is critical in the motion of any articulated object. With only muscles or joint torques the global position and orientation of the body is underactuated. It provides the fundamental mechanism for describing interactions with the world.

Models of locomotion I

Despite hundreds of years of research, the means by which humans walk and run are still not fully understood. Models have focus on:

- **Energetic Efficiency:** biological motion is efficient,
    - e.g., when walking at a given speed, people choose a step-length which minimizes metabolic energy.
    - In computer animation, the magnitude of joint torques is often used to measure the desirability of motions when designing controllers or entire motions.
    - However, the magnitude of joint torques is widely understood as a poor substitute for the energy expended.
    - Joint torques are the net result of a variety of phenomena it is impossible to distinguish the passive elements (e.g., tendons and ligaments) from the energy consuming muscle activations

Models of locomotion II

- **Angular Momentum:** minimize angular momentum
  - Herr and Popovic argue that during walking, the angular momentum of the entire body is a regulated quantity which is kept small.
  - This is done, at least in part, by moving the arms in opposite phase with the lower body.
  - However this is not for all motions.

Models of locomotion III

- **Monopode Model**: Blickhan and Full 1993.
  - The gross motion of the body can be primarily understood through by the linear motion of the center of mass.
  - The motion of the center of mass during legged locomotion of animals as diverse as cockroaches and horses can be explained by a point mass attached to a massless, retractable leg.
  - This model, called the monopode, is able to represent both walking and running. Inverted pendular motion.

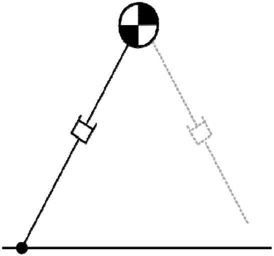


Figure: Monopode (Blickhan and Full 1993)

Models of locomotion IV

- **Passive dynamics:**
  - Energetic efficiency remains one of the primary differences between human locomotion and the motion of humanoid robots
  - Much of human locomotion can be explained through passive dynamics: motion entirely driven through momentum and gravity.
  - This was thoroughly demonstrated by a range of models which were able to walk down hill without active control strategies.
  - An example is the Anthropomorphic Walker

Models of locomotion V

- **Anthropomorphic Walker:**
  - point mass for torso and rigid bodies for legs
  - wide range of cyclic gaits can be found by varying spring stiffness and impulse
  - forces due to torsional spring between legs and an impulsive toe-off
  - Use for tracking by Brubaker et al. CVPR 07.

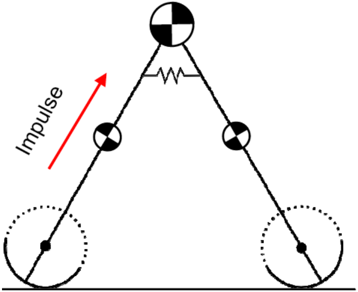


Figure: Anthropomorphic walker (McGeer1990)

Articulated tracking and physics

- Very few methods exist that use physics for constraining tracking.
- This is due to the level of difficulty of getting the equations of motion and simulation to work.
- Here we will focus on one particular example: The Kneed walker by Brubaker and Fleet, 2008.

The Kneed Walker

- A 5 DOF planar model of human locomotion
- Powered by joint torques and an impulsive toe-off.
- Capable of walking on hills or level ground, running,

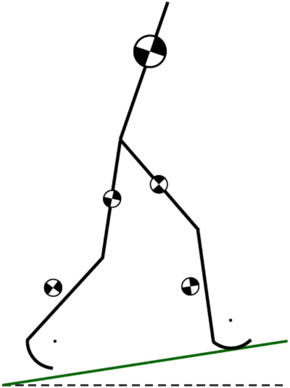


Figure: The kneed walker (Brubaker and Fleet 08)

Kneed Walker: Inertial parameters

- Kneed walker comprises: a torso, two legs (with knees) and rounded feet
- Specified by: part lengths l_i, masses m_i, mass centers c_i, moments of inertia I_i and stance / swing foot.

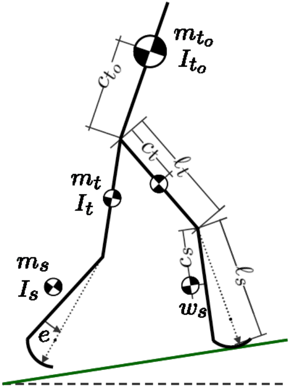


Figure: The kneed walker (Brubaker and Fleet 08)

Kneed Walker: Generalized coordinates

- Model state $\mathbf{x}^T = (\mathbf{q}^T, \dot{\mathbf{q}}^T)$ is given by the part orientations $\mathbf{q}^T = (\phi_{t_0}, \phi_{t_1}, \phi_{t_2}, \phi_{s_1}, \phi_{s_2})$, and their angular velocities $\dot{\mathbf{q}}^T = (\dot{\phi}_{t_0}, \dot{\phi}_{t_1}, \dot{\phi}_{t_2}, \dot{\phi}_{s_1}, \dot{\phi}_{s_2})$

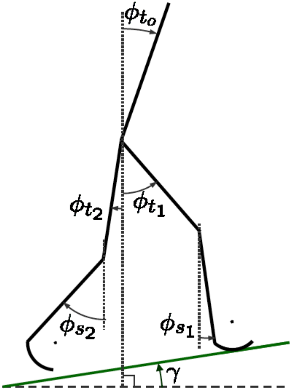


Figure: The kneed walker (Brubaker and Fleet 08)

Kneed Walker: Applied forces I

Dynamics due to

- Joint torques $\tau_{t_0}, \tau_h, \tau_{k_1}, \tau_{k_2}$ (for torso, hip, and knees)
- Impulse applied at toe-off (with magnitude ι)
- Gravitational acceleration (w.r.t. ground slope γ)

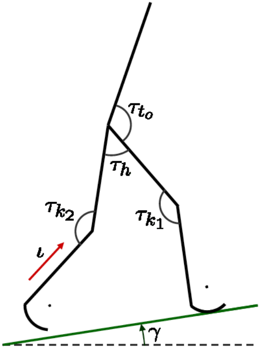


Figure: The kneed walker (Brubaker and Fleet 08)

Kneed Walker: Applied forces II

- Joint torques are parameterized as damped linear springs.
- For hip torque

$$\tau_h = \kappa_h(\phi_{t_2} + \phi_{t_1} - \phi_h) - d_h(\dot{\phi}_{t_2} + \dot{\phi}_{t_1})$$

with stiffness and damping coefficients, κ_h and d_h and resting length ϕ_h.

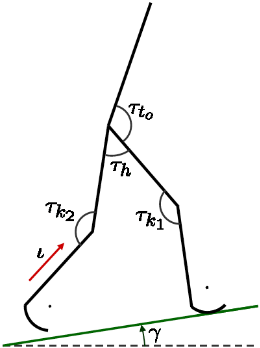


Figure: The kneed walker (Brubaker and Fleet 08)

- The equations of motion

$$\mathcal{M}\ddot{\mathbf{q}} = \mathbf{f}_s(\boldsymbol{\kappa}, \mathbf{d}, \phi) + \mathbf{f}_g + \mathbf{f}_c$$

  with $\mathcal{M}$ the generalized mass matrix, $\mathbf{f}_s$ the spring torques and $\mathbf{f}_g, \mathbf{f}_c$ the forces due to gravity and joints.
- Augment this with ground collisions and joint limits.

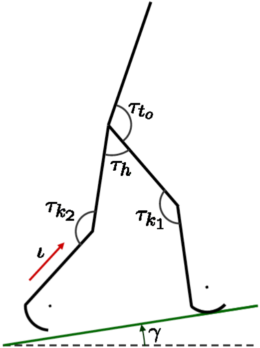


Figure: The kneed walker (Brubaker and Fleet 08)

Kneed Walker: Joint limits

- Joint limits easily expressed as constraints $\mathbf{a}^T\mathbf{q} \geq \mathbf{b}$
- When a joint limit violation is detected in simulation
  - localize constraint boundary (i.e., the time at which joint limit is reached) and treat as an impulsive collision
  - as long as constraint is active, include a virtual reactive force to ensure joint limits
  - augmented equations of motion

$$\begin{pmatrix} \mathcal{M} & -\mathbf{a} \\ \mathbf{a}^T & 0 \end{pmatrix} \begin{pmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\tau} \end{pmatrix} = \begin{pmatrix} \mathcal{F} \\ 0 \end{pmatrix}$$

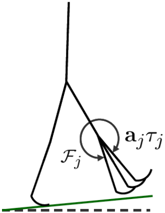


Figure: The kneed walker (Brubaker and Fleet 08)

Prior for the Kneed Walker

- How do we design a prior distribution over the dynamics parameters to encourage plausible human-like walking motions?
- Assumption: Human walking motions are characterized by efficient, stable, cyclic gaits
- Find control parameters that produce optimal cyclic gaits over a wide range of natural human speeds and step lengths, for a range of surface slopes.
- Assume additive process noise in the control parameters to capture variations in style.
- The minimization is with respect to the dynamics parameters $(\boldsymbol{\kappa}, \mathbf{d}, \phi, \iota)$ and the initial state $\mathbf{x} = (\mathbf{q}, \dot{\mathbf{q}})$.

Approach to tracking

- Generate stochastic dynamics
- From the stochastic dynamics generate samples for the 3D kinematics
- Evaluate image likelihood in a particle filter

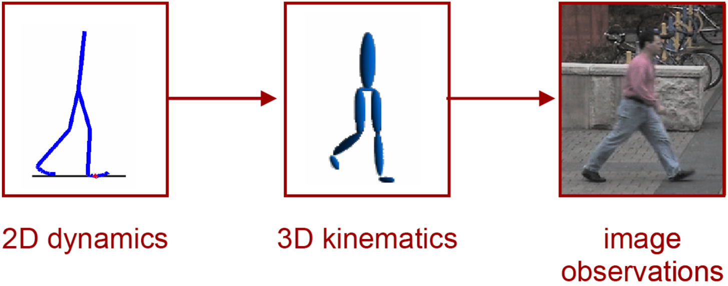


Figure: Tracking pipeline (Brubaker and Fleet 08)

Stochastic dynamics

The prior over human walking motions is derived from the manifold of optimal cyclic gaits

- assume additive noise on the control parameters (spring stiffness, resting lengths, and impulse magnitude).
- assume additive noise on the resulting torques.

3D kinematic model

- Kinematic parameters (15D)
- dynamics constrain contact of stance foot, two hip angles (in sagittal plane), and knee and ankle angles
- other parameters modeled as smooth, second-order Markov processes
- limb lengths assumed to be static

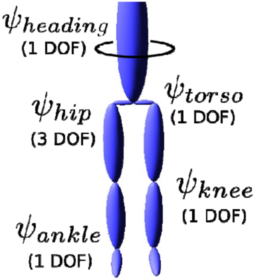
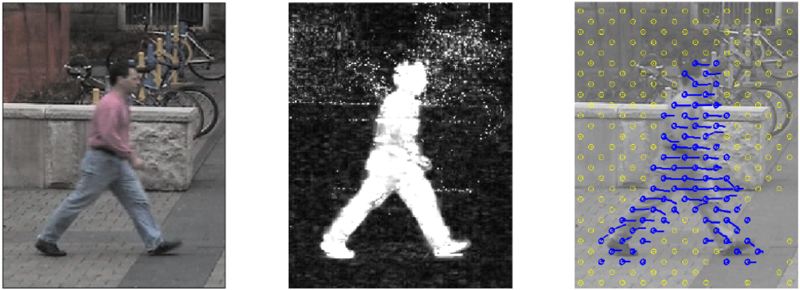


Figure: 3D kinematic model (Brubaker and Fleet 08)

Image likelihood



Foreground model   Background model   Optical flow

Figure: Image likelihood (Brubaker and Fleet 08)

Calibration and initialization

- Camera calibrated with respect to ground plane.
- Gravity assumed to be known relative to the ground
- Body position, pose and dynamics coarsely hand-initialized.

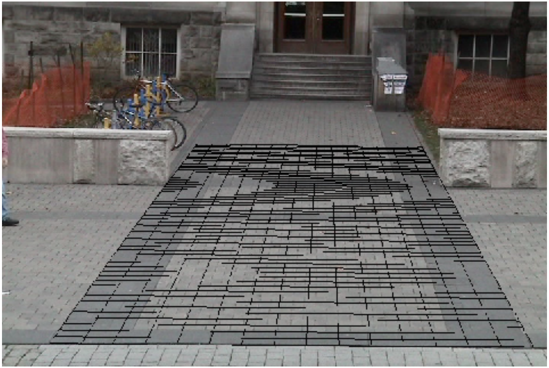


Figure: Calibration (Brubaker and Fleet 08)

Figure: (Brubaker and Fleet, CVPR 2008)

More?

- Including physics helps generalization beyond what mocap priors do
- Simple models since the equations of motion are very complicated!
- Difficult optimization problem: more dof than just kinematics.
- If you want to learn more, look at the additional material.
- Otherwise, do the research project on this topic!
- Next week we will look into discriminative prediction